Design of an efficient AC magnetohydrodynamic stirrer

A. Affanni[1], G. Chiorboli[1]

*[1] Dipartimento di Ingegneria dell'Informazione, University of Parma,
parco Area delle Scienze, 181/A – 43100 Parma, Italy
Phone: +39 0521 905809, Fax: +39 0521 905822,
E–mail: antonio.affanni@unipr.it, giovanni.chiorboli@unipr.it .*

Abstract-– Rapid mixing of two or more analytes in microchannel networks is essential in the design of most biochemical, immunoassays and DNA analysis microsystems. In this paper we report numerical simulations of a chaotic magneto-hydrodynamic (MHD) stirrer that exhibits fast mixing of steady pressure-driven flows in microchannels at medium-low Reynolds number. The proposed mixer is benchmarked against a MHD stirrer prototype recently reported in literature. Numerical results show that the proposed device has a higher and faster mixing efficiency with respect to recent literature.

I. Introduction

Microfluidic technology, allowing fast analysis, integration of multiple processing steps, and the requirement for very low sample and reagent volumes in the order of microliter to nanoliter, is becoming one of the most rapidly growing supporting technologies for innovation in analytical biosciences [1-3] . Integration of micro-total analytical systems (µTAS) in often inexpensive and disposable lab-on-a-chip is changing, in terms of functionality, time to result, and cost effectiveness, the life sciences industry. For instance, diagnosis low-cost microfluidic chips have been designed that, thanks to sensitive, rapid, and accurate genetic analysis, allow analysing nanoliter-size DNA samples [4] and could facilitate customized therapies tailored to match the vulnerabilities of any types of cancer [5]. Most microfluidic systems have several technical steps in common, including precise volume measurement of reagent, enzyme, and DNA-template solutions, mixing of solutions, controlled thermal reaction of the mixture, loading of the reaction products onto a microdevice, often followed by separation steps, and finally a way to detect, quantify, and collect the final product. In those components, a micromixer is essential and important for the homogenization of the reagents used in the chemical reactions. Mixing of two or more reagents in a reasonable amount of time is essential for many microfluidic systems used in biochemical and DNA analysis and for microreactors that involve complex chemical synthesis.

However, in most microfluidic systems, flow viscosity dominates over inertia and the Reynolds number is low (Re = UL/ν, where U is the average flow speed, L is the characteristic length of the channel, and ν is the kinematic viscosity of the fluid). Since, usually, $0.01 < Re < 100$, well below the critical Reynolds number that indicates the transition between laminar and turbulent flow, the rapid mixing process produced by turbulence is usually not available at the microscale due to the extreme weakness of inertial forces, and the flow is extremely laminar. Thus, some other mechanism, such as diffusion or chaotic advection must be employed to enhance mixing. If the mixing is dominated by diffusion, and the diffusion coefficient is of the order of 10^{-10} m^2/s, such as for analyte solutions containing DNA and proteins, complete and homogeneous mixing of fluids might take very long time, requiring large channel lengths, on the order of tens of centimetres, too long for many lab-on-chip applications. A useful solution in order to enhance diffusive micro-mixing is to generate non-axial flow by applying external forces, e.g. dielectrophoretic, magnetophoretic, electroosmotic, thermal fields or magnetohydrodynamic.

In this paper the FEM simulation of a magnetohydrodynamic AC driven stirrer is presented, in section II the equations system is discussed, in section III the state of the art for MHD stirrers is introduced, in section IV the novel simulated geometry is described and in section V the simulations results are shown with particular attention to the stirring efficiency.

II. Theory of operation

The theory of operation of MHD stirrers involves many equation systems because the MHD body force depends on electromagnetic equations; then the MHD body force gives rise to a pressure gradient to be solved via Navier-Stokes equation and finally the obtained velocity field can provide the concentration rate once the convective-diffusive problem is solved.

A. MHD fluid motion: the Navier-Stokes problem

Magnetohydrodynamic fluid motion is based on the Lorentz force and the equations system joints the fluid mechanics and electromagnetism problems. The interaction between a magnetic field and a non-aligned electric current produces a force orthogonal to both electromagnetic vectors.
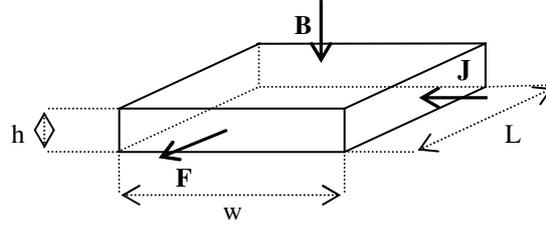

Figure 1. Scheme of a MHD microchannel

For the rectangular channel of Figure 1, where the magnetic field B is oriented along the z axis and the electric current flows in the y direction through an incompressible conductive fluid with isotropic properties, the Lorentz force per unit of volume is

$$\bar{f}_{MHD} = \bar{J} \times \bar{B}$$
$$\left(f_x = J_y B_z \right)$$

(1)

The force described in (1) gives rise to a pressure proportional to the pumping element length L

$$P = \int_0^L \bar{f} \cdot dx = \int_0^L (\bar{J} \times \bar{B}) \cdot dx = J_y B_z L = \frac{IB_z}{h}$$

(2)

where I is the electric current and h the channel depth. The Finite Elements simulator solves the Navier-Stokes equation

$$\rho(\frac{d\bar{u}}{dt} + \bar{u} \cdot \nabla \bar{u}) = -\nabla P + \bar{f}_{MHD} + \mu \nabla^2 \bar{u}$$

(3)

being ρ the mass density, P the pressure, u the fluid speed and μ the dynamic viscosity of the fluid; Navier-Stokes equation coupled with continuity equation

$$\nabla \cdot \bar{u} = 0$$

(4)

is integrated on each element and meshed over the entire geometry of the microchannel.

B. Mixing equations: the convective-diffusive problem

The finite elements simulator must solve the convective-diffusive equation

$$\frac{dc}{dt} + \bar{u} \cdot \nabla c = D\nabla^2 c$$

(5)

where c is the molar concentration and D the diffusivity. In microfluidic devices the Peclet number

$$Pe = \frac{uL}{D}$$

(6)

is usually very high ($10^3 \div 10^6$) so the concentration rate is mainly due to the convective term in (5) with respect to the diffusive term; for this reason the stirring process must be accelerated with artificial vortices created by appropriate paths along the stirring channel.

III. State of the art for MHD stirrers

In the recent past MHD stirrers have been presented in a wide variety of geometries [6-8]. Most recent solutions generate chaotic fluid motion through the generation of a quasi-axial MHD force whose direction is changed by a sequential switch of electrodes pairs. In this way it is possible to revert the fluid direction inside the mixing channel and obtain transitional vortices in the fluid. In figure 2 is shown a typical longitudinal section of the mixing channel; by sequential switching of the electrodes pairs EVEN+-ODD- and EVEN--ODD+, the fluids inside the channel are subjected to non aligned forces which create the growth of a vortex during the transient change of motion. The main advantage of such geometry is that it is easily realizable, the disadvantages regard the necessary stirring time (each electrodes pair needs to be active for 4 s) and the large dimensions of the mixing channel necessary to obtain a MHD force strong enough to generate transitional vortices.
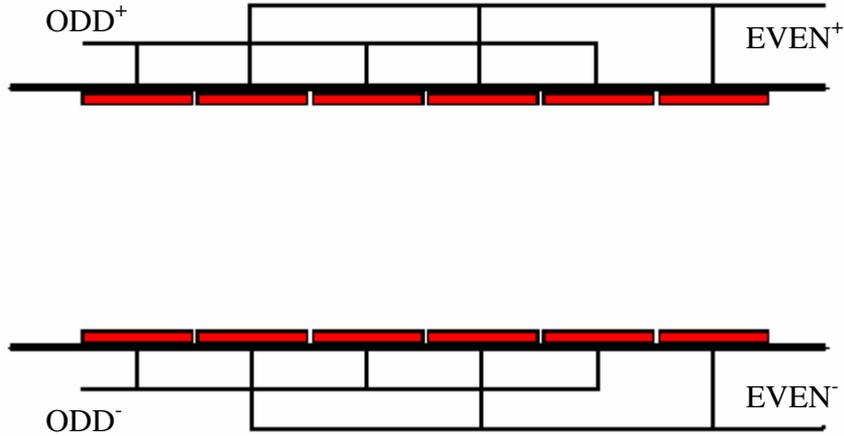
Figure 2. State of the art of MHD stirrers: longitudinal channel section.

IV. Innovative approach to MHD stirrers

The new approach to MHD stirrers presented in this paper has the aim to create vortices inside the mixing channel through MHD forces created by radial current densities whose centre is the centre of the vortex. This is made possible posing electrodes on the bottom of the mixing channel at appropriate positions with respect to channel centre and imposing appropriate voltages with respect to the channel walls. The simulated geometry is reported in figure 3; at the two inlets the electrodes are posed only on the walls of the microchannel, in this way it is possible to create a MHD pumping effect moving the two fluids into the central mixing channel. The voltages imposed at the electrodes (+2V and -2V in figure 2) are square waves at 10 Hz frequency with zero mean value and amplitude 2V in phase opposition in order to prevent electrolysis and bubble formation inside the fluid, this forces the use of an electromagnet providing a quasi-square wave magnetic field whose amplitude is 70 mT. On the bottom wall of the mixing channel are posed alternately five electrodes forced at potentials $+V_{EL}$ and $-V_{EL}$ (also referring to square waves at 10 Hz with zero mean value); if the magnitude V_{EL} is greater than 2 V, one can see that the current density assumes a radial behaviour with respect to the centre of each electrode. This means that inside the channel over each electrode a vortex grows making the fluid rotate orthogonally to the current density as reported in (1)
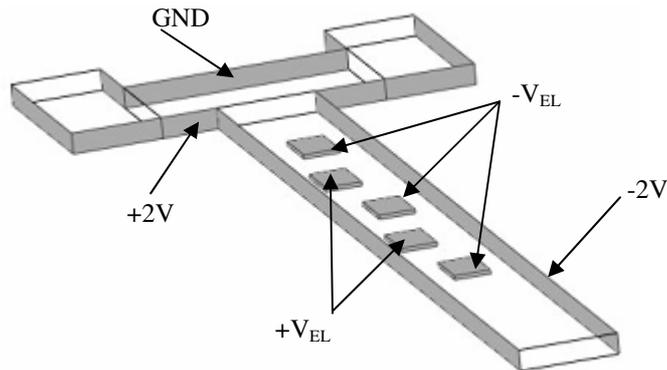

Figure 3. Simulated geometry of the stirrer. The electrodes are evidenced in grey

The channel dimensions are: 500 μm in depth, 2 mm in width and 22 mm in length (respectively *h*, *w* and *L* in figure 1) and the electrodes posed on the bottom of the channel are 800μm x 1mm; the distances of the central electrodes from the channel walls are opportunely dimensioned in order to obtain that the current density modulus is equal in each side of the electrode. If d_1 and d_2 are the distances from the channel walls to the electrodes then is assured that

$$| \overline{J} |= \sigma | \overline{E} |= \frac{\sigma(V_{EL} - 2)}{d_1} \cong \frac{\sigma(V_{EL} - (-2))}{d_2} \qquad (7)$$

where σ is the conductivity of the fluid inside the channel. The fluid physical properties have been

posed to values comparable with saline solutions, typical for the biological applications; the density ρ is 1000 kg/m^3, the dynamic viscosity μ is 1.1·10^{-3} kg/(ms), the conductivity σ is 2.5 S and the diffusivity D has been varied from 5·10^{-11} m^2/s to 5·10^{-9} m^2/s in order to verify that the mixing quality depends only on the presented geometry and not on the diffusivity of the fluids inside the channel; the amplitude of the square waves driving the central electrodes has been varied from 2V to 6V. The boundary conditions for Navier-Stokes simulations have been posed as "no slip" everywhere except to the inlets and outlet of the stirrer where the conditions are "neutral" (i.e. pressure is zero at inlets and outlet).

V. Simulation results

In figure 4 are plotted the streamlines of the current density inside the mixing channel; over each electrode it is possible to see the radial distribution implying that the velocity field of the fluid will be circular as stated in (1).
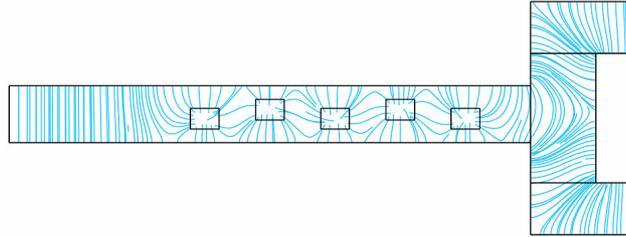

Figure 4. Streamlines of the current density distribution along the mixing channel

In figure 5 the velocities of the fluid along x axis (channel direction) and y axis (transverse the channel) are plotted.
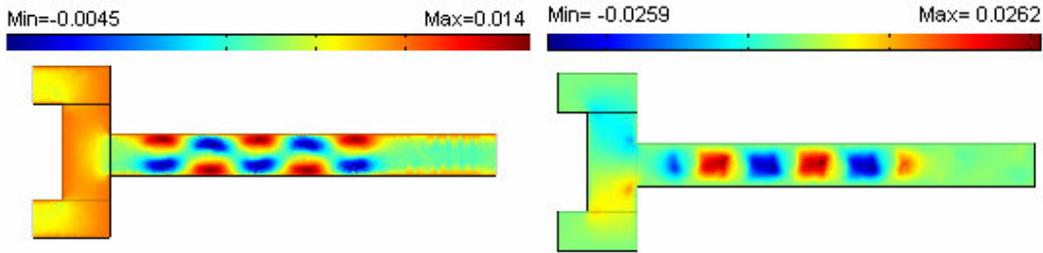

Figure 5. Velocities of the fluid along x (left) and y (right) axes

As one can see the velocity along the x direction is alternately positive and negative and so does the y velocity; this means that the vortices make the fluid rotate alternately clockwise and anticlockwise respectively. For the simulation of stirring efficiency a concentration of 16 mol/m^3 has been posed at the upper inlet and has been evaluated the mixing quality at the outlet of the stirrer. The stirring efficiency has been defined as in literature [6]

$$\alpha(x,t) = 1 - \frac{\sigma(x,t)}{\sigma(x,0)} \tag{8}$$

where $\sigma(x,t)$ represents the standard deviation of the concentration over the entire cross section of the channel:

$$\sigma^2(x,t) = \int_0^h \int_0^w (c(x,y,z,t) - C_{AVG}(x,t))^2 \, dydz \tag{9}$$

In (9) C_{AVG} represents the cross section mean value of the concentration at position x. In figure 6 is shown the qualitative behaviour of the stirrer mixing efficiency after 1 s, 2 s, 3 s, 4 s, 5 s and 10 s.

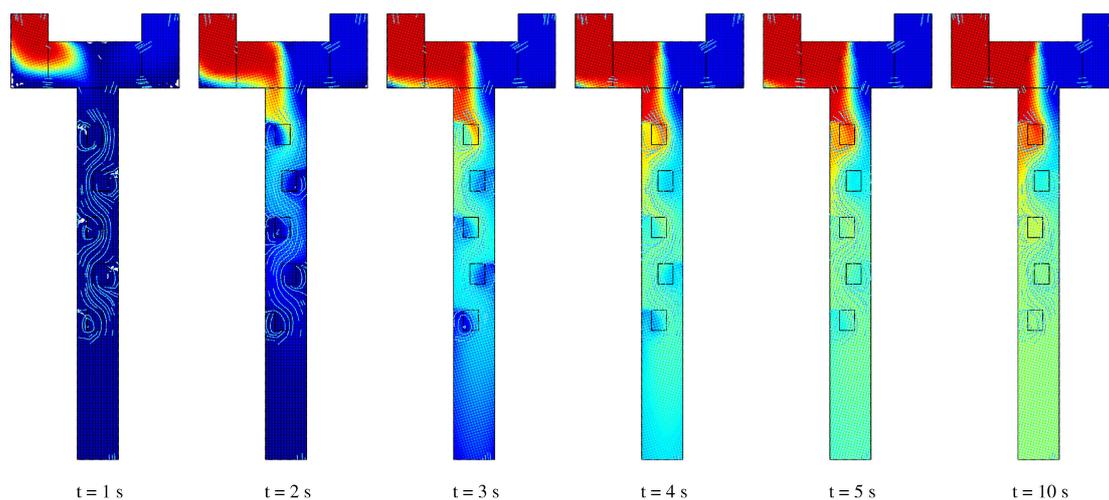| t = 1 s | t = 2 s | t = 3 s | t = 4 s | t = 5 s | t = 10 s |

Figure 6. Mixing quality after 1 s, 2 s, 3 s, 4 s, 5 s and 10 s; the streamlines represent the convective flux

The stirring efficiency has been evaluated at the outlet of the microchannel and in figure 7 is shown a parametric behaviour of the stirrer varying the electrodes potential from 2 to 6 V, in particular it is shown the quantity $1-\alpha(L,t)$ versus time.
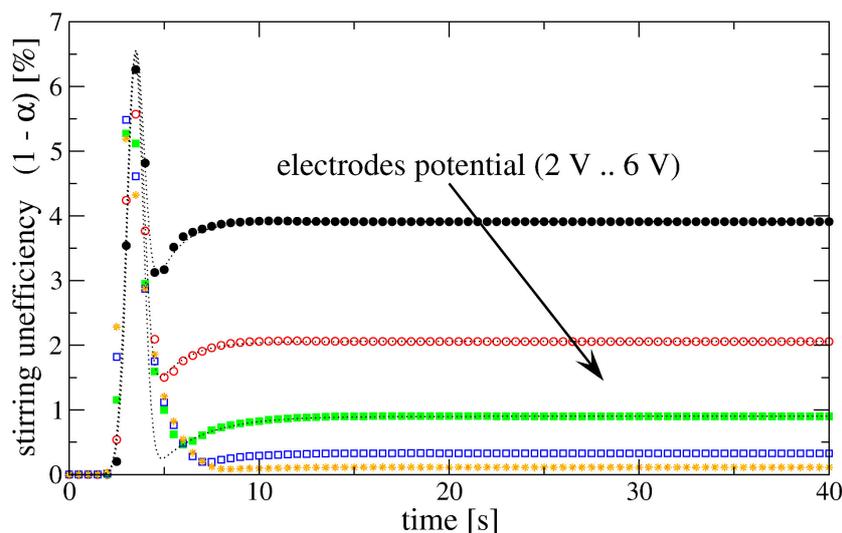

Figure 7. Stirring unefficiency at the channel outlet versus time varying the electrodes potential V_{EL}

As one can see from figure 7 the stirring efficiency increases increasing the electrodes potential from 96% to 99.95%. Finally the efficiency of the stirrer has been analysed over a wide range of diffusivities in order to prove that the efficiency does not depend on the diffusivity of the fluid inside the channel. In figure 8 is shown the stirring efficiency at the outlet of the channel as a function of the electrodes potential with the diffusivity as parameter.
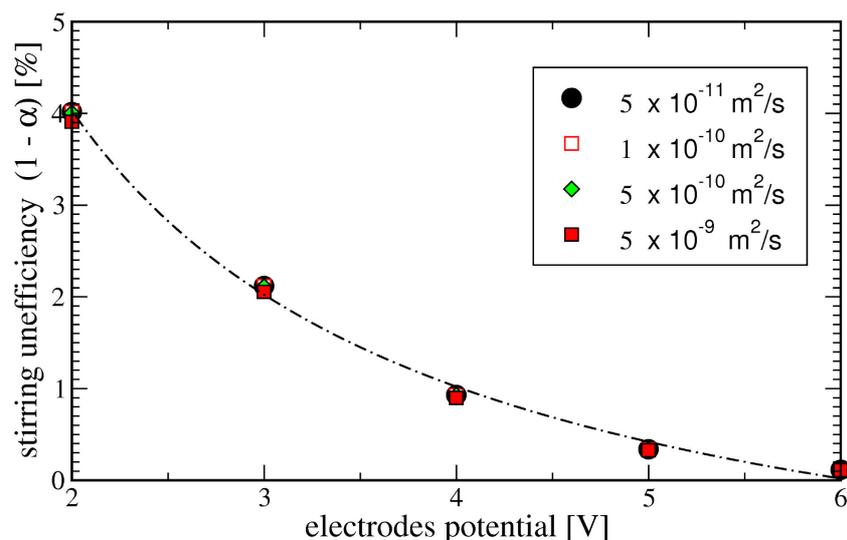
Figure 8. Stirring unefficiency over a wide range of diffusivities

As one can see the efficiency does not depend on the diffusivity of the fluid, this means that the stirrer efficiency is due to the simulated geometry whose convective term in (5) predominates.

VI. Conclusions

A novel geometry for an AC driven MHD stirrer is presented. The performed FEM simulations show a good behaviour in terms of efficiency and speed of mixing; FEM simulations have been also performed over a wide range of diffusivities in order to show that the mixing efficiency does not depend on the mixing fluids properties but on the convection generated by a novel electrodes disposition in the presented geometry.

References

[1] S. J. Lee, S. Y. Lee, "Micro total analysis system (µ-TAS) in biotechnology", *Appl Microbiol Biotechnol*, vol. 64, pp. 289–299, 2004.

[2] H.A. Stone, A.D. Strook, A. Ajdari, "Engineering flows in small devices: microfluidics towards a lab-on-a-chip", Annual Review of Fluid Mechanics, vol. 36, pp. 381–411, 2004.

[3] G. M. Whitesides, "The origins and the future of microfluidics", *Nature*, vol. 442, pp. 368–373, 2006.

[4] M. A. Burns et al., "An Integrated Nanoliter DNA Analysis Device", *Science*, vol. 282, pp. 484–487, 1998.

[5] J. VanDijken et al., "Microfluidic chips for detecting the t(4;14) translocation and monitoring disease during treatment using reverse transcriptase-polymerase chain reaction analysis of IgH-MMSET hybrid transcripts", *Journal of Molecular Diagnostics*, Vol. 9, pp. 358-367, 2007.

[6] J. Qian, H. H. Bau, "Magneto-hydrodynamic stirrer for stationary and moving fluids", *Sensors and Actuators B: Chemical*, Vol. 106, pp. 859-870, 2005.

[7] H. H. Bau, J. Zhong, M. Yi, "A minute magneto hydro dynamic (MHD) mixer", *Sensors and Actuators B: Chemical*, Vol. 79, pp. 207-215, 2001.

[8] J. West, J. P. Gleeson, J. Alderman, J.K. Collins, H. Berney, "Structuring laminar flows using annular magnetohydrodynamic actuation", *Sensors and Actuators B: Chemical*, Vol. 96, pp. 190-199, 2003.

[9] D. Erickson, "Towards numerical prototyping of labs-on-chip: modelling for integrated microfluidic devices", *Microfluidics and nanofludics*, Vol. 1, pp. 301-318, 2005.