Conversion from geometrical to electrical model of LVDT

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Abstract- The Linear Variable Differential Transformer (LVDT) is an inductive sensor which is used to measure linear displacement and finds uses in modern machine-tool, robotics, avionics, and computerized manufacturing. Its basic structure consists of a primary coil and two secondary coils like an electrical transformer. However, LVDT has a movable magnetic core that when the primary coil is excited with an AC voltage source, induced secondary voltages vary with the displacement of the core. In general, this accurate and reliable displacement-to-electrical sensor can be modeled in two forms: geometrical-parameter-based model and electrical-parameter-based model. Both are very used. However, research results based in geometrical-based model may become useless when only parameters from electrical one are known. In this paper, it is shown a way of conversion from geometrical- to electrical-based model in order to allow the interaction from one to other.

I. Introduction

The Linear Variable Transformer Differential (LVDT) is used to measure linear displacement or position. LVDT can be used in several applications as: automobiles' suspension [1], seismic shocks measurement [2], orthodontia [3], femoral prosthesis in medicine [4], deformations in concrete frames in civil engineering [5], position system of robotic arms [6], besides other physical measurements.
The basic structure of LVDT consists of a primary coil and two secondary coils like an electrical transformer. However, LVDT has a movable magnetic core that when is dislocated it varies the mutual inductances among secondary coils and the primary coil. In operation, it is applied an AC voltage, E_i in the primary coil, as is shown in Figure 1. The secondary coils are connected in series with opposing phase, so when the magnetic core is exactly at the physical center between them, then the output voltage, E_o, is zero, as the net output is the difference between the two secondary voltages, $E_o = E_{o1} - E_{o2}$. If the core is moved from the center, it is obtained an output voltage that is proportional to the core displacement, because there is a misbalance of magnetic flux intensities among the primary coil and the secondary coils and one secondary coil receives more flux than the other. This misbalance produces non-zero output voltage proportional to the core displacement [7].
On the other hand, mathematical models are very important tools for engineering and for previous developments. For LVDT, the determination of a model and the estimation of its parameters is an important way to develop new applications using this sensor. Also, when it is commercialized there is a dependence of the manufacturer with regard to the calibration and the precision of the sensor.

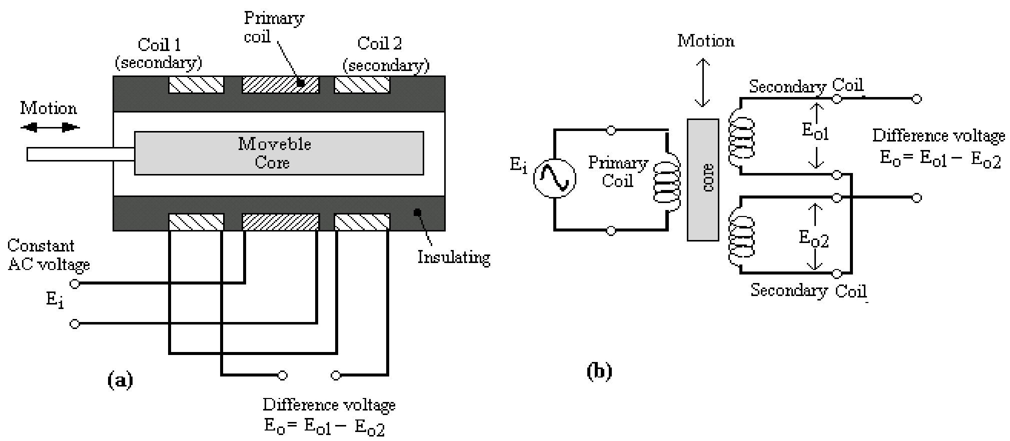

Figure 1. LVDT: (a) Internal Schematic. (b) Internal Model.

In general, LVDT is modeled in two ways: electrical-parameter-based model (EP) or geometrical-parameter-based model (GP). EP model uses electrical parameters as resistances and inductances, and GP model uses geometrical parameters based on physical dimensions of the LVDT. It is noticed that, some scientific works use EP model and others used GP ones. Thus, research results based in a certain model can become useless when only parameters from another model are known.
To consider the results of a model having parameters from other model, it is necessary to have tools to permute its parameters. In this way, it is presented a way of conversion from GP model to EP model in order to allow the interaction between this models and use results from one to other.

II. Electrical-Parameter-Based Model of LVDT

From Figure 2, an EP model of the LVDT can be obtained as follows, considering the following parameters. $\boldsymbol{R_p}$: total resistance of the primary coil, $\boldsymbol{R_{s1}}$ and $\boldsymbol{R_{s2}}$: resistances of the secondary coils, $\boldsymbol{L_P}$: self-inductance of the primary, $\boldsymbol{L_{s1}}$ and $\boldsymbol{L_{s2}}$: self inductances of the secondary, and $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$: mutual inductances between primary coil and the secondary coils [8] [9] [10]. Considering $\boldsymbol{R_s}$ as the total resistance of the secondary circuit, it is obtained $\boldsymbol{R_s = R_{s1} + R_{s2} + R_L}$ where $\boldsymbol{R_L}$ is a load resistance not showing. The induced voltage in the primary coil is given by:

$$E_i = R_p I_p + sL_p I_p + sM_1 I_s - sM_2 I_s \tag{1}$$

where M_1, M_2 are dependent of the core displacement and E_i, the excitation voltage. The net equation in the secondary circuit is given by:

$$0 = sM_1 I_p - sM_2 I_p + I_s R_s + sL_{s1} I_s + sL_{s2} I_s \tag{2}$$

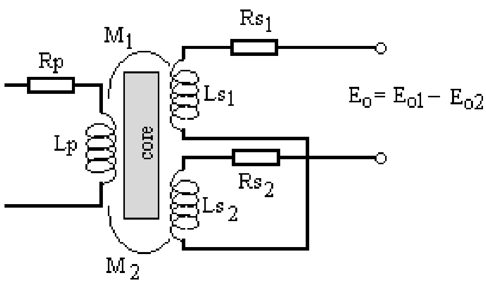

Figure 2. LVDT Electrical Schematic.

From (1), it is obtained

$$I_p(s) = \frac{E_i - sM_1 I_s + sM_2 I_s}{(R_p + sL_p)} \tag{3}$$

and from (2)

$$I_p(s) = \frac{-I_s(R_s + sL_{s1} + sL_{s2})}{s(M_1 - M_2)} \tag{4}$$

Comparing (3) and (4) and after some algebraic manipulations, it is obtained

$$I_s(s) = \frac{s(M_1 - M_2)E_i}{s^2 X + sY + R_p R_s} \tag{5}$$

where

$$X = L_p(L_{s1} + L_{s2}) - (M_1 - M_2)^2 \tag{6}$$

$$Y = L_1 R_s + R_p(L_{s1} + L_{s2}) \tag{7}$$

The output voltage, E_o, is given by

$$E_o(s) = I_s(s) R_L \tag{8}$$

Replacing (5) in (8), it is obtained

$$\frac{E_o}{E_i} = \frac{s(M_1 - M_2)R_L}{s^2 X + sY + R_p R_s} \tag{9}$$

This is a second-order system which indicates that the phase angle varies from +90º in low frequencies to –90º in high frequencies. When the core is in the physical center, $\boldsymbol{M_2 = M_1}$, and according to (9), $\boldsymbol{E_O = 0}$, as is expected.

III. Geometrical-Parameter-Based Model of LVDT

A general structure that shows the geometrical parameters (GP) of LVDT is shown in Figure 3(a) where: $\boldsymbol{h_1}$ is the primary coil length; $\boldsymbol{h_2}$ is the secondary coil length; $\boldsymbol{D}$ is the external diameter of the cylinder formed by the coils; $\boldsymbol{d}$ is the core diameter; $\boldsymbol{t_1}$ is the core length that is inserted in the secondary **coil 1**; $\boldsymbol{t_2}$ is the core length that is inserted in the secondary **coil 2**; $\boldsymbol{\delta_1}$ e $\boldsymbol{\delta_2}$ are the airgaps among secondary coils and the core; and $\boldsymbol{R}$ is the flux effective radius.

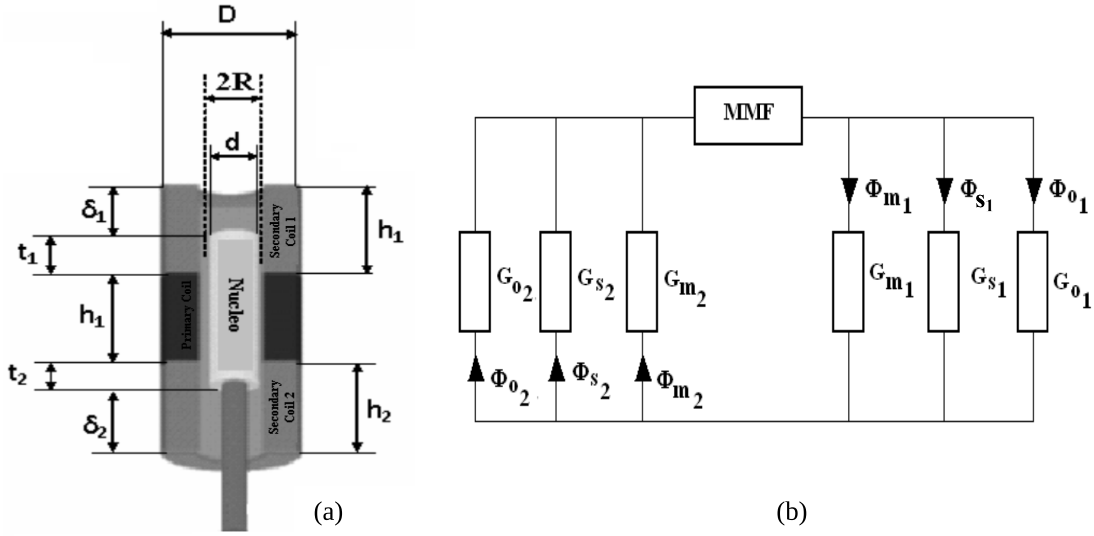

Figure 3. (a) LVDT Geometrical Schematic. (b) LVDT equivalent magnetic circuit.

In [11], it was used magnetic circuit theory to develop a math model that involves internal parameters like: numbers of coil turns, physical dimensions of the coils and material permeability of the core. This research has involved experimental procedures using known LVDT dimensions and comparing obtained theoretical calculation of output voltage and measured values. Using these dimensions, it is possible to calculate self-inductances and mutual inductances between primary coil and the secondary coils.

The equivalent magnetic circuit for an LVDT is illustrated in Figure 3(b) where ***MMF*** is the magnetomotive force and $\boldsymbol{G_{01}, G_{s1}, G_{m1}, G_{02}, G_{s2}, m_2}$ are magnetic conductances related to its respective magnetic fluxes: $\boldsymbol{\varphi_{o1}, \varphi_{s1}, \varphi_{m1}, \varphi_{o2}, \varphi_{s2}, \varphi_{m2}}$ where $\boldsymbol{\varphi_{o1}}$ is the main flux which goes to secondary 1; $\boldsymbol{\varphi_{s1}}$ is the secondary 1 sideflux; $\boldsymbol{\varphi_{m1}}$ is the main sideflux which crosses the secondary 1; $\boldsymbol{\varphi_{o2}}$ is the main flux which goes to secondary 2; $\boldsymbol{\varphi_{s2}}$ is the secondary 2 sideflux; and $\boldsymbol{\varphi_{m2}}$ is the main sideflux which crosses the secondary 2.

When $\boldsymbol{\delta_1}$ or $\boldsymbol{\delta_2}$ varies slightly due to a core displacement, $\boldsymbol{R}$ varies, and if the displacement is small, then $\boldsymbol{G_{01}}$ and $\boldsymbol{G_{02}}$ are not considered (because they are dependent of $\boldsymbol{\delta}$ and $\boldsymbol{R}$). Also, the flux $\boldsymbol{\varphi_m}$ does not cross the secondary coils and it is not able to produce mutual inductance. In this way, $\boldsymbol{G_{m1}}$ and $\boldsymbol{G_{m2}}$ are not considered as well [11]. So, only $\boldsymbol{G_{s1}}$ and $\boldsymbol{G_{s2}}$ are considered, which are the magnetic conductances crossed by the flux $\boldsymbol{\varphi_s}$, and they can be calculated, according to [11], by

$$G_{s1} = (h_2 - \delta_1)g = t_1 g \tag{10}$$

$$G_{s2} = (h_2 - \delta_2)g = t_2 g \tag{11}$$

where $\boldsymbol{g}$ is the specific magnetic conductance between cylindrical surfaces and is given by $2\mu_0\pi/ln(D/d)$ [12].

After compute the magnetic conductances, the self inductance of the primary coil L_P is given by

$$L_p = N_p^2 G \tag{12}$$

where N_p is the number of turns of the primary coil and G is the total magnetic conductance that is given by

$$G = G_{s1} // G_{s2} \tag{13}$$

The mutual inductances M_1 and M_2, crossed by the flux φ_m, among the primary coil and the secondary coils, can be calculated by

$$M_1 = N_p N_s g t_1^2 / (4h_1) \tag{14}$$

$$M_2 = N_p N_s g t_2^2 / (4h_2) \tag{15}$$

where N_s is the number of turns of the secondary coils.
According to [10], $E_o = \frac{(M_1 - M_2)E_i}{L_p}$. So, replacing (12), (14) and (15) in it, then E_o is given by

$$\frac{E_o}{E_i} = \frac{N_s g t_0 \Delta t}{h_2 N_p G} \tag{16}$$

IV. Geometrical- to Electrical-Parameter-Based Model Conversion

A relationship between electrical and geometrical parameters in a direct way is presented as follows. Considering as ideal the resistances of primary and secondary coils, i.e. $R_P = R_S = 0$, in frequency domain (replacing *s* by *iw*,), from (9) it is obtained

$$\frac{E_o}{E_i} = \frac{i\Delta M R_L}{-w(\Delta M^2 + L_p L_S) + iL_p R_L} \tag{17}$$

where $L_s = L_{s1} + L_{s2}$. The magnitude of (17) is given by

$$\left|\frac{E_o}{E_i}\right| = \frac{\Delta M R_L}{\sqrt{(w^2(\Delta M^4 + 2\Delta M^2 L_p L_s + L_p{}^2 L_s{}^2) + L_p{}^2 R_L^2}} \tag{18}$$

resulting in

$$\left|\frac{E_o}{E_i}\right| = \frac{\Delta M R_L}{wL_p \sqrt{\frac{\Delta M^4}{L_p^2} + \frac{2\Delta M^2 L_s}{L_p^2} + L_s + \frac{R_L^2}{w^2}}} \tag{19}$$

Denoting $Z = \frac{\Delta M^4}{L_p^2} + \frac{2\Delta M^2 L_s}{L_p^2} + L_s$, it is obtained

$$\left|\frac{E_o}{E_i}\right| = \frac{\Delta M}{L_p w \sqrt{\frac{1}{R_L^2}(Z + \frac{R_L^2}{w^2})}} \tag{20}$$

and

$$\left|\frac{E_o}{E_i}\right| = \frac{\Delta M}{L_p \sqrt{\frac{wZ}{R_L^2} + 1}} \tag{21}$$

In order to obtain a relation independent of the load resistance, it is done $R_L \rightarrow \infty$, then $\sqrt{wZ/R_L^2+1} \rightarrow 1$, then

$$\left|\frac{E_o}{E_i}\right| = \frac{\Delta M}{L_p} = \frac{M_1 - M_2}{L_p} \tag{22}$$

Hence, replacing (12), (14) and (15) in (22), it is obtained

$$\left|\frac{E_o}{E_i}\right| = \frac{N_p N_s (t_1^2 - t_2^2)}{4h_2 L_p} \tag{23}$$

As $L_P = N_P^2 G$, so

$$\frac{E_o}{E_i} = \frac{N_s g(t_1^2 - t_2^2)}{4h_2 N_p G} \tag{24}$$

Due to the LVDT symmetrical structure: $t_1 = t_0 + \Delta t$ e $t_2 = t_0 - \Delta t$, where t_0 is the length of the magnetic core inserted in the secondary coils, when the core is put at the LVDT center, and Δt is its displacement. In this way, it is obtained

$$\frac{E_o}{E_i} = \frac{N_s g}{4h_2 N_p G}[t_0^2 + 2t_0\Delta t + \Delta t^2 - t_0^2 + 2t_0\Delta t - \Delta t^2]$$

$$\frac{E_o}{E_i} = \frac{N_s g}{4h_2 N_p G}(4t_0\Delta t) = \frac{N_s g t_0 \Delta t}{N_p h_2 G}$$

which is the same as (16).

Denoting $k = \frac{N_2 g t_0}{N_1 h_2 G}$, then

$$\frac{E_o}{E_i} = K\Delta t \rightarrow E_o = KE_i\Delta t \rightarrow E_o = C\Delta t \tag{25}$$

where $C = KE_i$ and as E_i is considered constant, it shows the linearity of the transducer.

Considering at balance, $\Delta t = 0$, and knowing that $M_1 = \sqrt{L_p L_{s1}}$ and $M_2 = \sqrt{L_p L_{s2}}$ that comes from theory of coupled magnetic circuits between primary coil and the secondary coils, it is obtained

$$\frac{M_1^2}{L_p} = L_{s1} \tag{26}$$

and

$$\frac{M_2^2}{L_p} = L_{s2} \tag{27}$$

Replacing (12) and (14) in (26), it is obtained

$$L_{s1} = \left(\frac{N_s}{h_2}\right)^2 \frac{g(t_1 + t_2)t_1^3}{16t_2} \tag{28}$$

In the same way, replacing (12) and (15) in (27), it is obtained

$$L_{s2} = \left(\frac{N_s}{h_2}\right)^2 \frac{g(t_1 + t_2)t_2^3}{16t_1} \tag{29}$$

V. Results

As a result, according to (12), (14), (15), (28) and (29), it is possible to compute the electrical parameters from geometrical ones by using the conversion formulas given in Table 1.

Primary Inductance	Mutual Inductance 1	Mutual Inductance 1
$L_p = N_p^2 G$	$M_1 = \frac{N_p N_s g t_1^2}{4h_2}$	$M_2 = \frac{N_p N_s g t_2^2}{4h_2}$
Secondary Inductance 1	Secondary Inductance 2	
$L_{s1} = \left(\frac{N_s}{h_2}\right)^2 \frac{g(t_1 + t_2)t_1^3}{16t_2}$	$L_{s2} = \left(\frac{N_s}{h_2}\right)^2 \frac{g(t_1 + t_2)t_2^3}{16t_1}$	

Table 1. Conversion formulas from GP to EP.

Example: considering a real LVDT with geometrical parameters given by d = 3mm, D = 9mm, $h_1 = h_2$ = 4mm, $t_1 = t_2$ = 2mm (core in the initial position) e $N_P = N_S$ = 1000. Calculating g = $2\mu_0\pi/ln(D/d)$ and G, see Eq.13, it is computed the following electrical parameters

L_P	M_1	M_2	L_{S1}	L_{S2}
7,1mH	1,8mH	1,8mH	0,2mH	0,2mH

VI. Conclusions

In this work was described the models based on electrical parameters (EP) and on geometrical parameters of LVDT. It was presented a way of conversion from GP model to EP model in order to allow the interaction between both models and use results from one to other. Using the conversion formulas, research results based in a certain model can be used when only parameters from another model are known.

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