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Linking the kilogram to Electrical Quantities by the Energy Dissipated in a Pendulum

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Abstract- The development of system for linking the kilogram to electrical quantities is described in this paper. The method is based on a multifilar pendulum interacting with a coil, where a variable electromotive force that produces a current in the coil and in a suitable external resistor is generated. The electrical energy dissipated is measured and the mass is evaluated from this energy and from the kinematics of the pendulum.

I. Introduction

Monitoring the kilogram and measuring the related fundamental constants (Planck, Avogadro and Faraday constants) has assumed a great interest for the probable changes in the SI system. In fact, the last GCPM recommended that "National Metrology Institutes and the BIPM pursue the relevant experiments so that the International Committee can come to a view on whether it may be possible to redefine the kilogram, the ampere, the kelvin, and the mole using fixed values of the fundamental constants at the time of the 24th General Conference (2011)" [1]. Different systems for the measurement of the kilogram from electrical quantities have been utilized. In the last years, there have been two different approaches, based respectively on a levitation mass [2] [3] or on the so called "watt balance". The last method for the electrical determination of the kilogram, in particular, can be generated either by the magnetic interaction, which is the prevalent in technical literature [4], [5], [6], [7], or by the electrostatic one [8].
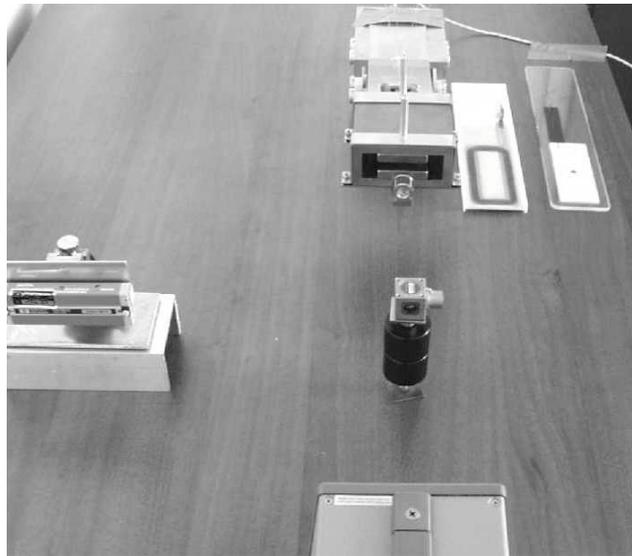

Figure 1. Picture of the prototype of the pendulum system for the determination of the kilogram from electrical quantities.

Recently, a pendulum method has been proposed at I.N.RI.M. The related theory has been explained in [9] and it can be employed to realize different systems, all having a pendulum or some other mechanical oscillators interconnected with an electromechanical system, which can be either of electrometric or of magnetic type. A prototype implementing this method has been set up [10], aimed at

showing the feasibility of the method and at acquiring the necessary know-how for realizing further experiments for high precision measurements.

In this prototype the mobile assembly is built with two neodymium magnets enclosed in a magnetic circuit, which has the purpose of reducing to a negligible amount the magnetic field outside the moving assembly. In this way, a strong static magnetic field is instead produced only in the gap between the two magnets, where a suitable coil can interact. The moving assembly is suspended as a pendulum by two couples of strings in Kevlar, attached to two stems that maintain the center of mass under the suspension points. For each couple, the strings are joined in one end to two points at a distance of about 500 mm, on a mechanical support located about 2 m above the moving assembly, while the other end is joined to each of the stems. In the hypothesis of stable length, each couple compels the attached point to oscillate in a plane and, if the two couples are parallel, the moving assembly will also be forced to move in this well defined plane, maintaining the same dynamic of a simple pendulum.

In the prototype the moving assembly oscillates with the gap between the two magnets, where the magnetic field is stronger, kept horizontal and the interaction is produced by a coil situated in the gap between the two magnets. The coil links a magnetic flux dependent on the position of the moving assembly along the moving direction. The position of the moving assembly is detected by a corner cube reflector rigidly joined to it. A laser interferometer provides the measurement of the position as a time function.

II. Measurements on the prototype

Preliminary measurements have been made in the normal environment of the laboratory. The position of the moving assembly has been measured by a laser interferometer and both the electromotive force and the current in the coils have been measured by a precision sampling voltmeter and a shunt. The instability of the position of the moving assembly at rest, due to the vibrations and the circulation of air, was about 4 μm (1 standard deviation). The Q of the pendulum is about 600 and the period is about 2.78 s. The position as function of time differs from an ideal cisoid by a residual rms error less than 5 parts in 10^5. No detectable difference has been observed between the conditions with and without the coil when it is kept open. With a coil of about 200 turns the maximum value of the electromotive force is about 150 mV for the displacement of 20 mm which requires a current of about 92 mA to be produced.

The method proposed in [10] requires two subsequent measurements, and is based on the reciprocal interaction between the electrical system and the mechanical system. The first measurement is performed by supplying constant of currents I_j to the coil and measuring the position x_j at rest. So, a characteristic $x(I)$ is obtained. In the second measurement, the pendulum oscillates freely and the electromotive force of the coil $e(t)$ and the position $x(t)$ are measured. By processing $x(I)$ $e(t)$ and $x(t)$ the equivalent mass of the pendulum M is evaluated. In the hypothesis of linear harmonic oscillator with a sinusoidal signal the equivalent mass of the moving assembly M is:

$$M = \left(\frac{T}{2\pi}\right)^2 \cdot \left|\frac{e/v}{x/I}\right| \tag{1}$$

where T is the oscillation period, e/v the ratio between the electromotive force and the velocity and x/I the ratio between the displacement and the current supplied (in the hypothesis of linearity both these ratios are constant).

The limits in the determination of the mass are mainly due to the e/v ratio, which is measured with variable signals. A repeatability test for the determination of the ratio has been performed at each half period, where the ratio between the maximum value of the electromotive force and the velocity has been computed. In each set the standard deviation of the mean value is at the level of 3 parts in 10^6.

III. A new approach based on the dissipated energy

With little modifications of the prototype shown in Figure 1 a new set of measurements is now being organized for measurements based on the generation of an electric signal dissipated energy on a resistor. This method takes into account the mechanical energy of the moving assembly and its gradual transformation into electrical energy in the external coil.

In the pendulum, which is almost a harmonic oscillator, the restoring force as a function of x is given by:

$$F(x) = -k_0 \cdot (1 - f(x)) \cdot x \tag{2}$$

where $f(x) << 1$ is a function of the order of the square of the ratio between the displacement and the length of the strings and can be evaluated approximately by modeling the pendulum behavior.

Considering the energy of the pendulum starting from an initial potential energy E_{p0} the mechanical energy of the moving assembly as a time function will be a combination of the kinetic and potential energy.

$$E(t) = E_{p0} - \int_0^t \frac{\partial E}{\partial t}(\dot{x}, x, t) \cdot dt = \left| k_0 \int_0^x (1 - f(x)) \cdot x(t) \cdot dx \right| + \frac{1}{2} \cdot M \cdot \dot{x}(t)^2 + \Delta E_k(x(t), \dot{x}(t)) \tag{3}$$

where M is the equivalent mass of the pendulum, ΔE_k is a correcting term for the kinetic energy (motion components different from x). The mechanical energy of the pendulum $E(t)$ can be evaluated at specific times t_i in term of the equivalent mass when the kinetic energy is at a relative maximum (every half a period). At these times the pendulum has a maximum value of the velocity and the energy is;

$$E(t_i) = \frac{1}{2} \cdot M \cdot \dot{x}(t_i)^2 + \Delta E_k(x(t_i), \dot{x}(t_i)) \tag{4}$$

So, when the pendulum is freely oscillating the energy lost between the initial time t_{in} and the final time t_{fin} will be:

$$\int_{t_{tin}}^{t_{fin}} \frac{\partial E}{\partial t}(\dot{x}, x, t) \cdot dt \tag{5}$$

which is different when the coil is open or closed on a known resistance.

For the first case, especially if the Q is high, it can be evaluated with a good approximation by both experiments as a model estimation. When, instead, the coil is closed on a known resistor, an additional energy is lost, equal to the electrical energy produced on the coil and the resistor, which is given by:

$$\int_{t_{in}}^{t_{fin}} (R_{ext} + R_{coil}) \left(\frac{v(t)}{R_{ext}} \right)^2 \cdot dt \tag{6}$$

R_{ext} is the external shunt connected where the voltage is measured and R_{coil} is the coil resistance.

Consequently, by evaluating the energy lost given by (5) and the electrical energy produced and dissipated with relation (6) it is possible to derive the variation of the kinetic energy of the moving assembly and then its inertial mass M.

The measurement of the equivalent mass also in this case can be made in two steps.

- In the first step the pendulum oscillates freely and the position is recorded as a function of the time. The natural energy loss due to all contribution apart from the electric generation is derived by the measurement of the relative reduction of the amplitude in each period.

- In the second step, the position as a function of the time is recorded and at a given time t_{in}, with velocity near to zero, the coil is closed on the shunt and the voltage on it is recorded. The energy transformed into electrical energy is evaluated by (6) while the energy loss due to other sources is evaluated by applying the losses as function of the amplitude derived in the first step.

The sum of these energies is the energy of the pendulum at the time t_{in}, which can be derived from the kinetic energy and so related directly to the inertial equivalent mass of the pendulum.

Then, the mass of the mobile assembly is evaluated from the equivalent mass M by taking into account the mass of the strings, which can be between two or three orders of magnitude less. However, in order to avoid disconnecting the strings and evaluating their contribution to the equivalent mass, the measurement can be made by a differential method. A proper additional mass (M_A) is added to the

oscillating assembly, a new measurement process is performed and a second equivalent mass (M_2) is derived. The additional mass will be then:

$$M_A = M_2 - M \qquad (7)$$

This new approach, based on the energy dissipated on a resistor, avoids the critical requirements of the previous method:
- the stability of the system characteristics during the two experiments for determining the mass,
- the precise alignment of the coil, and the symmetry of the magnetic field.

This method overcomes such difficulties even if the circulation of a current in the coil produces a small additional loss which has to be taken into account, evaluated and corrected.

A manganine 30x100 mm 100 turns coil has been built, in order to have a constant value of resistance. Its resistance is about 90 Ω and with a 100 Ω shunt in series and a maximum value of the oscillation of 25 mm the voltage on the resistor is about 40 mV.

A measurement campaign is now being performed to correlate the mechanical energy change of the pendulum with the energy produced and dissipated in the coil and the resistor. The level of accuracy will probably be not impressive (the goal is an accuracy at the level of 1 part in 10^5), because the system is not optimized for this purpose. However, even this level of accuracy will give further indications for evaluating the feasibility of the method and for supporting a project for the measurement of the kilogram referred to the electrical quantities based on it.

III. Difference between the two methods

The method based on relation (1) and the method described in Chapter 2 have different characteristics and are based on different assumptions. The first method is based on the equivalence between the static and the dynamic characteristics, which require repeatable mechanical constrains and correct positioning of the moving assembly, but the power measured is virtual. In such a way, the dissipated power does not need to be taken into account with high precision. Instead, in the method proposed here, which is based on transformation of the real energy from mechanical to electrical, any specific hypothesis on the trajectory of the moving assembly is not required and its stability is not a necessary condition. On the other hand, every type of energy lost must be considered carefully in the budget.

For this reason, a high Q of the pendulum is necessary, because it means very low energy losses.

Figure 2 shows the variation of the amplitude as a function of the amplitude of the oscillation in a measurement. As it can be seen, for the instability of the measurement in air, the identification of the energy losses is possible only in the limit of about 2 parts in 10^6 every cycle. In fact, the standard deviation around the second order fitting curve, in the peak amplitude range from 15 mm to 25 mm where the energy contribution is higher, is about 4 parts in 10^5 with about 300 measurements points.

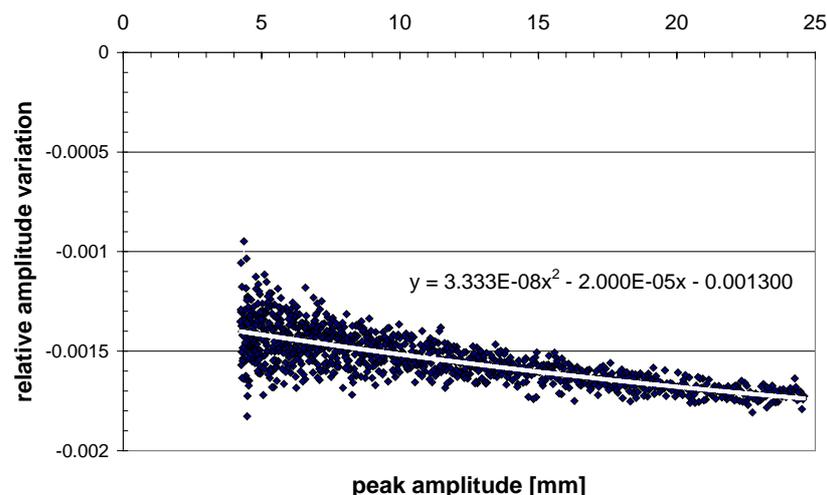$$y = 3.333\text{E-}08x^2 - 2.000\text{E-}05x - 0.001300$$

Figure 2. Measurement of the relative variation of the amplitude in each period of the pendulum as a function of the peak value of the amplitude, The equation on the graph is that of the fitting second order function which minimize the sum of the square of the residuals.

IV. Conclusion and future work

From a first prototype of a pendulum system for deriving the kilogram from the electrical quantities a new method for deriving the kilogram has been developed. In such a method only the general conservation energy principles are required. With some suitable modifications of the prototype, this new method can be used to derive very accurate measurement of the kilogram in term of electrical quantities and, consequently, to make the accurate measurements of the Planck constant. The implementation described here is not suitable, at the moment, for metrological measurements, but it is very encouraging. In a next step the measurements will be made on an anti-vibrating platform and in an evacuated container. Further developments using superconductive coils and a Josephson voltmeter are also now under investigation for a realization aiming at an accuracy better than 1 part in 10^7.

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