On the Uncertainty of Sensors Based on Magnetic Effects

E. Hristoforou, E. Kayafas, A. Ktena, DM Kepaptsoglou

National Technical University of Athens, Zografou Campus, Athens 15780, Greece
Tel: +302107722178, Fax: +302107722119, e-mail: eh@metal.ntua.gr

Abstract- In this paper we illustrate that magnetic hysteresis affects the uncertainty of sensors based on magnetic phenomena and materials. Testing various magnetic techniques, we conclude that sensor uncertainty can be optimized by both, the optimization of magnetic hysteresis and the operation in the magnetically reversible area of the material. Preliminary modelling results are in agreement with our experimental findings.

I. Introduction

Magnetic sensors are important in many fields of engineering [1]. They can be mainly used in the measurement of physical sizes, such as position, stress and field [2]. Their characteristics and range of measurement make them attractive for applications where other kinds of sensors cannot operate. They can be manufactured using hybrid or micromechanical techniques and can be easily packaged in miniaturized set-ups. Many of them have the intrinsic property of self-calibration, allowing accurate and repeatable measurements.

The most significant property of a sensor is its uncertainty of measurement, which is determined as the total deviation of the measured value from the near true value of measurement. The effort of all R&D groups working in the field of sensor development is mainly targeted towards the optimization of such uncertainty.

The optimization of the magnetic sensors response with respect to hysteresis has been our motivation for this work. In this paper we briefly present our theory on the sensor uncertainty optimization, based on the optimization of the magnetic hysteresis of the used material. We apply this theory to the calibration of various kinds of magnetic sensors, proving that the control of magnetic hysteresis is essential for the sensor uncertainty.

The principle of operation of magnetic sensors lies in the dependence on the applied field H of the magnetic flux density B, the magnetostriction λ, and the magneto-impedance Z, yielding the B-H loop, the λ-H loop and the Z-H loop respectively. The Z-H loop response corresponds to the dc and ac resistance response, *i.e.* the MR and MI response.

Considering a sensor based on the B-H loop response, *e.g.* a linear variable differential transformer (LVDT), two parameters are of importance: the dc biasing point and the ac periodic excitation. Usually, such sensor excitation is fixed in amplitude, while change of the dc biasing point results in a change of the sensor response, realized as a dynamic change of magnetization with respect to time. When operating in the irreversible area of magnetization, such dynamic change of magnetization is larger but hysteretic. In the reversible area of the B-H loop, hysteresis is practically absent and sensitivity is considerably decreased.

The same response can be observed in sensors based on the λ-H loop, as well as in sensors based on the Z-H loop, like the magneto-inductive sensors. From the description of these three main magnetic effects, it can be seen that magnetic hysteresis determines the hysteresis of a sensor.

The sensor uncertainty is a function of both the sensor sensitivity and the sensor hysteresis. Higher sensitivity reduces the uncertainty. However, both our theoretical and experimental results agree that higher sensitivity is achieved when operating in the hysteretic part of the curves against operating in the reversible anhysteretic region but it is also there that the uncertainty is higher due to hysteresis. Hence, good sensor design should take into account the hysteresis response of the material and either use it or avoid it.

II. Experimental

We have developed several kinds of sensors based on magnetic materials, allowing for all three kinds of magnetic responses presented in the previous section. For illustration purposes we shall provide the results of three kinds of sensors, using correspondingly the three above-mentioned different types of magnetic response. Each type of sensor was based on three different magnetic materials, performing at different magnetic hysteresis levels.

The first type of sensor was an LVDT [3], performing displacement measurements. The cores used to realize this sensor were polycrystalline Fe and Ni wires as well as amorphous field annealed FeSiB wires. The sensor operated in both the irreversible and reversible area of the B-H loop, resulting in the response shown in Fig. 1, illustrating results of Fe wires. From these results it can be seen that the sensor hysteresis is negligible for the case of the reversible area of operation of the material. In the irreversible area the signal is significantly enhanced but so is the hysteresis. For example, at 6mm displacement the signal is 10mV in the case of the reversible area and 21mV or 25mV in the irreversible. So, the signal is doubled but exhibits a ±7% deviation. It is noted that the hysteresis of the sensor using FeSiB wires was negligible in both cases of reversible and irreversible operation.

The second type of sensor was a magnetostrictive delay line (MDL) sensor [4], based on the λ-H loop, performing stress measurements. The same cores were used to realize this sensor, also operating in the reversible and irreversible areas of the λ-H loop. The response of the Fe and FeSiB wire, operating in the irreversible and hysteretic region of the λ-H loop, is shown in Fig. 2. It can be seen that hysteresis is smaller, almost negligible in the case of the FeSiB wire. Hysteresis in the sensor using Fe wire contributes ±10% uncertainty.

The third type of sensor was a magneto-inductive field sensor [5], also using Fe and amorphous FeSiB wires. The response of the Fe and FeSiB wire, operating in the irreversible and hysteretic region of the Z-H loop, is shown in Fig. 3. In this case also it is clear that hysteresis in the sensor using FeSiB is negligible. We note here, that in the case of the FeSiB magneto-inductive sensor the signal was amplified by 500 and the offset was deducted.

Following the above experimental results we have developed a macroscopic mathematical model of hysteresis describing the sensor sensitivity dependence on hysteresis.

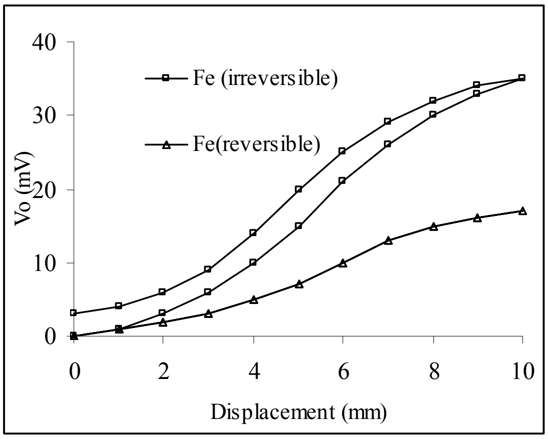

Fig. 1. Hysteretic response of LVDT, using Fe sensing cores, operating in the reversible and irreversible area of magnetization.

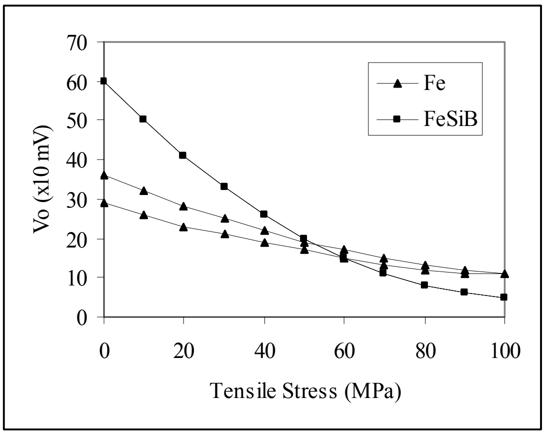

Fig. 2. Tensile stress hysteretic dependence of MDLs under tensile stress.

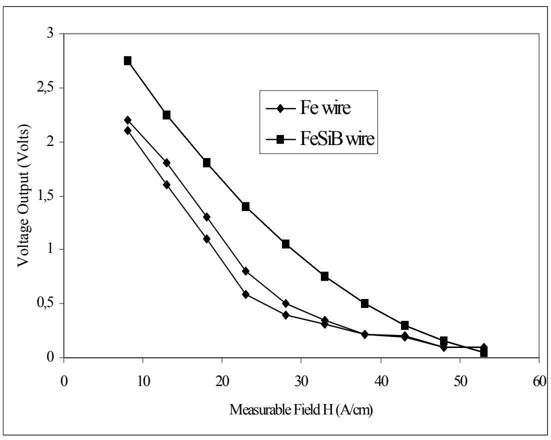

Fig. 3. Response of magneto-inductive sensors operating in the irreversible area.

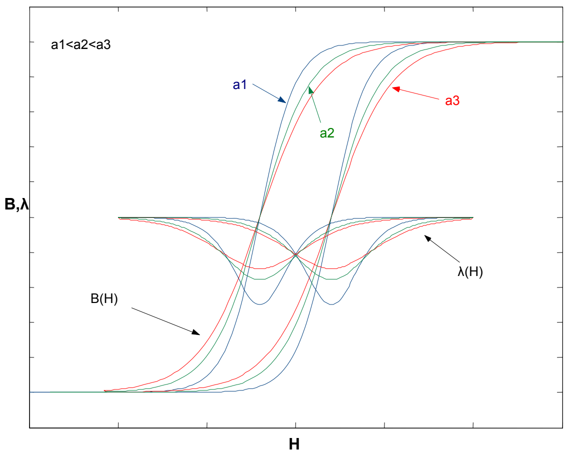

Fig 4. Calculated M-H and λ-H curves for a1<a2<a3.

III. The model

The phenomenological approach to the modeling of the M-H, λ-H and Z-H curves is based on the gaussian probability density function, $f(x) \propto e^{-x^2}$. The M-H curve can be modeled by the integral of f(x) while the other two curves by f(x) itself. To avoid numerical difficulties arising from the calculation of the integral of f(x) we use the "sigma" probability density function instead:

$$\sigma(x) = \frac{1}{1+e^{-x}}, -\infty < x < +\infty \qquad (1)$$

Its sigmoidal shape is appropriate for the modeling of the M-H loop:

$$M(H) = 2\sigma\left(\frac{H \pm H_c}{a}\right) - 1 = \frac{2}{1+e^{-\left(\frac{H \pm Hc}{a}\right)}} - 1 \qquad (2)$$

where H is the applied field, H_c is the coercive field defined as $M(H_c) = 0$, and a is a field related to the coercivity squareness of the loop. The plus or minus sign indicates the ascending or descending branch, respectively. Using the assumption that the minor loops of a given M-H loop belong to the same family of curves, eq. (2) can be parameterized and used to reproduce higher order curves as well. We have determined that

$$\lambda(H) = \lambda_s\left(1 - e^{\left(\frac{H \pm H_c}{a}\right)^2}\right)$$

which can be approximated by the first derivative of the sigma function:

$$\lambda(H) = \frac{1}{a}\sigma\left(\frac{H \pm H_c}{a}\right)\left[\sigma\left(\frac{H \pm H_c}{a}\right) - 1\right] \qquad (3)$$

Accordingly, the Z-H curve is given by the following Gaussian equation:

$$Z(H) = Z_{max} e^{\left(\frac{H \pm H_c}{a}\right)^2}$$

which can be approximated by:

$$Z(H) = \frac{2}{a}\sigma\left(\frac{H \pm H_c}{a}\right)\left[1 - \sigma\left(\frac{H \pm H_c}{a}\right)\right] \qquad (4)$$

Fig. 4 shows M-H and λ-H loops produced by (2) and (3) for different a-values. Fig. 5 shows a set of minor ascending curves for a given M-H curve. Similar minor loops can be obtained for the other two curves as well. The response of the sensors based on these three different loops may be calculated as

$$V \propto \frac{dB}{dt}, \text{ for the LVDT} \qquad (5)$$

$$V \propto \frac{d^2\lambda}{dt^2}, \text{ for the MDL} \quad \text{and} \qquad (6)$$

$$V \propto H, \text{ for the MI sensor} \qquad (7)$$

where V is the induced voltage and B the magnetic induction,

$$B = \mu_0 (H + M). \qquad (8)$$

Combining (2) and (8), for a sinusoidal excitation

$$H(t) = H_{dc} + H_{ac} \sin \omega t \qquad (9)$$

eq. (5) yields the response of the LVDT:

$$\left|\frac{dB}{dt}\right| = \frac{\mu_0 H_{ac}}{\omega}\left[1 - \frac{2}{a}\sigma(\cdot)[\sigma(\cdot) - 1]\right] \qquad (10)$$

where:

$$\sigma(\cdot) = \sigma\left(\frac{H_{dc} + H_{ac} \sin \omega t \pm H_c}{a}\right) \qquad (11)$$

We note that the induced voltage is proportional to the magnitude of the ac excitation and inversely proportional to the frequency ω=2πf. Depending on the region of operation on the M-H loop, the following three cases arise:

- Case I: $H_{dc} >> H_c$ and $H_{ac} << H_c$.

This case corresponds to operation in the reversible part of the curve, near saturation and eq. (10) yields, $\left|\frac{dB}{dt}\right| \to \frac{\mu_0 H_{ac}}{\omega}$.

The induced voltage is apparently small since ac-field is small.

- Case II: $H_{dc} \approx H_c$ and $H_{ac} < H_c + a$.

This case corresponds to operation in the highly hysteretic part of the M-H curve. The condition $H_{ac} < H_c + a$ ensures that the operation is limited to the approximately linear part of the curve and saturation is avoided. Because $\max\{2\sigma(\sigma - 1)\} = -0.5$,

$$\max\left\{\frac{dB}{dt}\right\} = \frac{\mu_0 H_{ac}}{\omega}\left[1 + \frac{0.5}{a}\right],$$

i.e. the sensitivity of the sensor is increased by a factor of $\frac{0.5}{a}$. The dependence on parameter a is two-fold: small a suggests steep loops and therefore dictates the application of a small H_{ac} while, on the other hand, the factor $\frac{0.5}{a}$ is increased. In any case, the sensitivity is higher than in Case I. We also note that the sin*ωt* dependence of σ(·), because of the prominent hysteresis effect, contributes to the uncertainty.

- Case III: $H_{ac} >> H_c$.

This case corresponds to operation between positive and negative saturation and

$$\left|\frac{dB}{dt}\right| \to \frac{\mu_0 H_{ac}}{\omega}.$$

The result is the same as in Case I but H_{ac} is now much larger and hence the response is enhanced. The same analysis has been carried out for the other two types of sensors with similar results. The response of the MDL sensor as given by (6) is illustrated in Fig. 6 where the first and second time derivative of $\lambda(H)$ is approximated by the first and second derivative of (3), respectively. The above analysis agrees with the experimental results presented in the previous section.

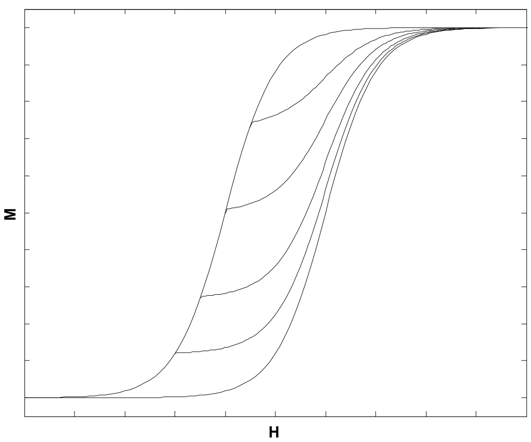

Fig 5. Calculated M-H loop with a set of minor ascending curves.

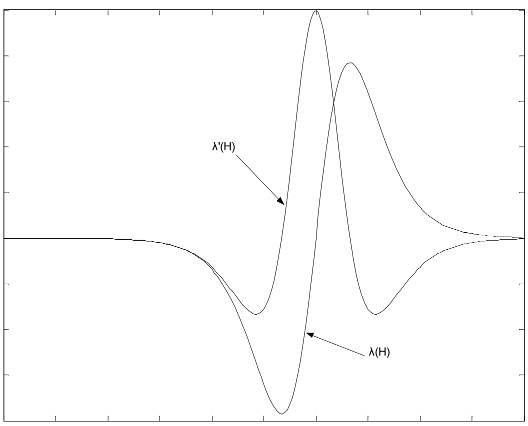

Fig. 6. The first and second time derivative of $\lambda(H)$

References

[1] R. Boll and L. Borek, “Magnetic Sensors of New Materials”, Siemens Forch.-u. Entwickl.-Ber.Bd. 10, p. 83-90, 1981.
[2] E.O. Doebelin, “Measurement Systems: applicatons and design.” Fourth Edition. McCraw-Hill, 1990.
[3] H. Chiriac, E. Hristoforou, M. Neagu, M. Pertanarian, F.J. Castano, J. Appl. Phys., 87, p. 5344-5346, 2000.
[4] E. Hristoforou, H. Chiriac, M. Neagu and I. Darie, Sensors and Actuators A, 67, p. 307-315, 1998.
[5] K. Mohri, K. Kasai, T. Kondo, H. Fujiwara, and M. Masumoto, IEEE Trans. Magn., 17, p. 3053-3058 1981.