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THD estimation by non-coherent sampling using various DSP algorithms and windows

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***Abstract*-**The paper inspects influence of the RMS-computing algorithm and the used window type on estimation of signal total harmonic distortion (THD). Computer simulations are used for evaluation of the THD bias. Three algorithms of THD estimation are investigated, differing by methods used for estimation of RMS values of the total signal and of the signal fundamental harmonic component. The signal RMS value is found by processing signal samples either in time domain or in frequency domain. The fundamental harmonic component RMS value is found either by processing DFT magnitude spectrum components within the used window spectrum mainlobe, or by using signal windowing and DFT spectrum interpolation in frequency domain. Figures show the relative bias (in percent) of the THD estimation for cosine windows of the first order and of the third order. Influence of signal quantization and of phases of the test signal harmonic components is also shown. The test signal is composed of 13 harmonic components with magnitudes corresponding to an international standard for compatibility levels of low-frequency voltage disturbances.

I. Introduction and objectives

Processing sampled signals in time or frequency domain for finding selected signal parameter is nowadays a common practice in measurement. Since signal sampling is most frequently non-coherent, windowing is used to reduce the energy leakage in frequency domain. Total harmonic distortion (THD) is estimated for a harmonic (sine or cosine) signal distorted by low-level higher-order harmonic components, i.e. for a multi-component signal. That is why the paper describes briefly the DFT spectrum of a windowed multi-component non-coherently sampled signal and reminds how the spectrum of such a signal relates to corresponding DTFT (Discrete Time Fourier Transform) spectrum. Energy leakage bound to the used window is the main cause of THD estimate bias. The paper objective is to inspect influence of the used window type and RMS-computing algorithm on estimation of signal total harmonic distortion (THD). Computer simulations are used. Signal quantization is also included – figures of the THD relative bias are shown for ideal quantizers with 12 bit and 16 bit resolution and an expression for THD uncertainty caused by signal quantization is also presented. The influence of test signal harmonic component phases on the THD estimate by non-coherent sampling is also mentioned.

II. Frequency spectrum of a non-coherently sampled windowed multi-component signal

As is well known, the Discrete Time Fourier Transform (DTFT) spectrum of a sequence of window samples *w(n)* is a periodic continuous function of frequency. Windows used in DSP practice are usually derived from originally even and real windows by a positive time shift by $(N/2)\cdot T$, N being window length and T being sampling period. This time shift results in shifted spectrum linear phase $\varphi = -(N/2)\cdot\theta$, being φ the window spectrum phase and θ normalized frequency ($\theta = \omega T$). The shape of the DTFT window phase spectrum depends on the position of zeros of corresponding DTFT window magnitude spectrum.

Windowing a discrete-time signal means convolving signal and window DTFT spectra in frequency domain. The DTFT windowed multi-components signal composed of M cosine components is therefore a superposition of $2M$ window spectra placed on M positive and M negative cosine components frequencies. The mutual influence of all these window spectra depends primarily on magnitude and frequency distance (in frequency bins) of signal components and on the used window spectrum side-lobes level and fall-off. The most frequently used windows are the well-known generalized cosine windows [1] – [4]. These windows are used also in this paper.

The DFT spectrum of finite-length discrete-time signal is the DTFT spectrum of the same signal sampled in normalized frequencies $\theta_k = k\cdot(2\pi/N)$, $k=0, 1, \ldots, N-1$.

Since the DFT theory assumes the signal and its spectrum being periodic with period N (the DFT length), windows used in DFT-based applications are the so-called *DFT-symmetric windows* [1]. The last sample of the originally even-symmetric window is left out.

III. THD estimation of a non-coherently sampled distorted sine (or cosine) signal

A. Basic information about the used THD simulation

All simulations have been performed in MATLAB environment. Total harmonic distortion characterizes content of higher harmonic components in a distorted harmonic (sine or cosine) signal. It is defined as a ratio of RMS value of signal harmonic components from the second one to the maximal one related to the fundamental harmonic component RMS value, and can therefore be estimated using formula

$$THD = \frac{\sqrt{\sum_{k=2}^{M} X_{kRMS}^2}}{X_{1RMS}} = \sqrt{\left(\frac{X_{RMS}}{X_{1RMS}}\right)^2 - 1} \qquad (1)$$

Here X_{RMS} is the signal RMS value and X_{kRMS} is RMS value of the signal k^{th} harmonic component. Various DSP algorithms in both time domain and frequency domain can be used for RMS value estimation [5]. The uncertainty of RMS measurement was theoretically analyzed in [6]. A general formula for THD measurement uncertainty estimation for heavily distorted waveforms where quantization errors can be disregarded and THD uncertainty is caused by signal transducers was published in [7]. The uncertainty is expressed there as a function of uncertainties of individual harmonic components. The influence of leakage by non-coherent sampling can be in all cases reduced by signal windowing [8-9]. The THD values are often expressed in percent.
In this paper we inspect the influence of the selected window and the selected RMS value estimation algorithm on the THD values by non-coherent sampling, as dependent on relative time of measurement (i.e. time of measurement expressed in non-integer multiples of fundamental signal period). This relative time is equal to frequency of DFT spectrum in (not only integer) frequency bins. Our analysis is aimed on cases where low values of THD compared to heavy current circuits with nonlinear loads, and the energy leakage and ADC quantization errors are taken into account by THD accuracy evaluation.
We have compared three different DSP algorithms for estimating THD computing accuracy. The test signal used for THD estimation includes the first 13 harmonic components with levels selected according to the international compatibility level standard [10]. The "true" THD value was calculated according to (1), RMS values of individual harmonic components being amplitude levels divided by $\sqrt{2}$ (that is valid for the integer number of signal periods sampled) and for our test signal it is $THD_T = 0.10737$, i.e. expressed in percent approximately 11%. The THD values were found by simulations for stepwise increasing sampling time expressed in (non-integer) number of signal periods. The THD bias is very high for sampling less than $2P+1$ signal periods, P being order of the used cosine window, in which case the main-lobes of neighbouring harmonic signal component spectra overlap. The figures in the paper present results for numbers of sampled signal periods between 15 and 20. Results for a classic window Hann (window order $P = 1$) and one of more advanced windows (Blackman-Harris minimum 4-term, denoted in figures as BH [1], window order $P = 3$) are presented in the paper. Rife-Vincent windows [3, 12] of the first class and of order 1 and 3 (denoted in figure titles as RV1-1 and RV1-3) are used for interpolation in frequency domain [3, 8] applied for estimating the RMS value of the fundamental harmonic component of the test signal in one of the investigated algorithms ("algorithm 2"). The window RV1-1 is identical with Hann window. Simulation results presented in figures were found for N=1024 samples. The test signal harmonic components have zero phases in simulations depicted in Fig.1 to Fig.14. The sensitivity of one of the investigated THD estimation algorithms to harmonic component phases ("algorithm 2") is shown in Fig. 15 and Fig. 16.

B. The three investigated THD computing algorithms

The *first algorithm* is based on the right-hand side of the expression (1) for THD computation, computing X_{RMS} in time domain [5] and X_{1RMS} value estimation in frequency domain by processing the magnitude DFT window spectrum main lobe components [11]. This main lobe is placed on frequency of fundamental signal harmonic component. X_{RMS} computed in time domain was found from the formula [5]

$$X_{RMS}(x(n)) = \frac{X_{RMS}(w(n)\cdot x(n))}{\sqrt{nnpg}} = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1}\left(w(n)\cdot x(n)\right)^2} \Big/ \sqrt{nnpg} \qquad (2)$$

Here *nnpg* is normalized noise power gain [11] and N is the signal (and window) length.

$$nnpg = \frac{1}{N}\sum_{n=0}^{N-1}\left(w(n)\right)^2 \tag{3}$$

X_{1RMS} value estimation is found by processing the magnitude DFT window spectrum main lobe components M(i) using the formula [11]

$$X_{1RMS} = \sqrt{\frac{2}{N^2 nnpg}\left(\sum_{i=k-P}^{k+P} M^2(i)\right)} \tag{4}$$

where P is window order and k is frequency bin index corresponding to frequency of the fundamental harmonic component (to the first local maximum in signal DFT magnitude spectrum).
The *second THD estimation algorithm* is based again on finding X_{RMS} in time domain (using (2)), but the X_{1RMS} value estimation was based on DFT interpolation in frequency domain, using windows Rife-Vincent of the 1st class and window order 1 and 3 [3, 12].
The *third THD estimation algorithm* uses computing both the X_{RMS} and X_{1RMS} by main-lobe frequency domain method (4). In X_{RMS} computing, the (4) is used repeatedly around all DFT magnitude spectrum maxima corresponding to frequencies of signal harmonic components and relevant values of k. A theoretical expression for THD standard uncertainty caused by transducer uncertainty suitable for heavily distorted signals was derived for this algorithm in [7]. This expression (5) is applicable also in our case, where effect of measured signal quantization is inspected.

$$\frac{u(THD)}{THD} = \sqrt{\frac{1}{THD^4}\cdot\sum_{k=2}^{M}\left(\frac{X_{kRMS}}{X_{1RMS}}\cdot\frac{u(X_{kRMS})}{X_{1RMS}}\right)^2 + \frac{u^2(X_{1RMS})}{X_{1RMS}^2}} \tag{5}$$

For the same standard uncertainty u_q of each harmonic component RMS value, expression (5) can be rewritten as

$$u(THD) = \frac{u_q}{X_{1RMS}}\sqrt{THD^2+1} \tag{6}$$

The uncertainty of the RMS value of a harmonic component u_q is [5]

$$u_q = \frac{u_n}{N \times nnpg}\sqrt{\sum_{n=0}^{N-1} w^4(n)} \tag{7}$$

Here u_n is quantization noise of the used quantizer, defined by the ADC voltage range and its effective number of bits (ENOB), N is total number of processed signal samples and $w(n)$ are values of the used window.
Algorithms using estimate of total signal RMS value by processing signal in time domain (denoted in the paper as algorithm 1 and 2) are not suitable for signals with inter-harmonic components.

C. Simulation results

All the figures presented in this paper depict dependence of THD estimation relative bias, expressed in percent, on the number of sampled signal periods, equal to frequency expressed in frequency bins. The percents of the THD relative bias are percents of the actual ("true") THD_T value that can be expressed also in percent (and it is in that case for our test signal approximately 11 %, see above part III A).
Figures 1 to 14 show the window order influence on the non-quantized signal THD estimation, the masking effect of the 12-bit signal quantization on THD estimation using higher-order windows, and the fact that bias of THD computed by algorithms 1 and 3 does not decrease with increasing number of sampled signal periods. This is a property of the spectrum main-lobe based algorithms of RMS finding and a difference compared to algorithm 2 using signal windowing and DFT interpolation in frequency domain (WIFD) for finding X_{1RMS}.
All figures are plotted for frequencies in bins higher than $2P$+1. Frequency band 15 to 20 frequency bins providing THD bias values of not too different values and allowing so good relative bias inspection in the whole depicted frequency band was selected for all the figures. This frequency band is also sufficient to show that using WIFD for estimation of the fundamental harmonic component RMS value (i.e. algorithm 2) allows achieving lowest THD bias for non-quantized signal for equal order windows and sufficient length of sampling time. As can also be seen from the figures, using algorithm 1 and the first order windows leads to the relative THD bias in the order of tens percents.
As can be seen from Fig. 8, Fig. 11 and Fig. 14, differences in the THD bias for the three investigated algorithms applied on non-quantized signal are totally masked by quantization noise in case of 12-bit ADC and the third-order windows. Differences by 16-bit quantization are not totally masked but they are also much reduced by the quantization noise (see Fig. 9 and Fig. 12).

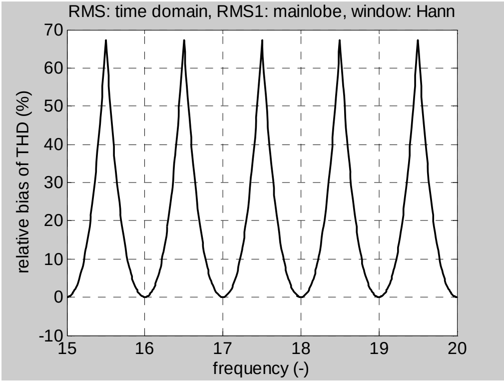

Fig.1. Algorithm 1, window Han, no quantization

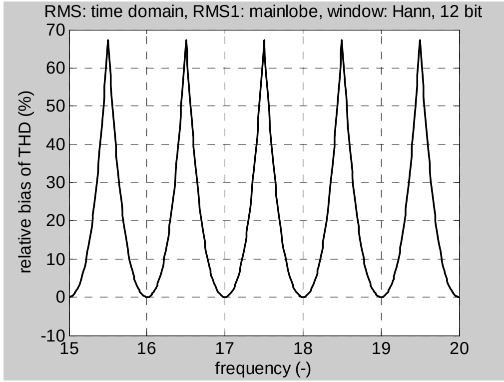

Fig. 2. Algorithm 1, window Hann, ADC 12 bit

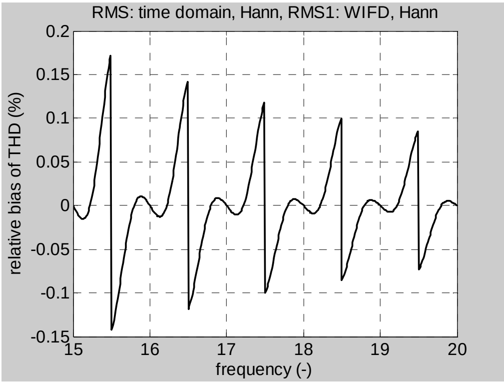

Fig.3. Algorithm 2, window Hann, no quantization

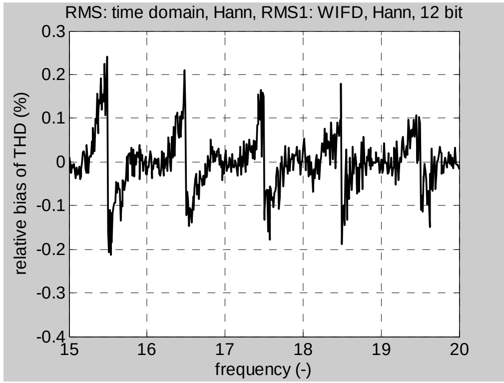

Fig. 4. Algorithm 2, window Hann, ADC 12 bit

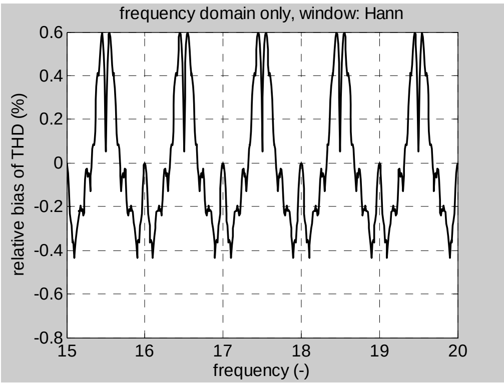

Fig.5. Algorithm 3, window Hann, no quantization

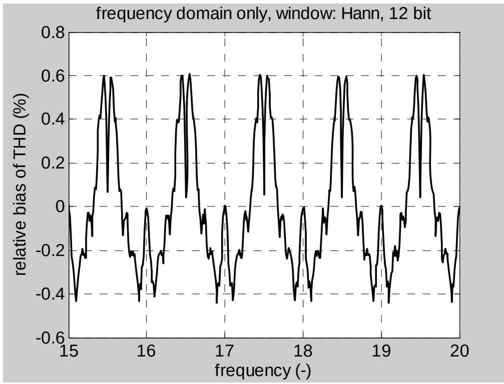

Fig.6. Algorithm 3, window Hann, ADC 12 bit

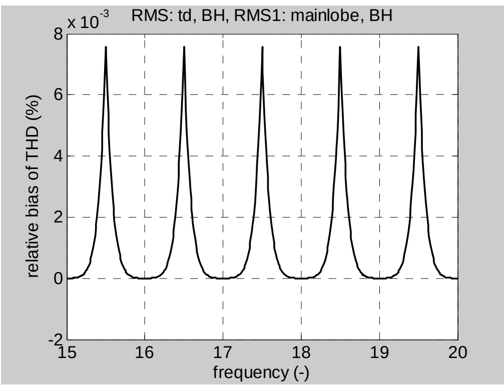

Fig.7. Algorithm 1, window BH, no quantization

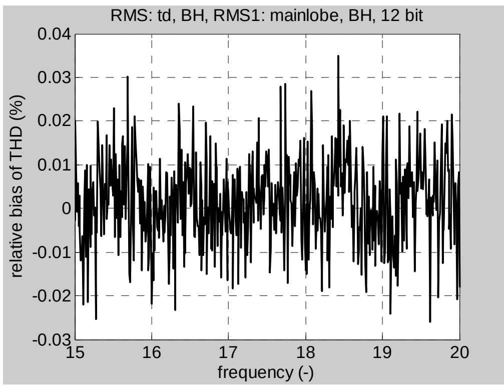

Fig. 8. Algorithm 1, window BH, ADC 12 bit

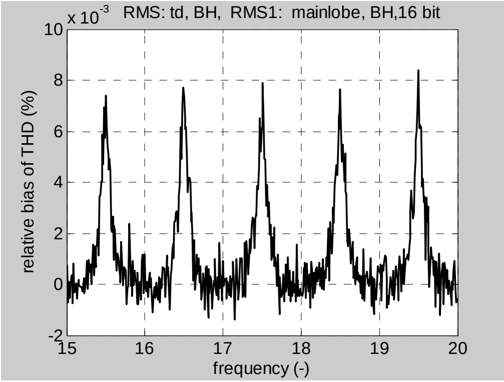

Fig.9. Algorithm 1, window BH, ADC 16 bit

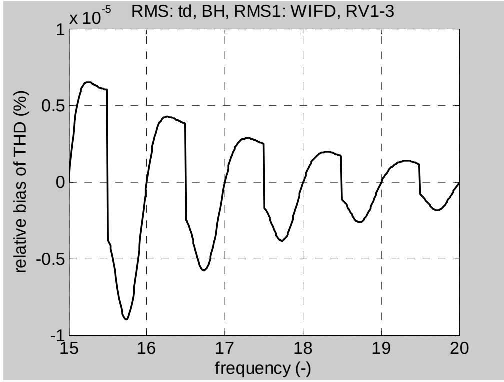

Fig.10. Algorithm 2, window BH and RV1-3

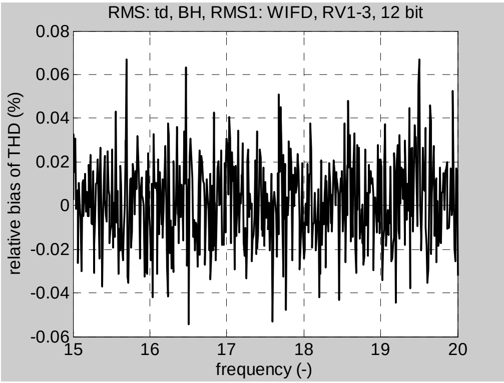

Fig.11. Algorithm 2, window BH and RV1-3, ADC 12 bit

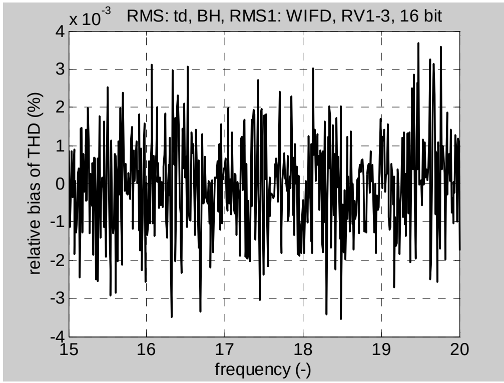

Fig. 12. Algorithm 2, window BH and RV1-3, ADC 16 bit

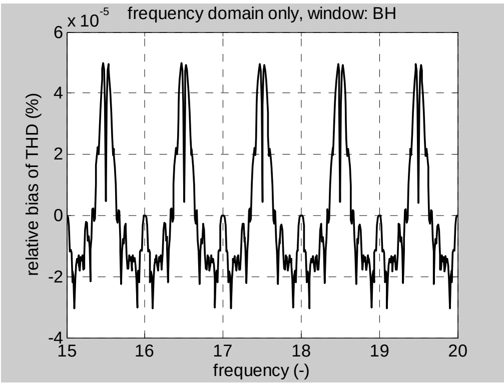

Fig.13. Algorithm 3, window BH, no quantization

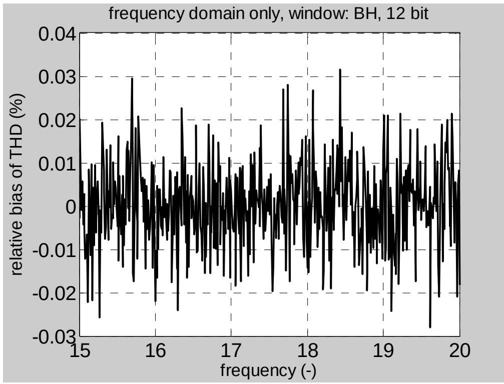

Fig.14. Algorithm 3, window BH, ADC 12 bit

Figures 15 and 16 demonstrate the sensitivity of the algorithm 2 to the phases of harmonic components of the test signal, namely on the phase of the fundamental signal component for the non-quantized signal. The phase of the fundamental component was zero in Fig.1 to Fig. 14, but it is $\pi/2$ in Fig.15 (denoted “phases 2” in figure title and figure caption) and $\pi/4$ in Fig. 16 (denoted “phases 3”). Phases of all other test signal harmonic components were zeros and their effect on the relative THD bias is much lower than the effect of the phase of the fundamental harmonic. Fig. 15 and Fig. 16 can be compared with Fig. 3 showing the THD bias for the test signal with zero phases of all harmonic components. Corresponding figures for algorithms 1 and 3 are not shown, since the influence of test signal phases on the total signal THD value is too small to be distinguished by inspection figures depicting the THD relative bias in the above used frequency band.

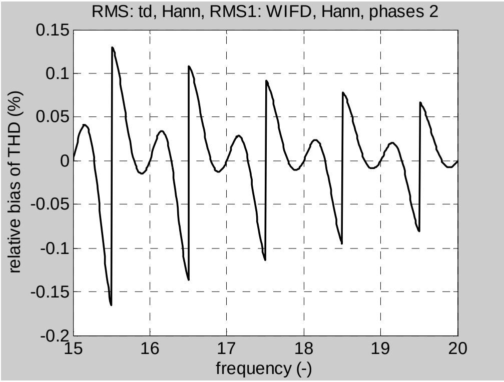

Fig.15. Algorithm 2, window Hann, phases 2

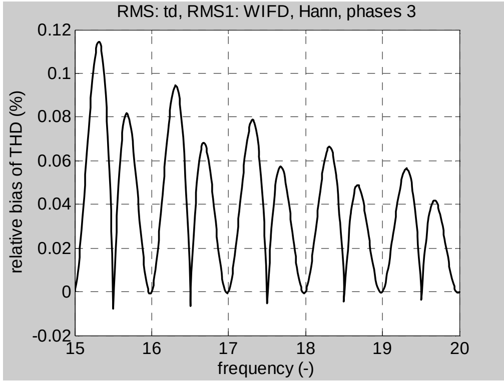

Fig. 16. Algorithm 2, window Hann, phases 3

IV. Conclusions

Three THD computing DSP algorithms were compared by computer simulations, and influences of window order, number of sampled signal periods, signal quantization and test signal harmonic components phases on the THD bias were shown. Information in the paper allow a reader to compare the accuracy of the three investigated THD estimation algorithms and to see the difference of these algorithms sensitivity to the number of sampled signal periods. It also allows to see that if 12-bit signal quantization of the measured signal is used, the difference in relative THD bias for the three investigated algorithms found by using non-quantized test signal is masked by quantization effect in case of using higher-order windows.

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