16th IMEKO TC4 Symposium
Exploring New Frontiers of Instrumentation and Methods for Electrical and Electronic Measurements
Sept. 22-24, 2008, Florence, Italy

New Approach to the Accuracy Description of Unbalanced Bridge Circuits

Zygmunt Lech Warsza[1] Jarosław Makal [2]

[1] *Polish Metrological Society Warszawa, phone/fax: +4822765510; email: zlw@op.pl*
[2] *Bialystok Technical University, Faculty of Electrical Engineering, Poland: email: jaromaka@pb.edu.pl*

Abstract. Accuracy measures of open circuit transfer functions of the arbitrary variable arm resistances of the bridge supplied by current or voltage, are presented. It includes the singular form of these measures introduced before in [1], [6] and also the new double form approach. Equations for measures of unbalanced bridges of equal arm resistances in balance and with sensors of variable resistances in four arms, in two arms or in one arm are given in Table 3. Some conclusions are formulated.

Introduction

The generalized accuracy description of the 4R bridge of arbitrary variable arm resistances was not existing in the literature, but is urgently needed mainly for:
- initial conditioning circuits of analogue signals from the broadly variable immittance sensor sets,
- identification of the changes of several internal parameters of the equivalent circuit of the object working as twoport X, when it is measured from its terminals for testing and diagnostic.

Near the bridge balance state, application of relative errors or uncertainties is useless as they are rising to $\pm\infty$. In [1], [6] this obstacle was bypassed by relating the absolute value of any bridge accuracy measure to the initial sensitivity of the current to voltage or voltage to voltage bridge transfer function. These sensitivities are valuable reference parameters as they do not change within the range of the bridge imbalance. In this paper the new double component approach to describing the bridge accuracy is developed. It has the form of sum of initial stage and of bridge imbalance accuracy measures. It is similar method of describing accuracy as is commonly used for digital voltmeters. Relation of each components to accuracy measures of all bridge arm resistances have been developed.

Basic formulas of bridge transfer functions

Four resistance (4R) bridge circuit with terminals ABCD working as passive twoport of variable internal resistances $R_1 - R_4$ is shown on Fig 1.

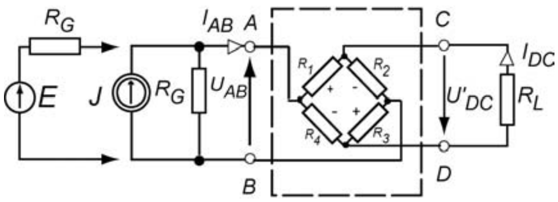


Fig 1 Four arms bridge as the unloaded twoport of type X
with the voltage or current supply source branch.

Commonly the bridge is supplied by current $I_{AB}\to J$=const., $R_G\to\infty$ or by voltage U_{AB}=const., R_G=0 and its output CD is unloaded, i.e.: $R_L\to\infty$, $U'_{DC}\to U_{DC}^{\infty}$. Formulas of U_{DC}^{∞} are given in table 1:

Table 1 Open circuit bridge voltage and its transfer functions	
$U'_{DC}\to U_{DC}^{\infty}=I_{AB}\,r_{21}$ (1)	$U'_{DC}\to U_{DC}^{\infty}=U_{AB}\,k_{21}$ (2)
$r_{21}\equiv\dfrac{U_{DC}^{\infty}}{I_{AB}}=\dfrac{R_1R_3-R_2R_4}{\sum R_i}\equiv t_0\,f(\boldsymbol{\varepsilon}_i)$ (3)	$k_{21}\equiv\dfrac{U_{DC}^{\infty}}{U_{AB}}=\dfrac{R_1R_3-R_2R_4}{(R_1+R_2)(R_3+R_4)}\equiv k_0\,f_E(\boldsymbol{\varepsilon}_i)$ (4)
where: $t_0\equiv\dfrac{mnR_{10}}{(1+m)(1+n)}$; $f(\boldsymbol{\varepsilon}_i)=\dfrac{\Delta L(\boldsymbol{\varepsilon}_i)}{1+\varepsilon_{\Sigma R}}$; $\boldsymbol{\varepsilon}_i=[\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4]^T$; $\Delta L(\boldsymbol{\varepsilon}_i)=\varepsilon_1-\varepsilon_2+\varepsilon_3-\varepsilon_4+\varepsilon_1\varepsilon_3-\varepsilon_2\varepsilon_4$; $\varepsilon_{\Sigma R}=\dfrac{\varepsilon_1+m\varepsilon_2+n(\varepsilon_4+m\varepsilon_3)}{(1+m)(1+n)}$	$k_0\equiv\dfrac{m}{(1+m)^2}$, n is arbitrary; $f_E(\boldsymbol{\varepsilon}_i)\equiv\dfrac{\Delta L(\boldsymbol{\varepsilon}_i)}{(1+\varepsilon_{12})(1+\varepsilon_{34})}$; $\varepsilon_{12}\equiv\dfrac{\varepsilon_1+m\varepsilon_2}{1+m}$; $\varepsilon_{43}\equiv\dfrac{\varepsilon_4+m\varepsilon_3}{1+m}$

where: I_{AB}, U_{AB} - current or voltage on bridge supply terminals A B, $R_i\equiv R_{i0}+\Delta R_i\equiv R_{i0}(1+\varepsilon_i)$ - arm resistance of initial value R_{i0} and absolute ΔR_i and relative ε_i increments; r_{21}, k_{21} – current to voltage and voltage bridge transfer functions of the open-circuited output;

$f(\boldsymbol{\varepsilon}_\mathbf{i})$, $f_E(\boldsymbol{\varepsilon}_\mathbf{i})$ - normalized imbalance functions of r_{21} and of k_{21}.

$t_0 = \frac{R_{10}R_{30}}{\sum_4 R_{i0}}$, $k_0 = \frac{R_{10}R_{30}}{(R_{10}+R_{20})(R_{30}+R_{40})}$ - initial bridge open circuit sensitivities of r_{21} and of k_{21},

$\sum R_i \equiv \sum_{i=1}^{4} R_i = (1+\varepsilon_{\Sigma R})\sum R_{i0}$ -sum of bridge arm resistances; $\varepsilon_{\Sigma R}(\boldsymbol{\varepsilon}_\mathbf{i})$, ΣR_{i0} - its increment and initial value.

Output voltage U_{DC} may change its sign for some set of arm resistances. If transfer function r_{21}= 0 or k_{12}= 0, the bridge is in balance and its conditions are the same: $R_1R_3=R_2R_4$. The bridge balance can occurs for many combinations of R_i, but the basic balance state is defined for all ε_i= 0, i.e. when:

$$R_{10}R_{30} = R_{20}R_{40} \quad (5)$$

Formulas of the bridge terminal parameters are simplified by referencing all resistances to their initial values in the balance, i.e. $R_i=R_{i0}(1+\varepsilon_i)$ and referencing initial resistances R_{i0} to one of the first arm, i.e.: $R_{20}\equiv mR_{10}$, $R_{40}\equiv nR_{10}$ and from (3) $R_{30}=mnR_{10}$. Both transfer functions can be normalized as in Table 1.

Accuracy description of broadly variable resistances

The accuracy of measurements depends on structure of the instrumentation circuit, values and accuracy of its elements and on various environmental influences of natural conditions and of the neighboring equipment. Two type of problems have been met in practice:

- description of circuits and measurement equipment by instantaneous and limited values of systematic and random errors, absolute or related ones, as well by statistical measures of that errors,
- estimation of the measurement result uncertainty, mainly by methods recommended by guide GUM.

Measures of accuracy (errors, uncertainty) of the single value of circuit parameter are expressed by numbers, of variable parameter - by functions. The measures of broadly variable resistance R_i, e.g. of the stress or of the temperature sensors, could be expressed for two their output components: for its initial value and for its increment as it is shown in table 2.

Table 2. Accuracy measures of the two component formula of sensor resistance

Sensor resistance R_i	$R_i=R_{i0}(1+\varepsilon_i)$ if $\varepsilon_i > -1$
Absolute error $\Delta_i \equiv R_i - R_{i\,\mathrm{nominal}}$	$\Delta_i = \Delta_{i0}(1+\varepsilon_i) + R_{i0}\Delta_{\varepsilon i}$ (6)
Relative error $\delta_{Ri} \equiv \frac{\Delta_i}{R_i}$	$\delta_{Ri} = \delta_{i0} + \frac{\Delta_{\varepsilon i}}{1+\varepsilon_i} = \delta_{i0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i}$ (7) where: $\delta_{i0} = \frac{\Delta_{i0}}{R_{i0}}$, $\delta_{\varepsilon i} = \frac{\Delta_{\varepsilon i}}{\varepsilon_i}$ - relative errors of initial value R_{i0} and of resistance increment ε
Relative limited error $\lvert\delta_{Ri}\rvert \equiv \frac{\lvert\Delta_i\rvert}{R_i}$	$\lvert\delta_{Ri}\rvert = \lvert\delta_{i0}\rvert + \frac{\lvert\Delta_{\varepsilon i}\rvert}{1+\varepsilon_i} = \lvert\delta_{i0}\rvert + \frac{\lvert\varepsilon_i\rvert}{1+\varepsilon_i}\lvert\delta_{\varepsilon i}\rvert$ (8)
Statistical measure standard deviation of δ_R (for random errors or uncertainty) $\bar{\delta}_{Ri} \equiv \frac{\bar{\Delta}_i}{R_i}$	$\bar{\delta}_{Ri} \equiv \frac{\bar{\Delta}_i}{R_i} = \sqrt{\bar{\delta}_{i0}^2 + \left(\frac{1}{1+\varepsilon_i}\right)^2 \bar{\Delta}_{\varepsilon i}^2 + 2k_i \frac{1}{1+\varepsilon_i}\bar{\delta}_{i0}\bar{\Delta}_{\varepsilon i}}$ (9) where: $\bar{\delta}_{i0}$, $\bar{\delta}_{\varepsilon i}$ – standard measures of initial value R_{i0} and of increment ε_i, $k_i \subset (-1...0..+1)$ – correlation coefficient
Particular cases of correlation:	$k_i=\pm1$ $\bar{\delta}_{Ri} = \left\lvert \bar{\delta}_{i0} \pm \left(\frac{1}{1+\varepsilon_i}\right)\bar{\Delta}_{\varepsilon i} \right\rvert$ (9a) $k_i=0$ $\bar{\delta}_{Ri} = \sqrt{\bar{\delta}_{i0}^2 + \left(\frac{1}{1+\varepsilon_i}\right)^2 \bar{\Delta}_{\varepsilon i}^2}$ (9b)

Instantaneous absolute error Δ_{Ri} and its relative value δ_{Ri} referenced to R_i are given by formulas (6) and (7), **relative limited error** $\lvert\delta_{Ri}\rvert$ of the poorest case of values and signs of $\lvert\delta_{i0}\rvert$ and $\lvert\Delta_{\varepsilon i}\rvert$ or $\lvert\delta_{\varepsilon i}\rvert$ – by (8), and **standard statistical measure** $\bar{\delta}_{Ri}$ for random errors or uncertainties - by (9).

If errors of increment and of initial value of resistance are statistically independent then correlation coefficient k_i = 0, but if they are strictly related each to the other then $k_i = \pm$ 1. Exact k_i value can only be find experimentally. From (8) follows that borders of the worse cases $\pm\lvert\delta_{Ri}\rvert$ of possible values of δ_{Ri} nonlinearly dependent on ε_i even if $\lvert\delta_{i0}\rvert$ and $\lvert\Delta_{\varepsilon i}\rvert$ or $\lvert\delta_{\varepsilon i}\rvert$ are constant [1], [6].

Distribution of the initial values and relative increments ε_i of the sensors' set resistances depends on their data obtained in the production process and its actual values on the environmental conditions.

Accuracy description of bridge transfer functions r_{21} and k_{21}

Instantaneous values of measurement errors of bridge transfer functions r_{21} and k_{21} result from the total differential of analytical equations (3) and (4) from Table 1.

After ordering all components of δ_{Ri} **absolute error of transfer function r_{21}** is:

$$\Delta_{r21} = R_1 \frac{R_3 - r_{21}}{\Sigma R_i}\delta_{R1} - R_2 \frac{R_4 + r_{21}}{\Sigma R_i}\delta_{R2} + R_1 \frac{R_1 - r_{21}}{\Sigma R_i}\delta_{R3} - R_4 \frac{R_2 + r_{21}}{\Sigma R_i}\delta_{R4} = \sum_{i=1}^{4} w_{Ri}\ \delta_{Ri} \tag{10}$$

where: $w_{Ri} \equiv R_i \dfrac{(-1)^{i+1} R_j - r_{21}}{\sum R_i}$ - weight coefficients of error δ_{Ri} components; - subscript i = 1,2,3,4 when j=3,4,1,2; - multiplier $(-1)^{i+1}$=+1 if i is 1, 3 or –1 if i is 2, 4.

If resistances are expressed as $R_i = R_{i0}(1+\varepsilon_i)$, $R_j = R_{j0}(1+\varepsilon_j)$ formula (10) is

$$\Delta_{r21} = \frac{t_0}{1+\varepsilon_{\Sigma R}} \sum_{i,j} \left[(-1)^{i+1}(1+\varepsilon_j) - \frac{r_{12}}{R_{j0}} \right](1+\varepsilon_i)\delta_{Ri} \tag{10a}$$

Absolute error of transfer function k_{21} has other forms, i.e.:

$$\Delta_{k21} = \frac{R_1 R_2}{(R_1+R_2)^2}\left(\delta_{R1} - \delta_{R2}\right) + \frac{R_3 R_4}{(R_3+R_4)^2}\left(\delta_{R3} - \delta_{R4}\right) \tag{11}$$

or

$$\Delta_{k21} = k_0 \left[\frac{(1+\varepsilon_1)(1+\varepsilon_2)}{(1+\varepsilon_{12})^2}\left(\delta_{R1} - \delta_{R2}\right) + \frac{(1+\varepsilon_3)(1+\varepsilon_4)}{(1+\varepsilon_{34})^2}\left(\delta_{R3} - \delta_{R4}\right) \right] \tag{11a}$$

From (10) and (11) one could see that if errors δ_{Ri} of the neighboring bridge arms have the same sign they partly compensate each other.

If errors δ_{Ri} of resistances R_i are expressed, as in (7), by their initial errors $\delta_{i\,0}$ and incremental errors $\delta_{i\varepsilon}$, then

$$\Delta_{r21} = \sum w_{Ri} \left(\delta_{i\,0} + \frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon\, i} \right) \tag{12}$$

where: $$w_{Ri} = \frac{t_0}{1+\varepsilon_{\Sigma R}}[(-1)^{i+1}(1+\varepsilon_j) - \frac{R_{i0}}{R_{j0}}\Delta L\,(\boldsymbol{\varepsilon}_\mathbf{i})](1+\varepsilon_i) \tag{12a}$$

If arm resistance R_i is constant, $\varepsilon_i = 0$, $\delta_{Ri} = \delta_{i0}$, but weight coefficient w_{Ri} of its component in Δ_{r21} still depends on other arm increments $\boldsymbol{\varepsilon}_\mathbf{j}$. In initial balance state, i.e. when all arm increments $\boldsymbol{\varepsilon}_\mathbf{j} = \mathbf{0}$, the nominal transfer function $r_{21}(0) \equiv r_{210} = 0$, but real resistances R_i have some initial errors δ_{i0} and usually $\Delta_{r210} \neq 0$, $\Delta_{k210} \neq 0$

$$\Delta_{r210} = t_0\ \delta_{210} \tag{13a}$$

$$\Delta_{k210} = k_0\ \delta_{210} \tag{13b}$$

where: $\delta_{21\,0} = \delta_{10} - \delta_{20} + \delta_{30} - \delta_{40}$

Relative errors are preferable in measurement practice, but it is not possible to use them for transfer functions near the bridge balance as the ratio of absolute error $\Delta_{r21} \rightarrow \Delta_{r210} \neq 0$ and the nominal $r_{21} \rightarrow r_{210} = 0$ (or for the voltage supplied bridge of Δ_{k21} and $k_{21} \rightarrow k_{210} = 0$) is rising to $\pm\infty$. Then other possibilities should be applied. There are two possible ways to describe accuracy of the bridge transfer function r_{21} (or k_{21}) in the form of one or of two related components:

- absolute error of the bridge transfer function may be referenced to initial sensitivity factor t_0 of r_{21} (or to k_0 of k_{21}) or to the range of transmittance $r_{21\max}$ - $r_{21\min}$ (or $k_{21\max}$ - $k_{21\min}$);
- initial error Δ_{r210} have to be subtracted from Δ_{r21} and then accuracy could be described by two separate terms: for zero and for transfer function increment, as it is common for digital instrumentation.

In **the first type method** it is preferable if error Δ_{r21} is referenced to the initial sensitivity factor t_0 as constant for each bridge, then to full range of r_{21} as it could be change. Such relative error δ_{r21} could be presented as sum:

$$\delta_{r21} \equiv \frac{\Delta_{r21}}{t_0} = \delta_{210} + \delta_{r21\varepsilon}(\varepsilon_i) \tag{14}$$

where: $\delta_{210} = \delta_{10} - \delta_{210} + \delta_{30} - \delta_{40}$ – initial (or zero) relative error of r_{21}=0; $\delta_{r21\varepsilon}(\varepsilon_i)$ –relative error of normalized imbalance function $f(\varepsilon_i)$ when $r_{21} \neq 0$, also referenced to t_0.

Error δ_{210} is similar for any mode of the supply source equivalent circuit of the bridge as twoport.
Zero of the bridge may be corrected on different ways: by adjustment of the bridge resistances, by the opposite voltage on output or by the digital correction of converted output signal. In such cases from (14) it is

$$\delta_{r21\varepsilon} = \frac{1}{t_0}\sum_{i=1}^{4}\left[w_{Ri} - (-1)^{i+1}\right]\delta_{i0} + \frac{1}{t_0}\sum_{i=1}^{4} w_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i} \tag{15}$$

From (15) follows that related to t_0 error $\delta_{r21\varepsilon}$ of r_{21} increment depends not only on increment errors $\delta_{\varepsilon i}$ of resistances R_i but also on their initial errors $\delta_{i0} \neq 0$ even when initial error of the whole bridge δ_{210}=0, because after (12a) weight coefficients of δ_{i0} in (15) depends on $\boldsymbol{\varepsilon}_i$. The component of particular error δ_{i0} despairs only when δ_{i0}=0. Functions of $\Delta_{\varepsilon i}$ or $\delta_{\varepsilon i}$ may be approximated for some ε_i intervals by constant values.

In the **second type method** absolute error of transfer function r_{21} after subtracting its initial value in (12) is

$$\Delta_{r21} - \Delta_{r210} = \sum_{i=1}^{4}\left[w_{Ri} - (-1)^{i-1}\right]\delta_{i0} + \sum_{i=1}^{4} w_{Ri}\frac{\varepsilon_i}{1+\varepsilon_i}\delta_{\varepsilon i} \tag{16}$$

And after referenced this difference of errors to r_{21}, and substitution w_{Ri} from (12a)

$$\delta_{r21r} \equiv \frac{\Delta_{r21} - \Delta_{r210}}{r_{21}} = \frac{\delta_{r21} - \delta_{210}}{f(\varepsilon_i)} = \sum_{i=1}^{4} w'_{r\,i0}\,\delta_{i0} + \sum_{i=1}^{4} w'_{r\,\varepsilon i}\,\delta_{\varepsilon i} \tag{17}$$

where : $w'_{r\,i0} = (-1)^{i-1}\dfrac{\varepsilon_i + \varepsilon_j + \varepsilon_i\varepsilon_j}{\Delta L} - t_0\dfrac{1+\varepsilon_i}{R_{j0}}$; $\quad w'_{r\,\varepsilon i} = \left[\dfrac{(-1)^{i-1}(1+\varepsilon_j)}{\Delta L(\boldsymbol{\varepsilon}_i)} - \dfrac{t_0}{R_{j0}}\right]\varepsilon_i$ (17a,b)

Weight coefficients (17a,b) are finite for any value of r_{21} including r_{21}= 0, if all $\varepsilon_i \to 0$ also $\Delta L \to 0$.
Error δ_{r21r} is equivalent to error $\delta_{\varepsilon i}$ of the resistance R_i increment ε_i in formula (7).
From (13a), (17) for r_{21} and from (13b) by the same way for k_{21} is:

$$\Delta_{r21} = t_0\,\delta_{210} + r_{21}\,\delta_{r21\mathrm{r}} \tag{18a}$$

$$\Delta_{k21} = k_0\,\delta_{210} + k_{21}\,\delta_{k21k} \tag{18b}$$

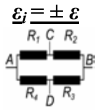
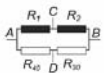
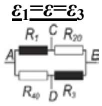
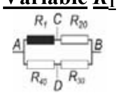
where: $t_0\,\delta_{210} = \Delta_{r210}$, $k_0\,\delta_{210} = \Delta_{k210}$ – absolute errors of initial value r_{21} or k_{21} e.g. r_{210} = 0 or k_{210}= 0
δ_{r21r}, δ_{k21k} - related errors of increments r_{21} - r_{210} or k_{21} - k_{210} from the initial stage.

Actual values of instantaneous errors of r_{21} or k_{21} could be calculated only if signs and values of errors of all resistances are known. In reality it happens very rare. More frequently are used their limited systematic errors (of the worst case) and statistical standard deviation measures. Formulas of this accuracy measures of r_{21} or k_{21} could be obtained after transformation of error formulas (10) – (18a,b). All these accuracy measures is possible to find in one component or two component forms. One component formulas for arbitrary and main particular cases of 4R bridge are given in tables in [1], [6]. The two-component method of the bridge transfer function r_{21} accuracy representation, separately for its initial value (eg. equal to zero) and for increment is similar like unified one used for digital instruments and of the broad range sensor transmitters. It is especially valuable if zero of the measurement track is set handily or automatically. Absolute measures could be transformed also by the sensor set linear or nonlinear function to the units of any particular measurand, e.g. in the case of platinum sensors - to ^{0}C.

Accuracy description of mostly used 4R sensor bridges

In the measurement practice the mostly used for sensors are four-arm bridges of resistances R_{i0} equal in the initial balance state. Formulas of accuracy measures for transfer functions r_{21} and k_{21} of these particular bridges are much simpler then general ones [1], [6]. They are presented below in Table 3. The transfer function r_{21} or k_{21} of the bridge including differential sensors of four or two relative increments $\pm\varepsilon$ and transfer function r_{21} of two equal ε in opposite arms may be linear but their measures depend on ε by different way each other. The single sensor bridge is less nonlinear for current supply.

Table 3 Accuracy measures of open-circuit 4R sensor bridges of arm resistances equal in balance.

No	Parameters of the bridge <u>*m*=1, *n*=1</u>	Measures of R_i	Accuracy measures of bridge transfer functions: a) of current to voltage r_{21} and voltages k_{21} transfer functions	b) of transfer functions' increments r_{21}-r_{210} and k_{21}-k_{210} only
1	**Variable R_1 - R_4** $R_i = R_{10}(1+\varepsilon_i)$	arbitrary	$\delta_{r21} \equiv \frac{\Delta_{r21}}{t_0} = \frac{1}{1+0{,}25\,\Sigma\varepsilon_i}\sum_{i=1}^{4}(1+\varepsilon_i)\left[(-1)^{i-1}(1+\varepsilon_j)-\frac{r_{21}}{R_{j0}}\right]\delta_{Ri}$ for $i=1, 2, 3, 4$ is $j=3, 4, 1, 2$	$\delta_{r21r} = \frac{\delta_{r21}-\delta_{210}}{f(\varepsilon_i)} = \sum_{i=1}^{4} w'_{r\,i0}\,\delta_{i0} + \sum_{i=1}^{4} w'_{r\,\varepsilon i}\,\delta_{\varepsilon i}$
2	**$\varepsilon_i = \pm\varepsilon$** 	arbitrary	$\delta_{r21} = (1+\varepsilon)(\delta_{10}+\delta_{30}) - (1-\varepsilon)(\delta_{20}+\delta_{40}) + \Delta_{\varepsilon1}+\Delta_{\varepsilon3}-\Delta_{\varepsilon2}-\Delta_{\varepsilon4}$	$\delta_{r21r} = \frac{1}{4}(\delta_{10}+\delta_{30}-\delta_{20}+\delta_{40}+\delta_{\varepsilon1}+\delta_{\varepsilon3}-\delta_{\varepsilon2}-\delta_{\varepsilon4})$
3	**R_1=R_3=R_{10}(1+ε)**	$\|\delta_{i0}\|=\|\delta_0\|$ $\|\Delta_{\varepsilon i}\|=\|\Delta_\varepsilon\|$	$\|\delta_{r21}\| = 4\,\|\delta_0\| + \|\Delta_\varepsilon\|$	$\|\delta_{r21r}\| = \|\delta_0\| + \|\delta_\varepsilon\|$
4	**R_2=R_4=R_{10}(1-ε)** $r_{21} = R_{10}\,\varepsilon$	$\bar\delta_{i0} \equiv \bar\delta_0$ $\bar\Delta_{\varepsilon i} \equiv \bar\Delta_\varepsilon$	$\bar\delta_{r21} = 2\sqrt{(1+\varepsilon^2)\,\bar\delta_0^2 + \bar\Delta_\varepsilon^2}$	$\bar\delta_{r21r} = 0{,}5\sqrt{\bar\delta_0^2 + \bar\Delta_\varepsilon^2}$
5	$t_0 = \frac{1}{4}R_{10}$ $f(\varepsilon_i) = f_E(\varepsilon_i) = 4\varepsilon$	arbitrary	$\delta_{k21} = (1-\varepsilon^2)(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}) + (1-\varepsilon)(\Delta_{\varepsilon1}+\Delta_{\varepsilon3}) - (1+\varepsilon)(\Delta_{\varepsilon2}+\Delta_{\varepsilon4})$	$\delta_{k21k} = -\frac{1}{4}\varepsilon(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}) + \frac{1}{4}(\delta_{\varepsilon1}+\delta_{\varepsilon3}-\delta_{\varepsilon2}-\delta_{\varepsilon4})$
6	$\varepsilon \le 1$	$\|\delta_{i0}\|=\|\delta_0\|$ $\|\Delta_{\varepsilon i}\|=\|\Delta_\varepsilon\|$	$\|\delta_{k21}\| = 4\left[(1-\varepsilon^2)\,\|\delta_0\| + \|\Delta_\varepsilon\|\right]$	$\|\delta_{k21k}\| = \varepsilon\,\|\delta_0\| + \|\delta_\varepsilon\|$
7	$k_{21} = \varepsilon$ $k_0 = 0{,}25$	$\bar\delta_{i0} \equiv \bar\delta_0$ $\bar\Delta_{\varepsilon i} \equiv \bar\Delta_\varepsilon$	$\bar\delta_{k21k} = 2\sqrt{(1-\varepsilon^2)^2\,\bar\delta_0^2 + (1+\varepsilon^2)\,\bar\Delta_\varepsilon^2}$	$\bar\delta_{k21r} = 0{,}5\sqrt{\varepsilon^2\,\bar\delta_0^2 + \bar\Delta_\varepsilon^2}$
8	**Variable R_1,R_2** **$\varepsilon_1 = \varepsilon = -\varepsilon_2$**	arbitrary	$\delta_{r21} = (1-0{,}5\varepsilon)\left[(1+\varepsilon)\delta_{10}+\Delta_{\varepsilon1}-\delta_{40}\right] - (1+0{,}5\varepsilon)\left[(1-\varepsilon)\delta_{20}+\Delta_{\varepsilon2}-\delta_{30}\right]$	$\delta_{r21r} = (1-\varepsilon)\delta_{10} - (1+\varepsilon)\delta_{20} + \delta_{30} - \delta_{40} + \delta_{\varepsilon1}(0{,}5-\varepsilon) - \delta_{\varepsilon2}(0{,}5+\varepsilon)$
9		$\|\delta_{i0}\|=\|\delta_0\|$ $\|\Delta_{\varepsilon i}\|=\|\Delta_\varepsilon\|$	$\|\delta_{r21}\| = 4(1-0{,}25\varepsilon^2)\,\|\delta_0\| + 2\|\Delta_\varepsilon\|$	$\|\delta_{r21r}\| = 4\|\delta_0\| + \|\delta_\varepsilon\|$
10	**R_1=R_{10}(1+ε)** **R_2=R_{10}(1-ε)** R_3=R_4 = R_{10}	$\bar\delta_{i0} \equiv \bar\delta_0$, $\bar\Delta_{\varepsilon i} \equiv \bar\Delta_\varepsilon$	$\bar\delta_{r21} = \sqrt{2}\,\sqrt{[1+(1-\varepsilon^2)^2]\,\bar\delta_0^2 + (1+\varepsilon^2)\,\bar\Delta_{\varepsilon1}^2}$	$\bar\delta_{r21r} = 2\sqrt{(1+0{,}5\varepsilon^2)\bar\delta_0^2 + (0{,}25+0{,}5\varepsilon^2)\,\bar\delta_\varepsilon^2}$
11	$r_{21} = 0{,}5R_{10}\,\varepsilon$	arbitrary	$\delta_{k21} = (1-\varepsilon^2)(\delta_{10}-\delta_{20}) + (1-\varepsilon)\,\Delta_{\varepsilon1} - (1+\varepsilon)\,\Delta_{\varepsilon2} + \delta_{30} - \delta_{40}$	$\delta_{k21k} = -0{,}5\varepsilon(\delta_{10}-\delta_{20}) + 0{,}5(1-\varepsilon)\,\delta_{\varepsilon1} + 0{,}5(1+\varepsilon)\,\delta_{\varepsilon2}$
12	$f(\varepsilon_i) = f_E(\varepsilon_i) = 2\varepsilon$ $\varepsilon \le 1$	$\|\delta_{i0}\|=\|\delta_0\|$ $\|\Delta_{\varepsilon i}\|=\|\Delta_\varepsilon\|$	$\|\delta_{k21}\| = 4(1-0{,}5\varepsilon^2)\,\|\delta_0\| + 2\|\Delta_\varepsilon\|$	$\|\delta_{k21k}\| = \varepsilon\,\|\delta_0\| + \|\delta_\varepsilon\|$
13	$k_{21} = \frac{1}{2}\varepsilon$	$\bar\delta_{i0} \equiv \bar\delta_0$, $\bar\Delta_{\varepsilon i} \equiv \bar\Delta_\varepsilon$	$\bar\delta_{k21} = \sqrt{2}\,\sqrt{[1+(1-\varepsilon^2)^2]\,\bar\delta_0^2 + (1+\varepsilon^2)\,\bar\Delta_{\varepsilon1}^2}$	$\bar\delta_{r21r} = \frac{1}{\sqrt{2}}\sqrt{\varepsilon^2\,\bar\delta_0^2 + (1+\varepsilon^2)\,\bar\delta_\varepsilon^2}$
14	**Variable R_1,R_3** **ε_1=ε=ε_3**	arbitrary	$\delta_{r21} = (1+\varepsilon)(\delta_{10}+\delta_{30}) - \delta_{20} - \delta_{40} + \varepsilon(\delta_{\varepsilon1}+\delta_{\varepsilon3})$	$\delta_{r21r} = 0{,}5(\delta_{10}+\delta_{30}+\delta_{\varepsilon1}+\delta_{\varepsilon3})$
15			$\|\delta_{r21}\| = (1+\varepsilon)(\|\delta_{10}\|+\|\delta_{30}\|) + \|\delta_{20}\| + \|\delta_{40}\| + \|\varepsilon\|(\|\delta_{\varepsilon1}\|+\|\delta_{\varepsilon3}\|)$	$\|\delta_{r21r}\| = 0{,}5(\|\delta_{10}\|+\|\delta_{30}\|+\|\delta_{\varepsilon1}\|+\|\delta_{\varepsilon3}\|)$
16			$\bar\delta_{r21} = \sqrt{(1+\varepsilon)^2(\bar\delta_{10}^2+\bar\delta_{30}^2) + \bar\delta_{20}^2 + \bar\delta_{40}^2 + \varepsilon^2(\bar\delta_{\varepsilon1}^2+\bar\delta_{\varepsilon3}^2)}$	$\bar\delta_{r21r} = \frac{1}{\sqrt{2}}\sqrt{\bar\delta_{10}^2+\bar\delta_{30}^2+\bar\delta_{\varepsilon1}^2+\bar\delta_{\varepsilon3}^2}$
17	**R_1=R_{10}(1+ε)** **R_3=R_{10}(1+ε)** R_2=R_4=R_{10}	$\|\delta_{i0}\|=\|\delta_0\|$ $\|\delta_{\varepsilon i}\|=\|\delta_\varepsilon\|$	$\|\delta_{r21}\| = 4(1+0{,}5\varepsilon)\,\|\delta_0\| + 2\|\varepsilon\|\,\|\delta_\varepsilon\|$	$\|\delta_{r21r}\| = \|\delta_0\| + \|\delta_\varepsilon\|$
18	$r_{21} = \frac{R_{10}}{4}2\varepsilon$	$\bar\delta_{i0} \equiv \bar\delta_0$, $\bar\delta_{\varepsilon i} \equiv \bar\delta_\varepsilon$	$\bar\delta_{r21} = 2\sqrt{(1+\varepsilon+0{,}5\varepsilon^2)\,\bar\delta_0^2 + 0{,}5\varepsilon^2\,\bar\delta_\varepsilon^2}$	$\bar\delta_{r21r} = \frac{1}{\sqrt{2}}\sqrt{\bar\delta_0^2 + \bar\delta_\varepsilon^2}$
19	$k_{21} = \frac{1}{4}\frac{2\varepsilon}{1-0{,}25\varepsilon^2}$	arbitrary	$\delta_{k21} = \frac{(1+\varepsilon)(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}) + \varepsilon(\delta_{\varepsilon1}+\delta_{\varepsilon3})}{(1+0{,}5\varepsilon)^2}$	$\delta_{k21k} = \frac{-0{,}25\varepsilon(\delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}) + \delta_{\varepsilon1} + \delta_{\varepsilon3}}{2(1+0{,}5\varepsilon)}$
20			$\|\delta_{k21}\| = \frac{1}{(1+0{,}5\varepsilon)^2}\left[(1+\varepsilon)\,\|\delta_{210}\| + \|\varepsilon\|\,(\|\delta_{\varepsilon1}\|+\|\delta_{\varepsilon3}\|)\right]$	$\|\delta_{k21k}\| = \frac{1}{2(1+0{,}5\varepsilon)}\left(0{,}25\|\varepsilon\|\,\|\delta_{210}\| + \|\delta_{\varepsilon1}\| + \|\delta_{\varepsilon3}\|\right)$
21	**Variable R_1** 	arbitrary	$\delta_{r21} = \frac{(1+\varepsilon_1)\,\delta_{10} + \varepsilon\,\delta_{\varepsilon1} + (1+0{,}5\,\varepsilon_1)^2\delta_{30} - (1+0{,}5\,\varepsilon_1)(\delta_{20}+\delta_{40})}{(1+0{,}25\varepsilon_1)^2}$	$\delta_{r21r} = \frac{(\frac{1}{2}-\frac{1}{16}\varepsilon_1)\,\delta_{10} + (\frac{1}{2}+\frac{3}{16}\varepsilon_1)\,\delta_{30} + \frac{1}{16}\varepsilon_1(\delta_{20}+\delta_{40}) + \delta_{\varepsilon1}}{1+0{,}25\varepsilon_1}$
22		$\|\delta_{10}\| \ne \|\delta_{20}\| = \|\delta_{30}\| =$ $= \|\delta_{40}\| \equiv \|\delta_0\|$	$\|\delta_{r21}\| = \frac{\frac{1}{2}\,\|1-\frac{1}{8}\varepsilon_1\|\,\|\delta_{10}\| + \frac{1}{2}(1+\frac{5}{8}\,\varepsilon_1)\,\|\delta_0\| + \|\delta_{\varepsilon1}\|}{1+0{,}25\,\varepsilon_1}$	$\|\delta_{r21r}\| = \frac{\frac{1}{2}\,\|1-\frac{1}{8}\varepsilon_1\|\,\|\delta_{10}\| + \frac{1}{2}(1+\frac{5}{8}\,\varepsilon_1)\,\|\delta_0\| + \|\delta_{\varepsilon1}\|}{1+0{,}25\,\varepsilon_1}$
23	**R_1=R_{10}(1+ε_1)** R_2= R_3= R_4=R_{10} $r_{21} = \frac{R_{10}}{4}\frac{\varepsilon_1}{1+0{,}25\varepsilon_1}$	$\|\delta_{i0}\|=\|\delta_0\|$ $\|\delta_{\varepsilon1}\| \ne 0$	$\|\delta_{r21}\| = \frac{\left[4(1+0{,}25\,\varepsilon_1)^2 + \varepsilon_1\right]\|\delta_0\| + \|\varepsilon\|\,\|\delta_{\varepsilon1}\|}{(1+0{,}25\,\varepsilon_1)^2}$	$\|\delta_{r21r}\| = \|\delta_0\| + \frac{\|\delta_{\varepsilon1}\|}{1+0{,}25\varepsilon_1}$ for $\varepsilon_1 \le 8$ $\|\delta_{r21r}\| = \frac{(1+0{,}75\varepsilon_1)\,\|\delta_0\| + \|\delta_{\varepsilon1}\|}{1+0{,}25\varepsilon_1}$ for $\varepsilon_1 \ge 8$
24	$t_0 = \frac{R_{10}}{4}$ $f(\varepsilon_i) = \frac{\varepsilon_1}{1+0{,}25\varepsilon_1}$	$\bar\delta_{i0} \equiv \bar\delta_0$ $\bar\delta_{\varepsilon1} \ne 0$	$\bar\delta_{r21} = \frac{\sqrt{\left[(1+\varepsilon_1)^2 + (1+0{,}5\,\varepsilon_1)^4 + 2(1+0{,}5\,\varepsilon_1)^2\right]\bar\delta_0^2 + \varepsilon^2\bar\delta_{\varepsilon1}^2}}{(1+0{,}25\,\varepsilon_1)^2}$	$\bar\delta_{r21r} = \frac{\frac{1}{2}\sqrt{(1+\frac{1}{2}\varepsilon_1+\frac{3}{16}\varepsilon_1^2)\,\bar\delta_0^2 + 4\bar\delta_{\varepsilon1}^2}}{1+0{,}25\,\varepsilon_1}$
25	$k_{21} = \frac{1}{4}\frac{\varepsilon_1}{1+0{,}5\varepsilon_1}$	arbitrary	$\delta_{k21} \equiv \frac{\Delta_{k21}}{k_0} = \frac{(1+\varepsilon_1)(\delta_{10}-\delta_{20}) + \varepsilon_1\,\delta_{\varepsilon1}}{(1+0{,}5\varepsilon_1)^2} + \delta_{30} - \delta_{40}$	$\delta_{k21k} = \frac{-0.25\,\varepsilon_1\,(\delta_{10}-\delta_{20}) + \delta_{\varepsilon1}}{1+0{,}5\,\varepsilon_1}$
26			$\|\delta_{k21}\| = \frac{1}{(1+0{,}5\varepsilon)^2}\left[(1+\varepsilon)\,\|\delta_{210}\| + \|\varepsilon\|\,(\|\delta_{\varepsilon1}\|+\|\delta_{\varepsilon3}\|)\right]$	$\|\delta_{k21k}\| = \frac{1}{(1+0{,}5\varepsilon)^2}\left[(1+\varepsilon)\,\|\delta_{210}\| + \|\varepsilon\|\,(\|\delta_{\varepsilon1}\|+\|\delta_{\varepsilon3}\|)\right]$
0	**Measures of balance accuracy**	**actual error** $\delta_{210} = \delta_{10}-\delta_{20}+\delta_{30}-\delta_{40}$	**limited error** $\|\delta_{210}\|_m = \sum \|\delta_{i0}\|$	**Mean square measure** $\bar\delta_{210} = \sqrt{\sum \bar\delta_{i0}^2}$

From Table 3 it is possible to compare the accuracy measure formulas of the current or voltage supplied 4R bridges of few variable elements sensor and of single element sensors. For example formulas of accuracy measures of similarly variable two opposite arm resistances are simpler then for the single variable arm. For small values of ε it is possible also to approximate last nonlinear formulas by polynomial of the first or higher order.

General conclusions

Two methods of describing accuracy measures of unbalanced bridges are presented and compared, i.e.:
- one component accuracy measures related to initial sensitivity of the bridge transfer functions [1],[6]
- the double component one of separately defined measures for zero and transfer function increment.

The second one simplifies the accuracy description and is similar as used for the broad range instruments, e.g. digital voltmeters. As accuracy measures of bridge arms are defined for initial resistances and for their increments, then they are independent from the sensor characteristic.

This methods are discussed using on few examples of 4R bridges of equal initial resistances, supplied by current or voltage source and with single, double and four element sensors.

Given formulas allow to find accuracy of the 4R bridge or uncertainty of measurements with bridge circuits if actual or limited values of errors or standard statistical measure of their resistances and sensors are known. For set of bridges in production or in application the systematic errors could be calculated also as random ones and if correlation coefficients are small enough obtained values are smaller than of the worst case of limited errors.

The good example of the broadly variable resistance are platinum temperature sensors Pt of A and B classes commonly used in industrial measurements. Tolerated differences from nominal characteristic are given in standard EN 60751÷A2 1997. Accuracy of 4R bridges with industrial Platinum sensors has been analyzed in [6] and more in detail including double-component method in [7]. Met in practice internal or external zero adjustment, negligible arm errors and calibration of the initial resistance are also considered. Single-component method was also used to describe accuracy of two-parameter measurements by cascade and unconventional double-current supplied bridges [3-5].

Similar formulas as presented for resistance four-arm bridges in [1], [4], [6], [7] and above could be formulated for any types of impedance sensor circuits as DC and AC passive and active bridge, also linearized by feedback or by multipliers, NASA Anderson loop and impedance converters with virtual DSP processing. Both methods may be also useful for accuracy evaluation in testing any circuit from its terminals as twoport, which is commonly used in diagnostics and in impedance tomography.

Double-component accuracy measures are invented also specially for describing unconventional AC double-current supply bridges just developed by the second author for doctor thesis supported by the 'habilitation grant' of Polish Ministry of Science and by project no S/WE/3/08 of TU Bialystok.

References

1. Warsza Z. L., "*Four-Terminal* (*4T*) *Immittance Circuits in Multivariable Measurements"* (*Immitancyjne układy czterobiegunowe* (*4T*) *w pomiarach wieloparametrowych*), Monograph of Industrial Institute of Control and Measurement PIAP, Warszawa 2004 (in Polish)
2. Sydenham P.H., Thorn R. (eds.), *Handbook of Measuring System Design*, Chapters: 126: "Electrical Bridge Circuits – Basic Information" and 127 " Unbalanced DC bridges" by Warsza Z. L., pp. 867 – 889 John Wiley & Sons, Ltd. New York ... 2005
3. Warsza Z. L., "Two Parameter (2D) Measurements in Four Terminal (4T) Impedance Bridges as the New Tool for Signal Conditioning". Part 1 and 2, *Proc. of the 14th International Symposium on New Technologies in Measurement and Instrumentation and 10th Workshop on ADC Modeling and Testing IMEKO TC-4*, pp. 31-42, Gdynia/ Jurata, Poland 2005.
4. Z. L. Warsza, "Two Parameter (2D) Measurements in Double-current Supply Four-terminal Resistance Circuits" *Metrology and Measurement Systems* vol. XIII, no 1 pp. 49-65, 2006
5. Warsza Z. L., "Backgrounds of two variable (2D) measurements of resistance increments by bridge cascade circuit". *Proc. of SPIE Photonic Applications…*vol. 6347 (Wilga Symposium, Poland), ed. Romaniuk R. S., pp. 634722R-1-10, 2006
6. Warsza Z.L.: Accuracy Measures of the Four Arm Bridge of Broadly Variable Resistances, part 1 and 2. *Proceedings of 15th IMEKO TC4 International Symposium on Novelties in Electrical Measurement and Instrumentations*, TU Iasi , Romania Sept.2007 vol.1 p.17-28
7. Warsza Z.L.: New Approach of the Accuracy of Industrial Pt Sensor's Bridge (Nowe ujęcie opisu dokładności mostka z przemysłowymi czujnikami Pt). *Proceedings of PPM'08 Conference*, Prace Oddziału PAN w Katowicach, Konferencje no 8, p. 155-164 (in Polish)