16th IMEKO TC4 Symposium
Exploring New Frontiers of Instrumentation and Methods for Electrical and Electronic Measurements
Sept. 22-24, 2008, Florence, Italy

Test frequencies selection criteria for parameter identification of anticorrosion coating using bilinear transformation

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Abstract- The paper presents comparison of new test frequencies selection criteria for the method of parameter identification of technical objects, modelled by two terminal networks. The method, based on reverse bilinear transformation of model impedance function, gives the possibility to reduce the identification time as compared to conventional impedance spectroscopy, by using limited number of test frequencies, equal to the number of parameters. As a test object, the model of anticorrosion coating has been chosen. The proposed criteria are based on high-signal impedance sensitivity, represented by length of bilinear transform identification curve on complex plane.

I. Introduction

The modelling of technical objects (anticorrosion coatings [1], materials [2], sensors [3], reinforced concrete constructions [4]) and biological objects (skin and physiological fluids [5], tissues [6]) with electrical circuits (multi-element two-terminal networks) has recently become very popular. The reason is that such modelling allows simulating of these objects, performance evaluation, monitoring and diagnosis of their state with well-developed tools and methods designed for electrical circuits.

In order to identify object impedance parameters, the impedance spectroscopy methods are used, based on impedance spectrum measurement in a wide frequency range. The measurement is usually carried point-by-point with a single frequency impedance analyser. The parameters are found by fitting the parameter dependant object model to the measured impedance spectrum [7], usually with Complex Non-Linear Least Square (CNLS) fitting algorithm [8].

Although very popular, the fitting method has some serious drawbacks. Firstly, without using the knowledge about an expected object topology and parameters, the required range of impedance spectrum begins with low or very low frequencies (in case of anticorrosion coatings order of mHz or μHz), and the usual number of impedance spectrum points is about 3-5/decade. As a result, the point-by-point impedance measurement method for such number and range of spectrum points is very time consuming. Secondly, the CNLS is an optimisation process, and for some input data, it can converge to local minimum or it can be inconvergent.

II. Objectives

There is a strong need to reduce the time of technical objects' impedance models identification measurement [9], as the long measurement time (order of hours) is inconvenient in field measurements, due to both technical and economical reasons. The authors have already proposed some approaches to achieve that goal: the acceleration of impedance spectrum measurement via multisine stimulation and various analysis methods [10-11] or modification of CNLS method by using limited number of selected test frequencies [12].

An alternative method of parameter identification is based on step-by-step calculating of model parameters using the reverse bilinear transformation [13]. The number of impedance spectrum measurement points is equal to the number of identified parameters. The method posses some interesting properties: not only it accelerates the measurement phase, but also, it simplifies the calculations, as the relations between measured impedance spectrum points and model parameters are analytical [14]. The method requires a precise selection of impedance measurement frequencies, based on equivalent circuit topology and expected values of parameters.

In the paper, we briefly present the bilinear identification method and compare some new frequencies selection criteria based on the length of bilinear transform identification curve on complex plane, representing the high-signal sensitivity of impedance function to model parameters.

The criteria are compared in terms of identification error of parameters of a test object – Beaunier's model of the anticorrosion coating in its early stage of degradation.

III. Parameter identification method based on bilinear transform

The parameter identification of objects with well-known equivalent circuit topology can be performed with the bilinear transformation, which is the basis of various diagnostic [14] and identification [13] methods.

The bilinear transformation allows to present the transfer function (in this case impedance of a linear multi-element two-terminal) for a given radial frequency ω_i, as a bilinear function of any of p_i parameter chosen from the vector of parameters $\boldsymbol{p}=[p_1, p_2, ..., p_k]^T$:

$$Z(j\omega_i, p_i) = \frac{A_i(j\omega_i)p_i + B_i(j\omega_i)}{C_i(j\omega_i)p_i + D_i(j\omega_i)}, \qquad i=1,2...k, \tag{1}$$

where: A_i, B_i, C_i, D_i are complex coefficients, calculated for a given parameter p_i, fulfilling the condition $A_iD_i - B_iC_i \neq 0$. These coefficients are dependant only on other parameters p_j ($j \neq i$).

The reverse bilinear transform allows to calculate the unknown parameter p_i from the A_i, B_i, C_i, D_i coefficients and object impedance $Z(j\omega_i)$, measured at frequency ω_i, according to formula:

$$p_i = \frac{D_i(j\omega_i) \cdot Z(j\omega_i) - B_i(j\omega_i)}{A_i(j\omega_i) - C_i(j\omega_i) \cdot Z(j\omega_i)}, \qquad i=1,2...k. \tag{2}$$

The image of bilinear transformation on a complex plane *Re Z*, *Im Z* is the fragment of a circle – so called the identification curve. A bunch of such curves, plotted for every object parameter changing in a given range, makes a family of identification curves, crossing each other at the point of nominal impedance.

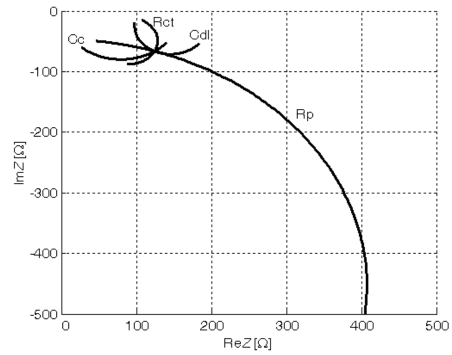

Figure 1. Exemplary bilinear transformation identification curves.

The shape, the length and the position of curves are dependant of the radial frequency ω_i. The frequency can be chosen in such a way, that one curve dominates over others, thus the high-signal sensitivity of impedance to that parameter is bigger than to other parameters. That situation, for model of anticorrosion coating is presented in the Figure 1. The R_p curve dominates; the other curves are rather short, representing small sensitivity of impedance Z to other parameters at that frequency.

The change of a single identification curve shape for different frequencies is shown in the 3D plot in the Figure 2. It can be seen, that the maximum length of C_c identification curve is at frequency about 100 mHz .

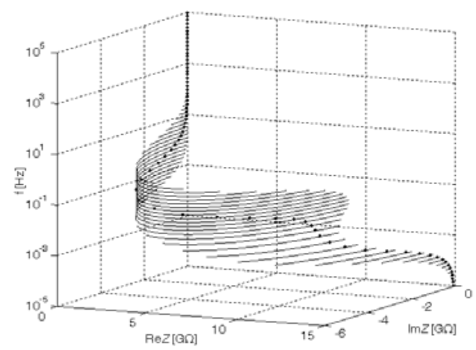

Figure 2. The variation of identification curve length as a function of a frequency
for the C_c element of test object.

The reverse bilinear transform based identification method [13] consists of 3 stages: pre-test stage, measurement stage and identification stage. In the pre-test stage, the vector of optimal test frequencies $\boldsymbol{f}=[f_1, f_2, ..., f_k]^T$ is being selected, for a given topology of equivalent circuit and expected parameter values $\boldsymbol{p}_s=[p_{s1}, p_{s2}, ...,$

$p_{sk}]^T$. The values can be default ones or can be taken from previous measurements of the object. In the measurement stage, the impedance of the object at k test frequencies is being measured. In the third stage, by using the reverse bilinear transformation (2), the k object parameters are calculated in k steps, from the k impedance values and matrix of coefficients A_i, B_i, C_i, D_i. The number of steps k is equal to number of impedance measurements and number of identified parameters.

In the first step of identification stage, the parameter p_1 is calculated using the coefficients A_1, B_1, C_1, D_2, which are computed from the expected values p_{s2} ... p_{sk}. In the next step, the p_2 is calculated using the coefficients A_2, B_2, C_2, D_2 computed from the expected values p_{s3} ... p_{sk} and value p_1 calculated in previous step. In the last step, the A_k, B_k, C_k, D_k coefficients are computed from parameters p_1 ... p_{k-1}, already identified in previous steps.

It is very important to choose impedance measurement frequency f_i for every k parameter, that the impedance function at that frequency is very sensitive to changes of that parameter and at the same time rather non-sensitive to changes of other parameters, especially the ones that are not yet identified and are chosen from expected values (in this case $p_{i+1}\ldots p_k$). That corresponds to the situation presented in the Figure 1 with dominating identification curve R_p. So, it is necessary to develop and test the selection criteria, which allows choosing the test frequencies automatically.

IV. Methodology

The developed frequency selection criteria have been tested by means of simulation on the example of the Beaunier's equivalent circuit of anticorrosion anticorrosion coating on a metal surface, presented in the Figure 3.

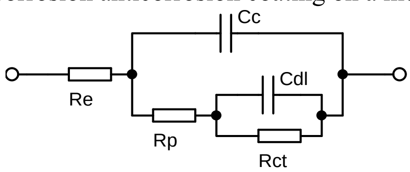

Figure 3. Test object - Beaunier's equivalent circuit of anticorrosion coating.

As the R_e element value is dependant on measurement cell properties and is several rank smaller then R_p and R_{ct}, the identification deals with a simplified 4-element equivalent circuit, which impedance is:

$$Z(j\omega) = \frac{R_p + R_{ct} + j\omega \cdot C_{dl} \cdot R_p \cdot R_{ct}}{1 - \omega^2 \cdot C_{dl} \cdot C_C \cdot R_p \cdot R_{ct} + j\omega \cdot C_{dl} \cdot R_{ct} + j\omega \cdot C_C \cdot R_{ct} + j\omega \cdot C_C \cdot R_p}, \quad (3)$$

dependant on vector of parameters $\boldsymbol{p}=[C_c\ R_p\ C_{dl}\ R_{ct}]^{\mathbf{T}}$. The values of parameters are presented in Table 1.

Table 1. Parameters of 4-element simplified Beaunier's model of anticorrosion coating.

Object	R_p	R_{ct}	C_c	C_{dl}
Stage A	100GΩ	100GΩ	10pF	100pF
Stage B	10GΩ	10GΩ	100pF	1nF

In order to compare frequency selection criteria, the sensitivity of the bilinear identification method to impedance measurement errors for different test frequencies vectors $\boldsymbol{f}$ was tested. The vectors $\boldsymbol{f}$ were calculated with several criteria from the parameters of the brand new coating (stage A). The identification process was simulated for the coating, which is being slightly penetrated by the water (stage B).

The nominal impedance measurement values have been calculated from the impedance function (3). Then, the measurement error has been taken into consideration, by adding a random multiplicative error of real and imaginary part of impedance, drawn from rectangular distributions. Such an "uncertaintization" of a simulated result has been repeated 100 times, thus producing a set of 100 simulated impedance measurement data. For the set of data, bilinear identification has been done, producing a series of identified parameters, followed by calculation of mean and standard deviation of relative identification error. The calculations were done in Matlab environment.

V. Description of Test Frequencies Selection Criteria

The criteria based on the high-signal impedance sensitivity to equivalent circuit parameters, represented by lengths of bilinear identification curves have been formulated and tested. The L length of the identification curve on a complex Z plane for a given frequency f_i and parameter changing from p_{min} to p_{max} was approximated by the sum of secants from nominal impedance to impedances computed for parameters p_{min} and p_{max}.

$$L = \left|Z(f, p_{\max}) - Z(f, p)\right| + \left|Z(f, p_{\min}) - Z(f, p)\right| \tag{4}$$

As the parameters change about a rank between degradation stages, it was assumed that:

$$p_{\max} = k \cdot p\,, \quad p_{\min} = \tfrac{1}{k} \cdot p\,, \quad k = 10\,. \tag{5}$$

All the considered criteria define the objective function $G_i(f)$ based on lengths L. The criteria proposed, allow to find the vector of test frequencies $\boldsymbol{f}=[f_1, f_2, f_3, f_4]^T$ of length equal to number of identified parameters, by assuming that each frequency is optimal for one parameter and we identify them from the one being near the terminals to the one being buried in the circuit's topology. So, the vector $\boldsymbol{f}$ is exactly a $\boldsymbol{f}=[f_{Cc}, f_{Rp}, f_{Cdl}, f_{Rct}]^T$. Each frequency in a vector can be determinated by finding a maximum of objective function $G_i(f)$, defined on a limited frequency range from f_{min} to f_{max}.

In the early investigations, the 3 criteria were formulated. The first one was the length of the identification curve itself: $G_i(f)$ equal to $L_i(f)$, as the $L_i(f)$ is proportional to the absolute impedance sensitivity $|\Delta Z|$. The second criterion was the identification curve length normalized by the modulus of impedance:

$$G_i(f) = \frac{L_i(f)}{\left|Z_{nom}\right|}\,, \tag{6}$$

as it is proportional to relative high signal impedance sensitivity:

$$S_{p_i} = \frac{\Delta Z / Z}{\Delta p_i / p_i}\,. \tag{7}$$

First 2 criteria find the maximum sensitivity to a given parameter, ignoring the influence of other parameters.

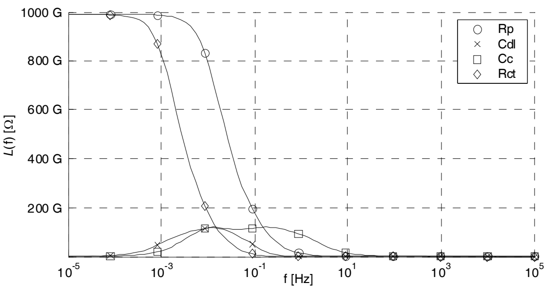

Figure 4. Plot of identification curve lengths against frequency for stage A model.

The third criterion was formulated to consider not only the maximum sensitivity for a given parameter, but also to minimize the influence of other model parameters [12]. Objective function was constructed by dividing the length for curve of parameter i by a sum of lengths of curves for parameters other than i:

$$G_i(f) = \frac{L_i(f)}{L_1(f) + \ldots + L_{i-1}(f) + L_{i+1}(f) + \ldots + L_k(f)}\,. \tag{8}$$

However, the simulations have shown, that these 3 "common-sense" criteria were absolutely inadequate for the bilinear method – the identification error exceeded 100% for some elements in every case. The reason is the form of bilinear transform coefficients matrix. For example, for the R_p element it is:

$$\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \begin{bmatrix} 1 + j\omega \cdot C_{dl} \cdot R_{ct} & R_{ct} \\ j\omega \cdot C_C - \omega^2 \cdot C_C \cdot C_{dl} \cdot R_{ct} & 1 + j\omega \cdot R_{ct} \cdot (C_{dl} + C_C) \end{bmatrix} \tag{9}$$

and its elements do not depend on R_p, but only on other elements, which values are already identified (C_c) or taken from the vector of expected parameters (R_{ct}, C_{dl}). As the expected values can vary from the nominal ones, this can implicate high identification error.

The next criterion, noted $\mathbf{C_4}$, allows computing optimal frequencies considering the sequence of parameter identification. In every step, we minimise the influence of the parameters not yet identified and taken from the vector of expected values. Formally, the optimal frequency for parameter i of a k-element circuit ($i \neq k$) is set by finding the **minimum** of the objective function:

$$G_i(f) = L_{i+1}(f) + \ldots + L_k(f)\,. \tag{10}$$

If the objective function has a minimum:

$$f_i = f_x \Leftrightarrow G_i(f_x) = \min_{f_{\min} < f < f_{\max}} \left[G_i(f)\right]. \tag{11}$$

In case of asymptotic function $G_i(f)$:

$$f_i = f_x \Leftrightarrow G_i(f_x) = (1+\varepsilon) \min_{f_{\min} < f < f_{\max}} \left[G_i(f)\right], \quad \varepsilon = 1\%\,. \tag{12}$$

As it can be seen, the sensitivity for parameters already identified $L_1(f)...L_{i-1}(f)$ is being ignored. The optimal frequency for the element being identified as the last one (without using expected values) is selected similar as in criterion **C₁** – maximum length of that element's identification curve, which is the same for elements k-1 and k.

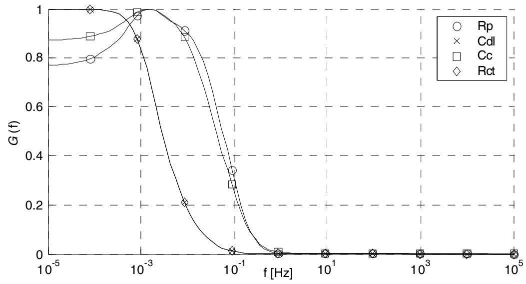

Figure 5. Plot of criterion $\mathbf{C_5}$ functions $G_i(f)$ against frequency.

A second version of the criteria, noted $\mathbf{C_5}$, has also been tested. In order to minimise the influence of expected parameters with the same weights, the curve lengths are normalized to values [0,1] before the summation (Figure 5):

$$G_i(f) = \|L_{i+1}(f)\| + \ldots + \|L_k(f)\|. \tag{13}$$

VI. Results and Conclusions

Test frequencies selected by the criteria described above for the test object in a 10-decade frequency range are presented in Table 2. As a reference, the vector of frequencies set by a sophisticated algorithm in [13] is presented ($\mathbf{C_R}$).

Table 2. Vectors of test frequencies for criteria $\mathbf{C_4}$, $\mathbf{C_4}$ and $\mathbf{C_R}$.

Criterion	Frequency optimal for:			
	R_{ct}	C_{dl}	R_p	C_c
$\mathbf{C_4}$	217.3 μHz	96.2 mHz	274.0 mHz	763.7 mHz
$\mathbf{C_5}$	217.3 μHz	94.9 mHz	596.4 mHz	813.4 mHz
$\mathbf{C_R}$	2.0 mHz	1.0 mHz	0.5 Hz	10.0 kHz

For the simulated series of 100 identification measurements, the mean relative identification errors of equivalent circuit parameters, for the test object in stage B are presented in Table 3.

Table 3. Mean relative identification error of equivalent circuit parameter identification

Criterion	Relative identification error [%]			
	R_{ct}	C_{dl}	R_p	C_c
$\mathbf{C_4}$	3.07%	-8.60%	-2.99%	-0.42%
$\mathbf{C_5}$	0.95%	-1.73%	-0.86%	-0.37%
$\mathbf{C_R}$	1.17%	-2.62%	-1.10%	-0.03%

The standard deviation of relative identification error was approximately 1% for parameters R_p and C_c, 2% for R_{ct} and 4% for C_{dl}. The spread of results is dependant on position of the element in circuit topology. The more the element is buried in the topology, the bigger the standard deviation of identification error is, which means that buried elements are harder to identify.

The frequencies selected by tested criteria are correlated with "common sense" approach to identification of Beaunier's model parameters. It can be seen, that the higher frequency is, the better the capacitor C_c can be identified, as it is shunting rest of the circuit and dominates the impedance. The R_{ct} identification frequency should be as low as possible, in order to minimise current flowing through C_c or C_{dl}, and thus R_p. The C_{dl} capacitor (with higher capacitance then C_c) should be identified at frequency low enough to limit influence of C_c, but higher than the frequency optimal for R_{ct}. The R_p identification frequency is localised at the point where that element influences on both real and imaginary part of impedance, due to C_{dl} and R_{ct} current flowing through it.

The results (both presented in Table 2 and the preliminary ones) have shown, that the bilinear identification method is highly sensitive to vector of test frequencies. They have to be chosen concerning the sequence of parameter identification and the properties of bilinear transform coefficients matrix. Ignoring it, as in the case of the preliminary criteria, leads to improper set of frequencies for which the identification method fails.

The criteria $\mathbf{C_4}$ and $\mathbf{C_5}$ allow identifying the object parameters correctly using bilinear identification method. The best results were achieved for criterion $\mathbf{C_5}$. It is worth to notice, that the presented procedure of finding vector $\boldsymbol{f}$ can be easily automated.

The method has been tested with parameters changing by a rank. Further plans include testing the method in a wider range of parameter change and application of the method to select the frequencies of a multisine stimulus in order to accelerate the impedance values measurement time.

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