A Helmholtz coil for high frequency high field intensity applications

S. Ciurlo, A. Mariscotti, A. Viacava

Dipartimento Ingegneria Elettrica (Università di Genova, Via all'Opera Pia 11A, 16145 Genova, +390103532169, +390103532700, andrea.mariscotti@ unige.it)

Abstract – In this work a general introduction to Helmholtz coil is presented and then attention is on a practical implementation of a wideband (50 kHz) coil for high magnetic flux density applications (several mT may be obtained). Field uniformity is within ±1.5% and ±0.35% inside cubic volumes of 25% and 10% of the coil side respectively, so suitable also for high accuracy applications such as sensor calibration.

I. Introduction

Generating fields of known and uniform strength is important in laboratory work. Such fields are necessary for accurate and repeatable immunity testing and also for equipment calibration.

Perhaps the simplest type of field generator seen is a planar coil. The magnetic field in the center of a circular coil is given by the formula $H = NI/2r$ where N is the number of turns, r the radius and I the current passed through the coil. The simple coil has its uses and its drawbacks. Simple coils are used in some EMC tests [1]-[3] to generate magnetic fields, either as small "field coils" placed next to the equipment under test (EUT), or in a larger "immersion" configuration. The benefits are simplicity of construction, small size and ease of installation; the drawback is non-uniformity of field: the field increases significantly at points near the wires and outside of the coil, the magnitude and direction of the field are sensitive to positioning and orientation.

There are several ways of producing a uniform magnetic field [4]: 1) a solenoid, 2) a toroid, 3) a spherical coil with variable winding, 4) a Helmholtz coil arrangement of stacked simple coils. The solenoid is a coil wound in a cylindrical fashion; for a long and narrow solenoid the field in the center region is H=NI/L, where L is the length. A toroid is a solenoid wound around in a donut so it closes on itself; the magnetic field is more uniform and it is still given by $H=NI/L$, where here L is the circumference of the toroid at the mean radius of the coil. There are two mechanical difficulties: size, because the toroid must be much larger than equipment inserted into its interior, and accessibility, because the toroid is a closed surface that must be opened for insertion. The spherical coils is a sphere wound so that the sheet current density is proportional to the sine of the angle relative to the coil axis, a uniform field results throughout the entire volume of the sphere. A sinusoidally distributed sheet current can be approximated either by multiple windings driven through resistors of different values, or by varying the winding density over the surface of the sphere.

The standard geometry for a Helmholtz pair is two parallel coils spaced one radius apart and driven in phase [6]. Variation of the spacing over a fairly wide range has some effect on the field amplitude and uniformity; the Helmholtz coil can be constructed with either circular or square coils and this choice has a slight influence on field uniformity. The field it generates is the sum of the fields generated by the two spaced coils; the surprisingly large volume of field uniformity results because there is a good deal of cancellation for the off axis field components generated by the coil.

The advantage of Helmholtz coils is that they have a simple geometry (with respect to configuration 3) and they allow large equipment to be fit within the two coils, that is impossible for all other solutions.

The rectangular geometry is convenient, especially for construction and installation. Single square coils have been used for calibration of extremely low frequency magnetic field meters for applications that require uncertainties of a few percent [5][7]. Multiple rectangular loops with a common axis have found applications in a number of fields, including biological exposure systems for in vivo and in vitro studies [4][5]. It is also noteworthy that a square Helmholtz coil produces a greater volume of nearly uniform magnetic field than a circular Helmholtz coil of comparable dimensions.

Finally, it is to be underlined that immunity standards [2][3] require single (or few) turn coils not only for high frequency but also for supply frequency testing. If this requirement is very important to limit stray capacitance and resonance effect at high frequency, it really asks for a high current capability amplifier. The presented design and implementation is a viable solution to keep the required current amplitude low while ensuring a large bandwidth (above 50 kHz).

II. Field equations

Only some expressions are reported to ease the analysis of the Helmholtz coil and the optimization of geometric parameters. For a single square coil of side $2a$ and $2b$, N turns and input current I, one can obtain the field B_z on coils axis by Biot-Savart law application and integration [8]

$$
\begin{aligned}
B_z &= NI\frac{\mu_0}{4\pi}\sum_{n=1}^{4}\frac{(-1)^n D_n}{r_n[r_n+(-1)^{n+1}C_n]}-\frac{C_n}{r_n(r_n+D_n)} \\
\text{where } & C_1=-C_4=a+x \quad C_2=-C_3=a-x \\
& D_1=D_2=b+y \quad D_3=D_4=-b+y \\
& r_1=\sqrt{(a+x)^2+(b+y)^2+z^2} \quad r_2=\sqrt{(a-x)^2+(b+y)^2+z^2} \\
& r_3=\sqrt{(a-x)^2+(b-y)^2+z^2} \quad r_4=\sqrt{(a+x)^2+(b-y)^2+z^2}
\end{aligned} \tag{1}
$$

With attention to the coordinate system with its origin on the center of the first coil and orientation of z along coil axis (see Fig. 1 below), one can add the second coil with the coordinated translation by d (coil spacing) and bring the second derivative to x of the total field (with $a=b$ in our case) to zero, to obtain the solution for maximum flatness $d \cong 1.07a$. Numerical simulations then indicate that the optimal spacing for minimum field variation along the x axis is slightly larger, around $d \cong 1.2a$.
Bronaugh [6] reports a useful formula (for circular coils) to locate the major sources of field error.

$$\frac{\delta B}{B} = -0.2\left(\delta r_1/r_1+\delta r_2/r_2\right)-0.6\,\delta s/s+\delta I/I+\delta N/N \tag{2}$$

The most important terms are $\delta I/I$ and $\delta N/N$, that are the relative errors on the input current and number of turns: the first may be made very small in the order of the accuracy of the shunt used for the measurement (about 0.1% or better over the entire frequency range, since stray capacitance and inductance terms are not relevant for a resistance value in the order of a fraction of Ω); the second term may be excluded since the number of turns is exact (128 in our application). The coil frame is made of plywood with reinforcement wood patches and bars; the machining tolerance is below 1 mm for edges, profiles and holes and we may estimate an equivalent size relative error of 1 mm / 520 mm (half the average side length of each coil). Spacing is controlled twice before each test on all sides of the parallelepiped and in this case the error is related to the reading error on a mm scale, roughly about 0.5 mm / 500-800 mm. The calculated theoretical worst case error is less than 1% and the expanded k=2 uncertainty is approximately 0.2%.

III. Coil design

Helmholtz coils are square with 1 m nominal side length. To account for the effective winding thickness, eight layers of 3 mm thickness, the internal side length was set to about 0.98 m, so that the average side length is approximately 1 m. Each coil is composed of N_t = 16 turns and N_l = 8 layers.
Winding geometry was designed to minimize stray capacitance. The analysis of inter-turn stray capacitance C_t and inter-layer stray capacitance C_l terms suggest that the most important term of the total coil capacitance C_c is C_l: because of Miller effect C_l for each turn pair is increased approximately by the ratio of the effective voltage difference between two turns on adjacent layers V_{ll} and the voltage difference between two adjacent turns in the same layer V_t.

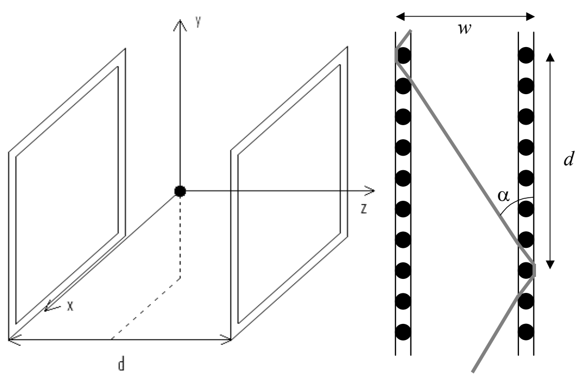


Figure 1. Helmholtz coil geometry with coordinate system and pattern of the single layer

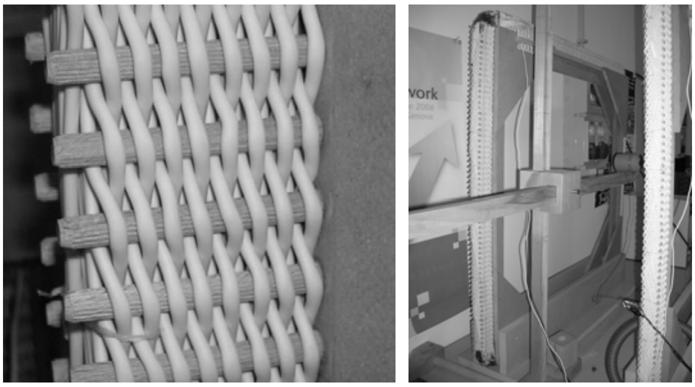
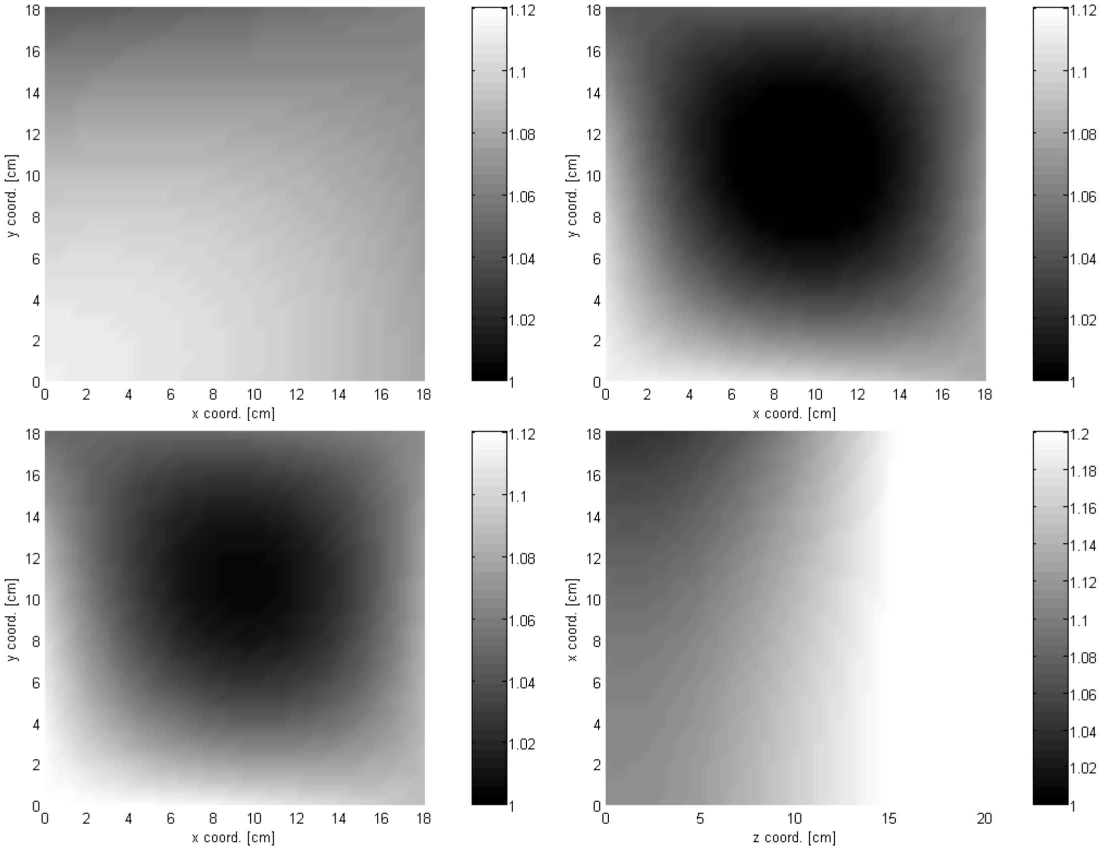


Figure 2. Construction of the Zig zag winding and final implementation with the positioning system

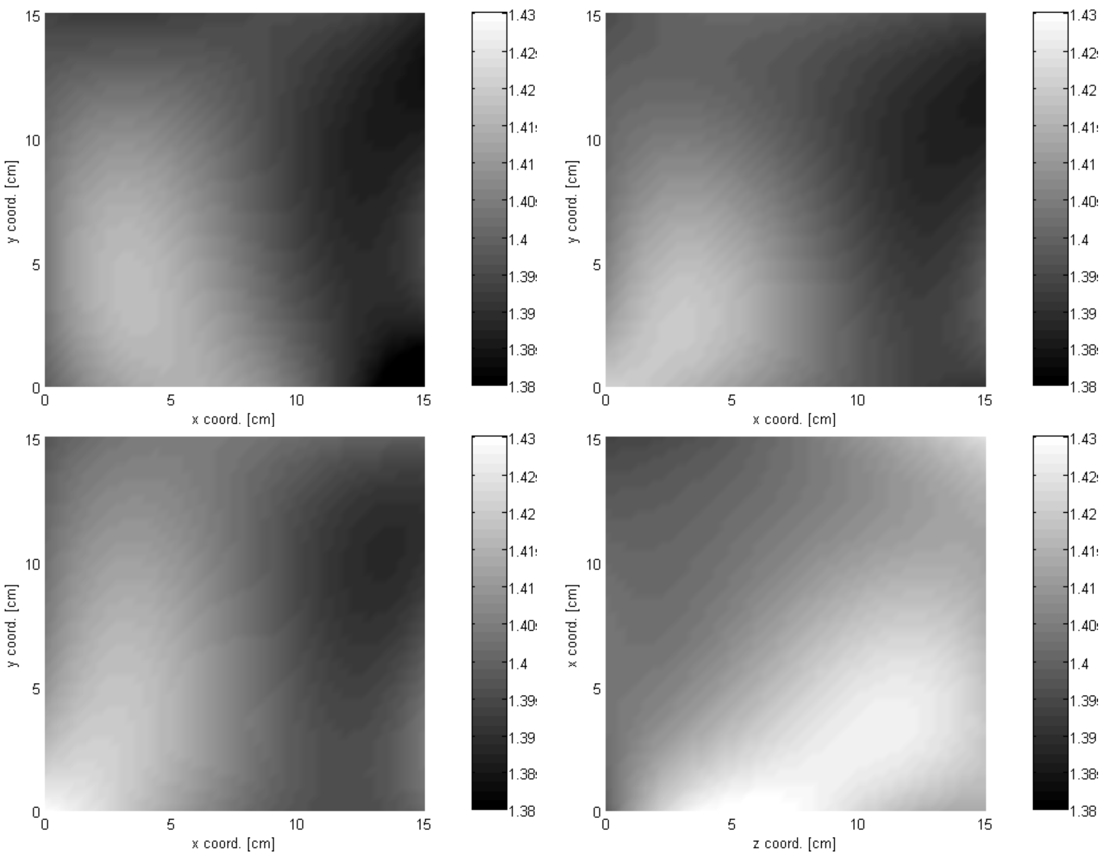
The winding structure is as shown in Fig. 1: the turn is wounded with a zig-zag path; the skew angle α may be computed from the pitch d and the winding width w. The total turn length L_t is $d/\cos\alpha$, and for usual values of α (about 20-25°) this represents an increase with respect to the straight length d of the frame of about 6-10%. In our case 128 turns with an average perimeter of 4.16 m took about 580 m of wire for each coil. The turn resistance R_t (and so the total coil resistance R_c) is a linear function of turn length and cross section ratio; it was found for both coils R_c=8.30 ± 0.11 Ω.
Coil inductance and capacitance measured at input terminals are L_1=43.8±0.8 mH and L_2=43.3±0.8 mH, C_1=170 ± 10 pF and C_2=176 ± 10 pF respectively for the two coils. The stray capacitance was determined based on the measurement of the coil self-resonance, found at 61.8±0.1 kHz for both coils.
Coil inductance is not an issue for large field tests at a single frequency at each time, since inductive reactance may be almost canceled out by the series capacitive reactance of the tuning capacitor bank, reaching magnetic field levels well above 1 mT at audiofrequency. Transient field tests or fast sweeping field tests may be accomplished by means of a current source amplifier or a high voltage source followed by a large series resistor (much larger than the coils inductive reactance), because of field uniformity with respect to frequency (see next Section).

IV. Experimental characterization

In addition to the determination of the basic electrical parameters of the two coils, a series of measurements have been done in order to: first, define the coil input impedance Z_c; second, check uniformity of bulk current I_b (the sum of the currents in the individual wires, as defined in [9]) with respect to input current I_i; third, evaluate magnetic field level and uniformity, with the definition of regions of space of given field uniformity, where experimental values are compared with those derived from magnetic field equations (referred to as "theoretical values").
The coil impedance Z_c has been measured with a voltamperometric method and the results are those reported at the end of Section III to illustrate coil design.
Uniformity of bulk current was qualitatively checked by moving a Rogowski coil along coil perimeter and no appreciable difference was detected (within approximately 1%). A Rogowski coil with improved common mode and external electric field rejection [10] has been used also and it confirmed the first measurement results up to the resonance frequency.
Magnetic field uniformity was tested for different coils separations d, operating frequencies (attention is focused on 55 kHz, a very high value close to coil resonance, and a smaller value, 10 kHz, representing low frequency behavior) and positions with respect to coil center.
Other tests at lower frequency values have shown that, excluding the small uncertainty related to the search of the resonance frequency for each capacitor bank configuration, the flatness of the frequency response is within about 0.3%.
Some sample test results are shown in Fig. 3 to illustrate the activity. Two coil spacing values (found in the literature as suggested optimal values) have been investigated: 80 cm (indicated by the CENELEC standards [2][3]) and 60 cm (found as indicated in Section II); the field values are normalized per 1 A of bulk current.



(a)



(b)

Figure 3. Measured values of B_z [μT] / I_b [A] for coil spacing of (a) 80 cm, (b) 60 cm: z=0 cm (upper left), z=2 cm (upper right), z=4 cm (lower left) and y=0 cm (lower right)

For d=80 cm spacing (Fig. 3a) it can be observed that in the central portion of space the B field is very uniform and attention is concentrated on a cubic space of 11 cm side (5.5 cm in the half plane plots), where any magnetic field sensor may be placed for calibration. The average field H_{av}=1.0942 μT/A and the dispersion is 2%. If the considered portion of space is now a sphere (that fits much better the shape of the majority of magnetic field sensors [11][12]) with diameter of 11 cm, the average field increase slightly to 1.1032 μT/A, but the dispersion reduces to 0.5%.
Field uniformity is even better for d=60 cm (Fig. 3b) over the considered cubic space (11 cm side): H_{av}=1.4178 μT/A and the dispersion is 0.35%; for the sphere with diameter of 11 cm, H_{av}=1.4162 μT/A and the dispersion is still 0.35%. This clearly ensures a better uniformity with respect to larger spacing, and a 35% higher field (if the average field values are considered). It is worth to underline that the maximum spread of values, i.e. H_{max}–H_{min} vs. the average field is only 2.33%.

The Type B uncertainty from declared uncertainties of used equipment and assumed errors in positioning and geometry of the test setup is around 2%, but a direct evaluation of Type A uncertainty with k=2 from repeated measurements (9 for each point in space) indicates a value always better than 0.5%. This leaves only positioning and geometry errors of less than 1% (computed at the end of Section II); to give a better estimate reproducibility needs to be tested with several complete tests taken with different environmental conditions (especially temperature and humidity, taking into account that the coils supports and the positioning system is made of wood).

Conclusions

The main elements for the construction of a wideband Helmholtz coil pair and related test results have been presented and discussed. The target applications are the magnetic field immunity testing of equipment of considerable size, the application of fairly constant and uniform magnetic field to laboratory specimen (such as during biological experiments) and finally the calibration of magnetic field sensors with a satisfactory field uniformity and repeatability within a fraction of % for a wide range of sensor size. The Helmholtz coil is attractive with respect to solenoids and similar architectures because it is an open architecture that doesn't impose constraints on the geometry of the device under test, so that one equipment fits several applications, with a very satisfactory accuracy.

References

[1] MIL STD 461E, *Requirements for the control of Electromagnetic Interference Characteristics of Subsystems and Equipment*, 1999-08.

[2] EN 61000-4-8, *Electromagnetic compatibility (EMC) – Part 4: Testing and measurement techniques. Section 8: Power frequency magnetic field immunity test – Basic EMC publication*, 1993-09

[3] EN 61000-4-9, *Electromagnetic compatibility (EMC) – Part 4: Testing and measurement techniques. Section 9: Pulse magnetic field immunity test – Basic EMC publication*, 1993-09

[4] I. Straus, "Magnetic field calibration: unwinding the Helmholtz coil", *Conformity*, May 2002, pp. 40-44 (www.conformity.com)

[5] M. Misakian, "Equations for the Magnetic Field Produced by One or More Rectangular Loops of Wire in the Same Plane", J. Res. Natl. Inst. Stand. Technol., vol. 105 n. 4, July/Aug. 2000, pp. 557-564.

[6] E.L. Bronaugh, "Helmholtz coils for calibration of probes and sensors: limits of magnetic field accuracy and uniformity", IEEE EMC Symposium, 1995, pp. 72-76.

[7] IEEE Std. 291, *IEEE standard methods for measuring electromagnetic field strength of sinusoidal continuous waves, 30 Hz to 30 GHz*, 1991.

[8] IEEE Std. 644, *IEEE standard procedures for measurement of power frequency electric and magnetic fields from AC power lines*, 1994.

[9] C. Carobbi, S. Lazzerini and L. Millanta, "High frequency operation of the coils for standard magnetic field generation using the bulk-current technique", *IEEE Transactions on Instrumentation and Measurements*, vol. 54 n. 4, Aug. 2005, pp. 1427-1432.

[10] A. Mariscotti, "A Rogowski coil for high voltage applications", International Instrumentation and Measurement Technology Conf. I^2MTC, Victoria, Vancouver Island, Canada 12-15 May 2008.

[11] IEEE Std. 1140, *IEEE Standard Procedures for the Measurement of Electric and Magnetic Fields from Video Display Terminals (VDTs) from 5 Hz to 400 kHz*, 1994

[12] A. Mariscotti, "A Large Bandwidth and Dynamic Range Magnetic Field Probe", IMTC 2007, Warsaw, Poland, May 2-4, 2007.