Comparison of measurement accuracy characteristics and estimates via various mathematical models

V. A. Granovsky, T. N. Siraya

Central Scientific-Research Institute "ELEKTROPRIBOR",30, Malaya Posadskaya St., 197046, St. Petersburg, RUSSIA, e-mail: elprib@online.ru

Abstract-The problem of estimation of measurement accuracy characteristics is formulated as a significant methodological problem for metrology.
The main stages of the procedure of accuracy characteristics estimating are presented and discussed.
The vast variety of the possible estimation of measurement accuracy characteristics is described, taking into account structural aspects.
A practical approach for comparison of various accuracy estimates is proposed, which is based on the developed methodology of certification for data processing algorithms.

I. Introduction

Modern measurement problems need for considering a number of aspects of measurement results accuracy, which require for application a great variety of accuracy characteristics. In other words, there is an urgent necessity to expand the set of accuracy characteristics and their estimates in order to represent various properties of data and measurement results [1].
Really, the choice and estimation of accuracy characteristics is just a permanent problem of the metrology, and methodological foundations of the subject are always of great interest. These problems, both methodological and practical, also seem to be extremely vital nowadays [2].
Thus, the problem arises of the rational choice of the accuracy characteristics for the measurement problems, and there is also a task of joint employment of several ones. Furthermore, for the chosen characteristic there is a problem of rational estimation, taking into account the aims of measurement and resources.
These problems are extremely significant for the modern complex measurement systems, which include multi-step data processing algorithms. For instance, these are of great importance for wide range of GPS/GLONASS systems applications [3].
So investigation of the whole range (scope) of accuracy characteristics and estimates is the main aim of this paper. The main tasks under consideration are the following. The multidimensional variety of accuracy characteristics and estimates is to be described and structured. Besides, the tools for comparison of different accuracy characteristics and estimates within this variety are to be proposed.

II. General methodology

A. General measurement result models

As soon as comparing accuracy characteristics, one should take into account basic models of measurement procedures, which are usually obtained from the formal structure of measurement and thorough presentations of all influential factors for experimental data.
Measurement procedure is investigated as a complex system of various elements; for instance, it may schematically be presented as general schemes in [1, 4], with two associated and interactive rows of elements. The first (upper) row consists of the real objects, such as physical bodies and measuring instruments, and also of the bodies' properties, operations and processes. Mathematical models for the respective elements of the first row form the second row; these include measurand, models for measuring instruments and experimental data, data processing algorithms, measurement results and errors (uncertainties). Naturally, the latter kinds of models are the most significant for the investigations of measurement accuracy problems. The practical choice of the model completeness and the level of its specification are determined by some factors, such as requirements for the measurement accuracy, available experimental data, and also the limited resources.

Thus the measurand is usually defined as a location parameter, and likewise, accuracy characteristics are the corresponding scale parameters within the model assumed. Of course, the model should be valid for measurement problem under consideration, and mathematical tools for parameters estimation should be correct within these models.
A general scheme of model relations in the procedure of the measurement accuracy estimating is presented in Fig. 1.

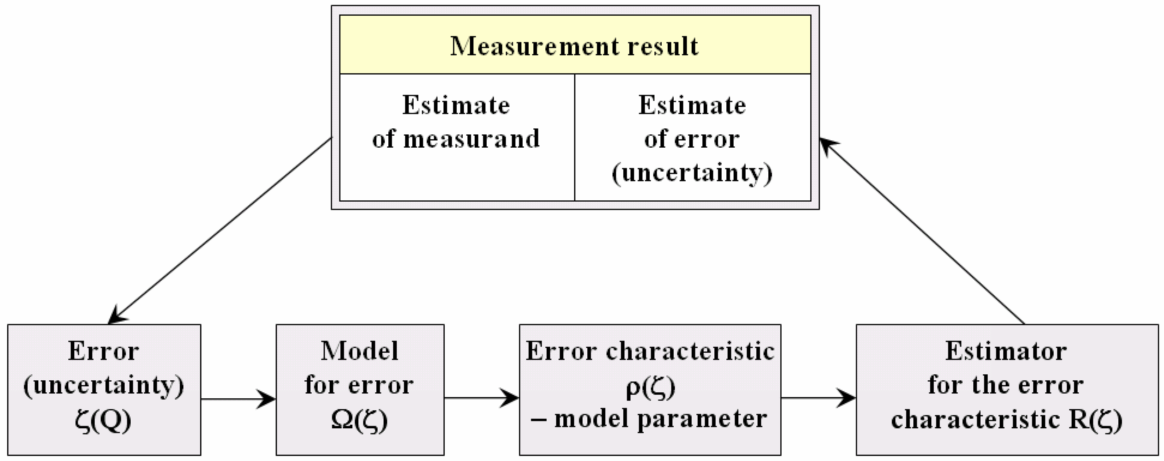

Figure 1. General scheme of measurement accuracy estimating

Taking into consideration both traditional and modern approach to data processing, there is a great variety of mathematical tools: along with classical statistics, these include modern methods of data analysis, robust and distribution-free statistics [5, 6], and also time series statistics. As a result, a great variety of accuracy characteristics and estimates are obtained; these are to be compared and moreover, to be integrated in rational ways.

B. Methodology of comparison of various accuracy estimates

Comparison of various accuracy estimates is based on the developed methodology of certification for data processing algorithms in measurements [7]. Originally, this approach was created for the comparison of the algorithms for measurement results computing (that is, measurand estimates); but it may be as well employed for the investigation of the accuracy characteristic estimates.
Therefore, for a certain class of accuracy estimates, which are intended for a given accuracy characteristic and a group of measurement problems, one has to choose:

a) $\{ \mu_1, \ldots, \mu_n \}$ - set of typical data models;
b) $\{ \varepsilon_1, \ldots, \varepsilon_m \}$ - set of efficiency characteristics for estimates.

The set of typical models $\{ \mu_1, \ldots, \mu_n \}$ is to be rather extensive in order to include all the general cases and special situations, which are practically important for a given accuracy characteristic, and also for a group of measurements. On the other hand, all the models μ_i are to be quite simple, so as not to cause difficulties while analyzing properties of accuracy estimates.
The set of efficiency characteristics $\{ \varepsilon_1, \ldots, \varepsilon_m \}$ is to be formed in order to represent all kinds of accuracy estimates properties. So these should firstly include traditional characteristics, which are just similar to the accuracy estimates for measurement results (under accepted data models). These should also contain some robustness characteristics, which would allow for the various deviations from the usual data models. For instance, while studying sample variance, one has to consider its standard deviation (SD) under Gaussian distribution of sample, and the additional SD, due to the data departures from the strict Gaussian model. The efficiency characteristics, which represent the range of application and computational complexity of the accuracy estimates, are also included into the set.
So, the efficiency matrix for an accuracy estimate λ is obtained as a result of estimation for all the efficiency characteristics ε_i , i=1…m, under data models μ_j , j=1…n :

$$E (\lambda , \mu) = \| \varepsilon_i (\lambda , \mu_j) \|, \quad i=1\ldots m, \quad j=1\ldots n. \qquad (1)$$

The efficiency matrix completely represents quality of the accuracy estimate λ, thus it is very useful for comparing this estimate with some similar ones. But it is usually unreal to choose "the best" estimate for a given characteristic of measurement accuracy with the use of efficiency matrices $E(\lambda, \mu)$ for various estimates. The matter is that in practice one usually cannot determine data model μ_j definitely, so to be quite sure about quantities ε_i (λ , μ_j). Thus, looking for "the best" estimate is rather misleading.

III. Principles for estimating of measurements accuracy characteristics

The procedure of measurement error (uncertainty) estimating, as presented in fig. 1, may be considered as a multi-step process [1], which is realised as follows:
1) One chooses the model of measurement error within a certain mathematical type, for instance, random value, random process, fuzzy set or function, interval function.
2) The accuracy characteristic is defined as a scale parameter within the assumed model of measurement error.
3) An estimator of the selected accuracy characteristic is chosen which can be calculated based on obtained experimental data.
4) The accuracy characteristic estimate is calculated according formulas assumed.

The major part of modern procedures of measurement error (uncertainty) estimating, which are graphically presented in fig. 1, may be formally presented as follows:

$$\Omega = \Omega\{ \zeta (t) = \xi (t) + \vartheta(t); \sigma(t), R(s,t), \Theta(t), \Delta(t); S(t), \Theta_p(t), \Delta_p(t) \},$$

where $\zeta (t) = \xi (t) + \vartheta(t)$ – decomposition of the total error into the components, such as
$\xi (t)$ – random error (with model as a random process);
$\vartheta(t)$ – systematic error (with model as an interval function).
Accuracy characteristics are presented as the scale parameters of error models:
$\sigma (t)$ – standard deviation as the scale parameter of the random value;
$R(s, t)$ – correlation function as the scale characteristic of the random process;
$\Theta(t)$ – limits of systematic errors;
$\Delta(t)$ – limits of total errors.
Further, the accuracy characteristics estimates are presented for the selected scale parameters of error models:
$S(t)$ – estimate of standard deviation (SD) of the random error;
$\Theta_p(t)$ – estimate of limits of systematic errors;
$\Delta_p (t)$ - estimate of limits of total errors.

In the classical case this general scheme is specified as follows:
- random error model is the random value with Gaussian distribution;
- the main accuracy characteristic is standard deviation (SD) of random value;
- the estimator of accuracy characteristic is the classical sample standard deviation.

This classical scheme is just the most popular in practice, and it is often taken for granted.
Nevertheless, various non-classical schemes for error estimating are possible, and moreover, the classical case is rarely occurring in pure type.
The extensions of the classical scheme may be carried out in several directions, when there are allowed:
a) various (non-Gaussian or even non-random) models of errors;
b) various scale parameters (accuracy characteristics);
c) various estimators for the selected parameters.
The possible directions for the such extensions are presented in fig. 2.

In general, above-listed groups of accuracy parameters and estimates allow to represent (depending on model assumed) all the aspects of accuracy, including all kinds of "precision" [8] and also "trueness" of data.
Naturally, various estimates are sensitive to diverse properties of data and data models. For instance, one can single out estimates, which are mainly representing data scattering (in purely stochastic sense), on the other hand, there are estimates connected with data trends and systematic effects. In addition, several estimates are primarily reflecting spectral properties of data (as time series).
As a result, several basic groups of accuracy characteristics are considered in this paper, including the following:
- classical scale parameters of random values, such as variance and standard deviation;
- non-classical parameters of scale, like mean absolute deviation, absolute moments of orders p, $1<p<2$, and median deviation [5,6];
- scale parameters of time series, such as Allan variance and several similar ones [9, 10].

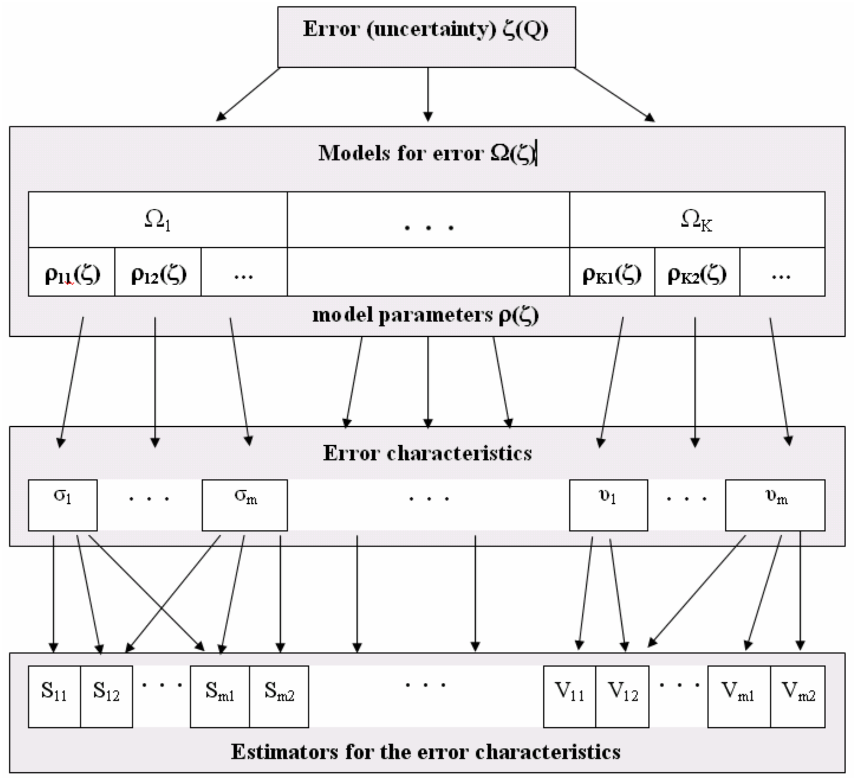

Figure 2. Variety of measurements accuracy characteristics and estimates

There are several groups of estimates which are possible for each characteristic. So, for the classical scale parameters one can use the following groups:
(C) classical estimates, such as sample variance or SD;
(N) non-parametric estimates (mainly based on order statistics) [5];
(R) robust estimates [6].
The classical estimates are optimal under the classical model, which is random sample with Gaussian distribution. But in practice some deviations from this model often happen, such as other distributions or mixed samples; in these cases non-parametric or robust estimates are preferable. On the other hand, for the cases of non-classical scale parameters just the proper non-parametric or robust estimates are optimal.
The scale parameters for time series are also diverse. Along with classical variance, these include Allan variance and some similar ones [9, 10]. It is significant, that Allan variance was originally introduced as empirical estimate [9], omitting definition of proper model parameter.

The multidimensional variety of accuracy characteristics and estimates, as described above, is to be structured, and the various estimators are to be compares with practical purposes. A way for comparison of different accuracy characteristics and estimates is proposed above, based on the methodology of data processing algorithms certification [7].
So the efficiency matrices $E(\lambda_i, \mu)$ represents the quality of the accuracy estimates λ_i, and these can be useful for comparing the estimates in practical cases. But one has to consider that the a priori information is not complete and exact, so "the best" choice is usually unreal.
On the other hand, it is promising to employ simultaneously several different accuracy estimates, being calculated on the same data; that would provide more substantial information on the measurement accuracy. It is reasonable to employ pairs or sets of estimates, which have distinct properties; so they are complementary in a certain sense. The efficiency matrices $E(\lambda, \mu)$ allow to select the rational sets of estimates, being relevant for a wide range of practical cases. The most useful are as following sets of estimates:
1) sample variance and median estimate of variance;
2) sample variance and Allan variance;
3) sample variance, median estimate of variance, and Allan variance.
Sometimes there are practical opportunities to employ the appropriate sets of estimates in different cases, and then it is possible to obtain more reliable accuracy estimates.

In some cases comparison of several accuracy estimates may furnish valuable information concerning data model. So it could be also used for justification and validation of the rational estimate of measurand. For instance, it is essential for the choice between classical estimate of measurand - arithmetic mean, and some non-parametric or robust estimates (such as median or M-estimates [6]).

IV. Conclusions

These results are of wide range of application, including rational choice of accuracy estimates for traditional measurement problems, and also development of complex multi-step data processing algorithms for GPS/GLONASS systems [3].

This paper is supported by RFFI grant 08-08-00678-a.

References

[1] Granovsky V. A., Siraya T. N. Methods of data processing in measurements. Leningrad, Energoatomizdat, 1990 (in Russian).

[2] Pavese F. To which degree the concepts for the expression of uncertainty are consistent in the present International written Standards and Guides for metrology and testing? - Abstracts of conference “Advanced mathematical and computational tools in metrology and testing”, 23-25 June 2008, ENS Cachan, France.

[3] Misra P., Enge P. Global positioning system. Signals, measurements, and performance. – Lincoln, Massachusetts, 2006

[4] Granovsky V. A., Siraya T. N. Model adequacy in measurements: estimation and improvement. - Abstracts of conference “Advanced mathematical and computational tools in metrology and testing”, 23-25 June 2008, ENS Cachan, France.

[5] David H. A. Order statistics – N. Y., Wiley, 1970.

[6] Robustness in statistics. – Ed. R. L. Launer, G. N. Wilkinson – Acad. Press, N. Y., 1979.

[7] Chelpanov I. B., Granovsky V. A., Siraya T. N. Certification of data processing algorithms in measurement: Bridge between algorithms and programs. – In: Advanced Mathematical and Computational Tools in Metrology, IV. – Edited by P. Ciarlini, A.B. Forbes, F. Pavese, D. Richter – 2000, World Scientific Publ.

[8] ISO 5725-1: 1994 Accuracy (trueness and precision) of measurement methods and results. - Part 1: General principles and definitions.

[9] Allan D. W., Ashby N., Hodge C. C. The Science of Timekeeping. – Application Note 1289, Hewlett-Packard Company, 1997.

[10] Granovsky V. A., Siraya T. N. Noise characteristics in electrical measurements: metrological approach. – Proc. 12th IMEKO TC4 International Symposium “Electrical Measurements and Instrumentation” - 2002, Zagreb, Croatia.