Measurement simulation and data assessment of circular scales

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A b s t r a c t - Analysis of circular scales accuracy calibration methods based on the use of computer modelling and data processing including information entropy assessment is presented. The computer simulation means for angular position measurement are presented using the circular scale simulation on the PC screen and giving a wide range of opportunities to change or select various scales' parameters for their further analysis. Accuracy analysis of the circular scales is available applying different methods of measurement and making experimental trials of newly developed methods. Various parameters of the scale can be simulated and an analysis of systematic and random errors can be performed as well. Methods of error correction for systematic errors can be applied, and information entropy assessment can be made. The means and model of simulation provide the possibility to apply a wide range of parameters change with subsequent their analysis what is impossible and highly expensive in case of real experimental trials in-situ. The software developed permits to calculate accuracy parameters of thousands of angular values after simulation of measurement using newly developed method of measurement of circular scales.

Key words: circular scale, angle, measurement, information, simulation

Introduction

Linear and angle measurement occupies the most important position in metrology and instrumentation. Data processing of all kinds of measurement are regulated by mathematics statistical methods and normative documents [1]. It can be stated that angle measurement has been less researched, and there are fewer research results in this field of metrology presented comparing with the linear ones. It is especially relevant for circular scales measurement as many companies producing angular measuring instruments (theodolites, tacheometers and other geodetic and machine engineering instruments) keep this field of metrology closed not making efforts for their publication. Review of the methods and means of circular scales metrology here are presented, and means for its analysis in the form of computer simulation are proposed. Meanwhile, the circular scales make the basis for modern rotary encoders, rotary tables for CNC machines and metal cutting tools, angle measuring systems, coordinate measuring machines (CMMs), robots, etc.

The methods of angle measuring using a rotary encoder, autocollimator and polygon, reference circular scale and microscopes are used for the calibration of raster, coded and ordinary circular scales. The multiangular prism – autocollimator system is considered as the main reference measure of the flat angle against which all other measuring systems are compared [2, 3]. An evident problem in this is the restricted number of angle positions determined due to the number of reference angles of the polygon, i. e., 12, 20, 24, 36, sometimes, 72 angle sides of the polygon. The problem exists due to the great amount of information that is gained in the calibration of scales, information – measuring systems of the CMMs in their total volume. It is technically almost impossible task to calibrate the enormous number of points available, e.g. the 32 400 pulses of the rotary table in one revolution [4] or 1 296 000 pulses/rev or more of the rotary encoder. According to the research performed [3, 4], some larger values of the systematic error can be omitted during the calibration including significant ones.

Intervals for metrological calibration of measuring system in mechanical engineering should not be longer than 25 mm for the linear measurement for the displacement of travel length of 250 mm or less. For longer travels – up to 1,000 mm – the interval should be no shorter than 25 mm or no longer than a 1/10 of the length of travel. There is a traditionally selected interval for angle standards calibration that is usually equal to 3°. It is impossible to perform such calibration using the polygon. Therefore, it is important to determine the information quantity during the calibration of the scale or encoder that has been evaluated providing a more thorough result assessment. These requirements stated by written standards show the same problem – information about the scale's or encoder's accuracy inside the interval of measurement during the calibration remains unknown.

Modern angular displacement control systems, such as "ring laser" or high accuracy rotary encoders have an information measuring systems that produce the amount of information exceeding gigabytes rather in the average complexity of the system. At the same time, the main angle standard of measure – the polygon has quite restricted number of reference positions. A general picture of the raster scale calibration is presented in Fig. 1. Here polygon and the circular scale are placed concentrically on the same axis of rotation of high accuracy. The position of the strokes on the scale is read by photoelectric microscope (not shown), and those of the sides of the polygon – by photoelectric autocollimator (not shown). The polygon is used as the standard of measure and the accuracy of the scale is determined as difference between the angular positions of the scale and the polygon.

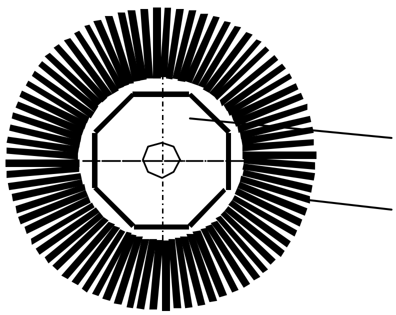

Fig. 1. Circular raster scale's calibration at the pitch of the standard measure – the polygon with 12 angles: 1 – polygon, 2 – circular raster scale

A computer simulation system is developed for investigating the circular scales parameters including accuracy and visualization of the object in conjunction of mathematic equations for an assessment of statistical accuracy parameters and information entropy characteristics for an assessment of what part of the information has been determined from the total amount of information generated from the object under testing by an information – measuring system of angle. The circular raster scale as a real example to be simulated is selected for this purpose.

I. Data processing in angular scales measurement

Classic exposition of the result of measurement is given by many authors presenting it as estimate of the value of the specific quantity, the measurand, having the systematic components and uncertainty. Uncertainty of the result is evaluated grouping it into two categories according to the method used to estimate their numerical values. Here we consider dealing with the numerical values which are evaluated by the statistical analysis of series of observations, e. g., category A [1]. For the purpose of statistical evaluation of the uncertainty the confidence interval is chosen defining a level of confidence at normal distribution of variable. For other types of distribution (uniform, rectangular or triangular) an appropriate factor for the uncertainty calculation is used [1]. In general, expression of the measurand looks:

$$X = \overline{X} \pm u_c \ , p \qquad (1)$$

where *X* is the result of measurement; $\overline{X}$ is the systematic component, u_c is the combined standard uncertainty of the result that is expressed as $u_c = \frac{ts}{\sqrt{n}}$, where *t* is the Student coefficient, *s* is the estimate of standard deviation, *n* is number of trials, and *p* is the probability distribution.

Estimate of the result of measurement is obtained from the sample of variables chosen freely or depending on the number of etalon values in use. The latter feature usually has no exposition in the final expression of the result. So, adding the quantity of information entropy to this expression would show the indeterminacy of the result [4, 5, 6] having in mind that there are rare occasions when all volume of data is covered by assessment process. According to [5, 6] full information entropy is expressed as

$H_0 = -\sum_{\Delta} \log_a \frac{1}{m} = \log_a m$. The information received after the calibration of the scale by measuring the part b of the strokes of the scale will be $H_1 = \log_a b$, where $b = \frac{m}{k}$ is the number of calibrated strokes in the scale having m number of strokes and k is the number of the strokes covered by the interval of the calibration. Every position is measured c times each for the statistical evaluation. Then the reduction in the information uncertainty (indeterminacy) due to the information received after the calibration will be:

$I = H_0 - H_1 = \log_a m - \log_a b$; and $\log_a b = \log_a m - I$; and $b = a^{(\log_a m - I)} = m \times a^{-I}$.

Supplementing the result of measurement (1) by an information entropy parameter will show the indeterminacy of the result, at some extent a validity of the measuring process in the total volume of data in question. Since the total number of measurements is $n = bc$, (each measurement during the calibration is performed c times), the expression for the measurement result (1) at a given probability distribution and the reduction in information indeterminacy becomes :

$$X = \overline{X} \pm \frac{t \cdot s}{\sqrt{a^{-I} mc}}; p; I(H_0, H_1) . \qquad (2)$$

It means that the measurement result is determined with the uncertainty assessed by probability distribution p and with the *indeterminacy* of the result assessed by the entropy $I(H_1, H_0)$ by the evaluation only the part of total data. It gives a better information of assessment of the result of measurement allowing one to know which part of total information is covered by this process [7].

In Fig 2 the relationship between the total number of strokes on the scale m, information I received after its evaluation of the number of strokes already assessed on the scale b is displayed. It is assumed that the base of logarithm is 2, and this base is used in the formulae.

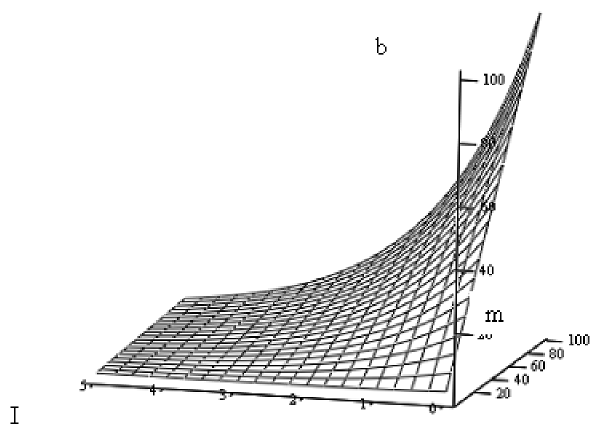

Fig. 2. The relationship between the total and assessed number of strokes in the view of the gained information from the scale: m is the number of strokes, b is the number of the assessed strokes on the scale, and I is the quantity of information

Information entropy permits the presentation to the user what part of the information that is available from the measurement data is determined with appropriate statistical estimate. An additional value to this approach would give an indication of the sampling value in the result of the measurement.

II. Computer simulation of the angle measurement system

The main task of the computer simulation [8] was to create a computer means for the circular scale's calibration modelling. Using the computer grid as reference measure for measurement some scale's parameters were introduced for the selection a different number of strokes of the circular scale to be simulated, change the width of their strokes, to put the positions of 2 or 3 microscopes, usually used in the circular scales measurement, to simulate the strokes at larger scale for the strokes under microscopes view, select the values of systematic and random errors of the scale and develop a software for great number of the strokes to be analysed using a special methods of the circular scale measurement.

Diagram of the circular scale's measurement applying a classical scales calibration method using constant angle placement in the full circle is presented in Fig. 3. Two microscopes M1 and M2 are placed on the strokes of the raster scale. The distance between the microscopes is φ_t, usually restricted by width of the microscope's box. The measurement is performed by rotating the scale under the microscopes by steps equal to φ_t. Error introduced by eccentricity of the scale according to the axis of rotation is eliminated by accurate centring or after the measurement by means of calculation when the first harmonics (from the eccentricity) is eliminated.

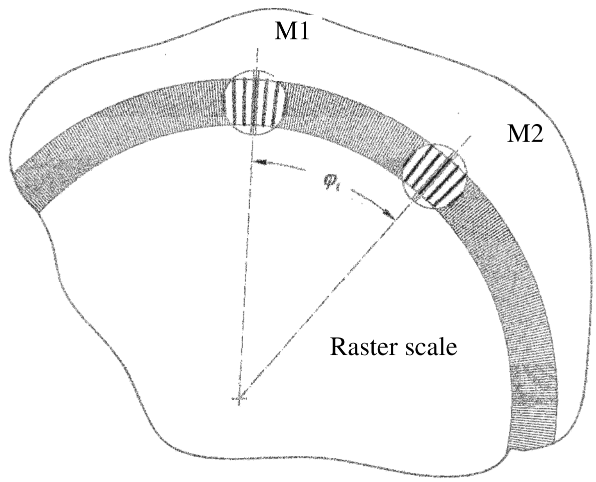

Fig. 3. Diagram of the circular scale's measurement applying the calibration method by constant angle placement in the circle. M1 and M2 – the strokes position reading microscopes, φ_t – interval of angle between the microscopes

The simulation screen of measurement is shown in Fig. 4. The simulation of angular scale measurement starts at the *Measurement* control group at the upper left corner or the program window. The number of strokes to simulate can be chosen virtually. Visualization can also be adjusted by choosing a line width for strokes from the Line width list on the right of the control group.

The program is written in *Microsoft Visual Basic*, the language not known for its flexibility but convenient for writing fairly simple and brief programs with a visual interface. The full code for the program is not provided due to the restricted space of the paper. This program is characterized by visual and data manipulation adjustments that can easily be made with the controls located on the left-hand side of the screen. The adjustment that makes the most evident impact on the program is changing the width of the scale strokes being displayed. In general, the greater the number of strokes being used, the smaller the width of strokes should be used. The first stroke width on the list has been optimized for the maximal number of strokes that could be displayed so that the visualization remains legible. The list of stroke widths is preset in the code of the program, but is would be easy to delete existing values or add new ones if such a need ever arises. Other adjustments that influence data processing are located in the controls group *Measurement* at the lower left corner of the screen. The first control at the top moves individual strokes to the position of Microscope 1. Pressing the minus button moves the scale by one stroke counter-clockwise, while pressing the plus button moves the scale by one stroke clockwise. The next control, an option list, is involved in with performing measurement by certain microscopes. The errors can be cancelled in relation to one of the three microscopes or the error difference between split the microscopes in the last option in the list. The default option is to perform no correction in relation to Microscope 1. It is used for the beginning of the readings from the scale. And the last control in the *Measurement* group introduced a random element into data processing. The user can indicate the random error that is included in calculating the results.

The code portion of the program as opposed to the visual portion consists of several major subroutines. The subroutine *Draw( )* displays the scale graphics on the right side of the screen and is called from inside other subroutines. Another important subroutine that loads initial data from a text file and writes the results to another text file is *From_File_Click( )*.

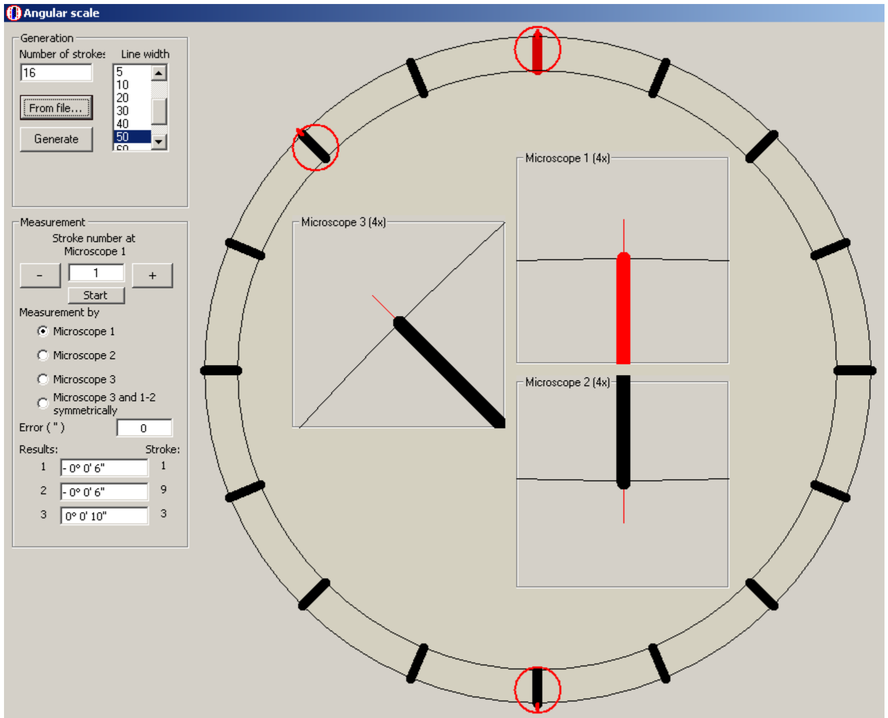

Fig.4. The program screen after generating results for 16 strokes. The width of stroke lines has been increased to 50 for a better visual representation

There is also an option to generate initial data automatically with some parameters for randomization provided; this job is performed by the subroutine *Generate_Click( )*, which uses the subroutine for randomization *K_Gen(En)* internally. The utility function *In_Degrees( )* is used to convert radians, used for drawing graphics, into degrees for result output. Note that in *Visual Basic* the difference between subroutines and functions is that subroutines do not return a value, while functions do. *Visual Basic* is a format-sensitive programming language, meaning that its statements end at the new line symbol, so in the code of the program the continuance of lines are indicated by a tab shift to the right in order to save space and to make clear the structure of the program.

After the measurement simulation some of the results are displayed in chart, an example shown in Fig 5. To make the visualization legible only 16 values have been taken, but the upper limit currently set in the program is 720 values; this limit can easily be changed in the code for this program without affecting program's functionality. Table 1 also lists a basic statistical summary for the series of 16 data values.

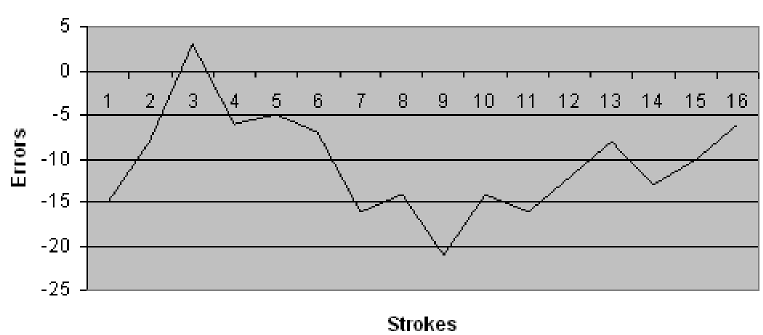

Fig 5. An example of systematic error distribution for angular scale simulation data: errors – systematic error, sec. of arc; strokes – number of the stroke in the scale

Table 1. Statistical summary of the data in Fig. 5

Statistical summary, sec. of arc	
Mean value	-10.5
Standard Error	1.446259543
Median	-11
Mode	-8
Standard Deviation	5.785038173
Sample Variance	33.46666667
Kurtosis	0.731776715
Skewness	0.471059917
Range	24
Minimum	-21
Maximum	3
Confidence Level (95.0%)	3.082629232

III. Conclusions

- Computer simulation means are proposed permitting the investigation of circular scales accuracy parameters, testing the new calibration methods and to develop newly proposed accuracy evaluation means adding the information entropy assessment in circular scales measurement along the generally accepted statistical estimates.
- The computer simulation developed permits the variation of the scales' features, e. g., the strokes number, their length and width, calibration method applied and scales' error estimation means and methods.
- Data acquisition and interactive mode of processing permit to implement a wide range of metrological tasks. Measurement simulation can be used for the development of new instrumentation means for circular scales measurement and data assessment structure.

Acknowledgements

This work has been funded by Lithuanian State Science and Studies Foundation, Project No B-32/2008.

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