Identification of Unified ADC Error Model by Triangular Testing Signal

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Abstract- Modelling of the integral nonlinearity by the unified behavioural error model expressed as one dimensional image in the code *k* domain requires a minimal number of the error parameters. The unified error model consists of low and high code frequency components. This paper presents a new approach how to determine the parameters of the low code frequency component by the same signal, that was proposed earlier for determining the high code frequency component. The authors prove that the triangular testing signal with exact DC component allows determining the model parameters with a sufficient accuracy.

I. Introduction

Differential and integral nonlinearities *(DNL*(*k*), *INL*(*k*)) of analogue-to-digital converter (ADC) are the functions of the code bin *k* and depend mainly on its architecture, the chip layout and on the technological procedure. Various non standardised ADC testing techniques can be found in [1][2][3][4]. The improvement in the technology suppressed the regularity in the error behaviours [3]. Because of this fact authors proposed the error model expressed as one dimensional image in the code *k* domain [1] consisting of two components:

a) The low code frequency component (LCF) represented by modelled polynomial approximation ${}^{LCF}INL_m(k)$ can be expressed by a polynomial of *L*-th order described by the formula

$$INL_m(k) = A_0 + A_1 k + A_2 k^2 + ..A_L k^L \qquad (1)$$

b) The high code frequency component (HCF) ${}^{HCF}INL_m(k)$ caused by significant deviations from the mean value of the differential nonlinearities $DNL_m(k)$. The code bins with significantly different nonlinearities have both the regular occurrence of the modelled values of $DNL_m(k)$, and a casual appearance. The periodical occurrence of various types of DNL similar to the Rademacher function system is the most frequent situation. [1] The advance in the ADC technology suppresses the main regularity in the DNL behaviours caused by the mismatching of the weighting components in the subcircuit on the chip.

The modelled shape of the integral nonlinearity using both components is as follows

$$INL_m(k) = {}^{LCF}INL_m(k) + {}^{HCF}INL_m(k) = {}^{LCF}INL_m(k) + \sum_{l=0}^{k} DNL_m(l) \qquad (2)$$

While the component ${}^{LCF}INL_m(k)$ represents the averaged nonlinearity of the ADC, the superimposed ${}^{HCF}INL_m(k)$ component describes major discontinuities in INL shape. The easiest description of HCF component rises from the measured significant values of the modelled values $DNL_m(k)$.

II. The proposed testing approach

The first step in the estimation of both components is the histogram test over full scale range (FS) using triangular testing voltage with the peak-to-peak value larger than FS of ADC under test. The scope of this first rough test is to estimate code bins k_H with an extreme value of the *DNL*(*k*). The HCF component is measured around these code bins in the second step. The significant deviations in the *DNL*(*k*) values comparing with the average DNL value cause the remarkable singularities in the INL shape. The singularities in the code bins k_H increase the estimation uncertainty of the average *INL*(*k*) value around the points using low-pass moving-average (MA) as it was proposed in [1],[3].

The second test step is the estimation of the HCF component by the histogram test using triangular voltage with reduced peak-to-peak value. The differential nonlinearity of the code bin k_H under consideration is calculated by the formula

$$DNL(k_H) = O(k_H)\frac{X_{pp}}{M.Q'} - 1 \qquad (3)$$

Here $O(k_H)$ is the number of samples collected within the k_H-th bin of the histogram. The input of the ADC is excited by a triangular voltage with the peak-to-peak value equals to $X_{pp} = x_{max} - x_{min}$. The value Q' is the mean code bin width value. In order to assure uniform distribution of the input signal, an integer number of periods with number of samples M is chosen from the whole record.

The equation (3) could be simplified by excluding occurrences $O(k_1)$ $O(k_2)$ for code bins k_1, k_2 at both ends of the acquired histogram. Those occurrences represent code bins not fully covered by the testing signal. Let consider K to be a number of code bins fully covered by the testing signal. The average occurrence of samples of one code bin is $\overline{O} = P'/K$. Differential nonlinearity as a deviation from the equidistance of every code bin could be estimated by the

$$DNL(k_H) = O(k_H)\frac{1}{\overline{O}} - 1 \qquad (4)$$

For this reason X_{pp} has no significant metrological impact on $DNL(k_H)$ determination. The differential nonlinearities in the other code bins are estimated with value DNL_{av} representing the average DNL value similar to the "measurement noise". The sum of the differential nonlinearities over the whole ADC range must satisfy the condition that it is equal to 0 if the end-points straight line definition is used [5]. Authors in the preceding papers [2] proved that the decreasing the peak-to-peak value of the testing signal proportionally reduces the signal distortion and the uncertainty of $DNL(k_H)$ estimation.

The LCF component is being measured in the third testing step. In order to avoid the requirement on the harmonic signal generator with extremely low distortion, authors are proposing a new method based on the testing of average ${}^{LCF}INL_m(k)$ around important code bins k_L of the LCF component by the same triangular signal generator with low peak-to-peak value. The required $\overline{{}^{LCF}INL(k_L)}$ is represented by the average value

$$\overline{{}^{LCF}INL(k_L)} = \frac{1}{2s+1}\sum_{j=-s}^{s} INL(k_L + j) \qquad (5)$$

where P=2s+1 is the length of averaging window. The real ADC can be considered as a cascade of two blocks: a nonlinear distortion circuit and an ideal quantisation block (Fig.1.)

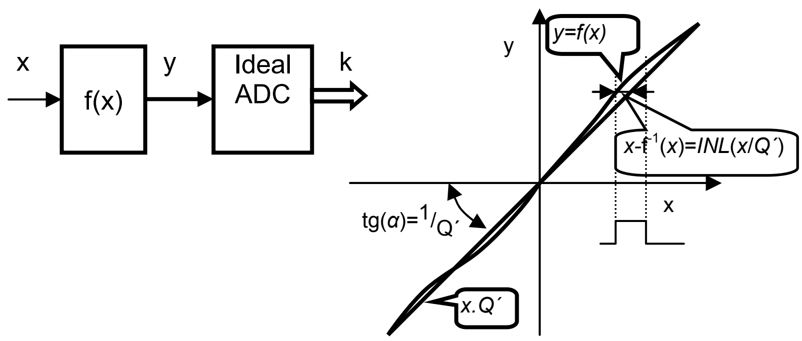

Figure 1. Block diagram of ADC and the transfer function of the first distortion block.

The average output value $\overline{y}$ for the input signal x with uniform $pdf(x)$ function is determined by the convolution taking in the account the condition, that the differential nonlinearity of ADC under test in the tested input range $<x_{min}, x_{max}>$ does not exceed the average value DNL_{av}.

$$|DNL(k)| \le DNL_{av} \qquad \text{for} \quad k \in \left\langle {x_{min}}/{Q'}, {x_{max}}/{Q'} \right\rangle \qquad (6)$$

Then the mean value

$$\overline{Y(k_L)} = \overline{pdf(y)} = \overline{pdf(x) \otimes f(x)} = \int_{x_{min}}^{x_{max}} f(x)\,dx = \frac{1}{x_{max} - x_{min}} \sum_{x_{min}}^{x_{max}} \left(\frac{x}{Q'} + {}^{LCF}INL\left(\frac{x}{Q'}\right) \right) \qquad (7)$$

The testing signal in the proposed method is the triangular signal with integer number of periods taken into processing. A half-period of the signal can be described by the formula $x(i) = X_0 + i\frac{(x_{max} - x_{min})}{I}$.

Because of the signal ergodicity, the $\overline{Y(k_L)}$ value could be calculated by the averaging of the data on the output of ADC.

$$\overline{Y(k_L)} = \frac{1}{I}\sum_{-I/2}^{I/2} k\left(x(i)\right) = \bar{k} = \frac{1}{I}\sum_{-I/2}^{I/2}\left(\frac{X_0 + i\frac{x_{\max}-x_{\min}}{I}}{Q'} + {}^{LCF}INL\left(\frac{X_0 + i\frac{x_{\max}-x_{\min}}{I}}{Q'}\right)\right) \qquad (8)$$

Formula (8) contains effect of the quantisation noise averaged and shows the shift under LSB. The integral nonlinearity is being determined for the code transition levels $T(k_L)$. Let consider the input voltage $X_0 = k_L.Q' + \Delta X$, which does not fit with the closest code transition levels $T(k_L)$. Here the value ΔX corresponds to $\Delta X < Q'$. The triangular shape of the testing signal with the shifted DC value X_0 causes that the mean value follows the linear interpolated shape of the ideal transfer function corrupted by the integral nonlinearities at two borders of the measured code bin.

$$\bar{k} = k_L + \frac{\Delta X}{Q'} + \overline{{}^{LCF}INL\left(k_L\right)} + \frac{\Delta X}{Q'}\left[\overline{{}^{LCF}INL\left(k_L+1\right)} - \overline{{}^{LCF}INL\left(k_L\right)}\right] \qquad (9)$$

The averaged values of the integral nonlinearities in two neighbouring code bins, $\overline{{}^{LCF}INL\left(k_L+1\right)}$ and $\overline{{}^{LCF}INL\left(k_L\right)}$ are nearly equal, the interpolating contribution could be neglected and, comparing the equations (8) and (9), the average value for closest code levels k_L is

$$\overline{{}^{LCF}INL(k_L)} = \bar{k} - \frac{X_0}{Q'} = \bar{k} - \left(k_L + \frac{\Delta X}{Q'}\right) \qquad (10)$$

When the values of $\overline{{}^{LCF}INL\left(k_L\right)}$ in some code bins – node points are known, the whole LCF component will be determined by the LMS algorithm or by spline interpolation. The pros and cons for various interpolation methods have been described in [1].

The testing generator, suitable for testing in all three steps, has to meet following requirements:

- Exact average DC value X_0. The X_0 value is only parameter which must be known with metrological accuracy for $\overline{{}^{LCF}INL\left(k_L\right)}$ testing.
- Peak-to-peak value X_{pp} of the input voltage $<x_{\min}, x_{\max}>$ must be stable only during measuring $\overline{{}^{LCF}INL\left(k_L\right)}$ for one k_L. Its exact value has no metrological impact on the $DNL(k_H)$ estimation.
- Distortion of the triangular testing signal could be average. The reduction of instability of amplitude increases proportionally the testing accuracy. The sufficiently small amplitude drift (few tenths of code bins) allow to estimate the $\overline{{}^{LCF}INL\left(k_L\right)}$ with a required accuracy.

The conceptual block diagram of the testing generator which meets the requirement of all testing steps is shown in Fig.2. The peak-to-peak value of testing signal is set by the $R_2/(R_3||R_1)$ ratio and offset X_0 by the $R_1/(R_3||R_2)$ ratio, respectively.

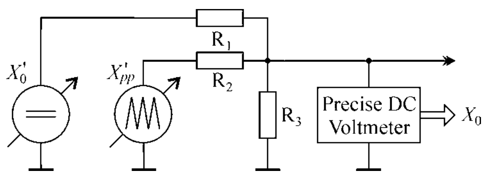

Figure 2. Block diagram of stimulus signal generator

III. Simulation and experimental results

The suggested new test method was verified by simulations and experiments The experimental setup consisted of multifunction DAQ card Lab-PC-1200 with embedded 12bits ADC, analogue function generator NG1.81, and the Agilent 34970A multimeter in function of precise voltmeter.

The acquired results for the first testing step are showed in Fig. 3. Fig. 3a presents the histogram obtained from 400k samples and Fig. 3b DNL calculated from this histogram.

Experiment indicated that the testing points with the significant DNL values lays in the eights of the ADC full scale range (7) and in some additional irregular casual points (4).

In the second step, the triangular testing signal with the reduced peak-to-peak value around previously chosen codes k_H (7, 11 respectively) was applied to acquire histograms determining DNL with higher accuracy. Fig. 4a and 4b present the obtained histograms and Fig. 4c and 4d show $^{HCF}INL_m(k)$ modelled from DNL values calculated from the related histograms.

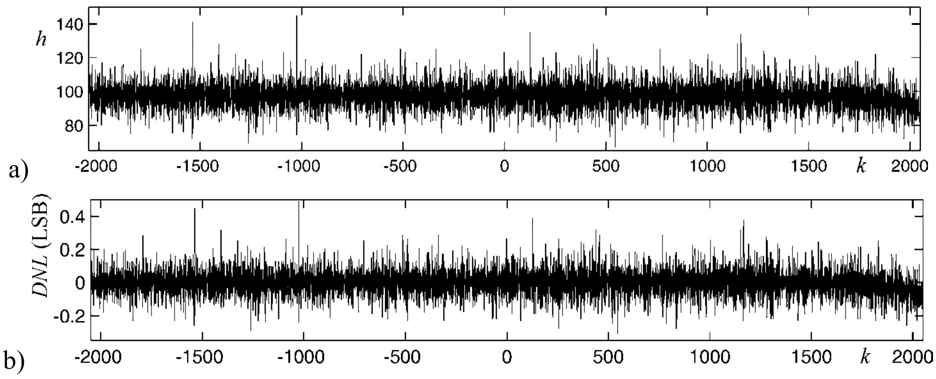

Figure 3. a) Histogram of 400k samples for Lab-PC-1200 b) related DNL calculated from histogram

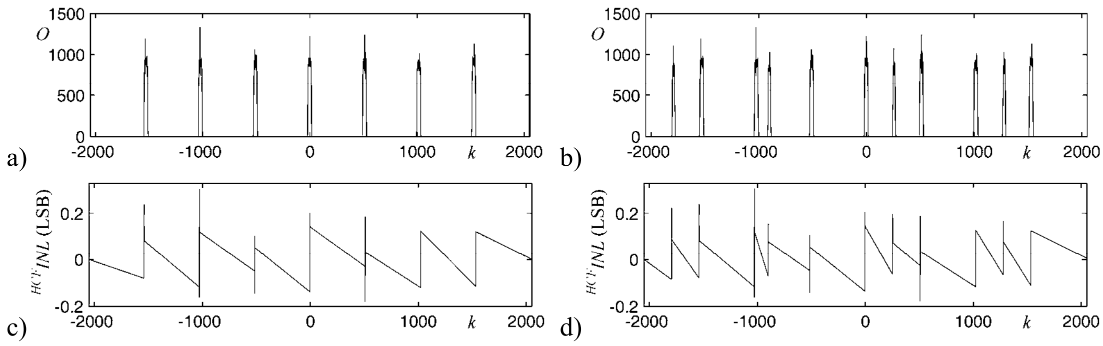

Figure 4. Histograms for reduced peak-to-peak value triangular testing signal a), b) and related $^{HCF}INL(k)$ c), d).

The third testing step contains the main novelty in suggested new test method. In contrary to the approach proposed in [2] the third testing step utilizes the same testing signal as is used in the second testing step for the LCF component determination. That is why the idea was verified by simulations at first and then by experiment. The simulations come from known INL of 12 bit ADC embedded in microcontroller ADuC812 and INL of DAQ card Lab-PC-1200 previously measured by the standardised static test method. The significant nodes points k_L were determined and the averaged values $\overline{Y(k_L)}$ were calculated for P=50. The influence of the node points position on $^{LCF}INL(k)$ was examined. The simulations indicate that two of node points have to be located near the borders of ADC FS, i.e. the extrapolation of $^{LFC}INL(k)$ should be minimised (Fig. 5). Increasing number of the node points decreases the sensitivity of the method on the node points placing (Fig. 5c, 5d).

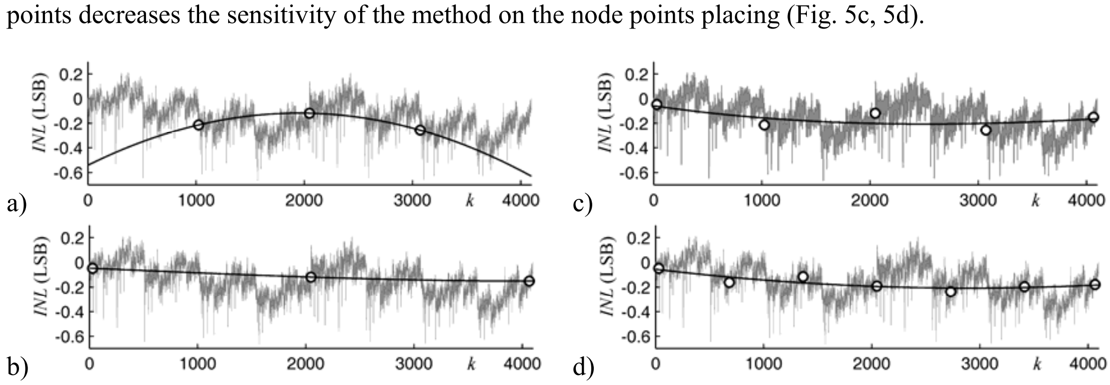

Figure 5. Influence of the node points number and position on LCF component determination. Simulation with INL of ADuC 812, P=50.

The next arising question was position of node points within the ADC FS. This problem was analysed by the next simulation coming from the same condition as the previous ones. As first the position of the third node point was moved between the beginning and the end node point. The LCF component was calculated by the LMS algorithm. Standard deviation σ (11) as function of the position of the third node point is plotted in Fig. 6a, 6c. for both known INLs. The function of the standard deviation shows the minimums around the middle of the FS. For the ADCs under test the standard deviation plots have small local minimum if the node point is placed exactly to the significant points k_H in the middle of FS. In the following simulation the position of the third node point was set into the middle of the ADC FS and the forth point was moved along the testing range. Graphs of standard deviation (Fig. 6b, 6d) show minimums near to the 1st and 3rd fourths. Analogously the equidistant distribution of the node points can be considered to be optimal for the LCF component determination with the maximal accuracy. The accuracy of modelling was assessed by standard deviation σ between the measured $INL(k)$ and modelled $INL_m(k)$ function according to formula

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{k=1}^{N-1}\left(INL(k) - INL_m(k)\right)^2} \tag{11}$$

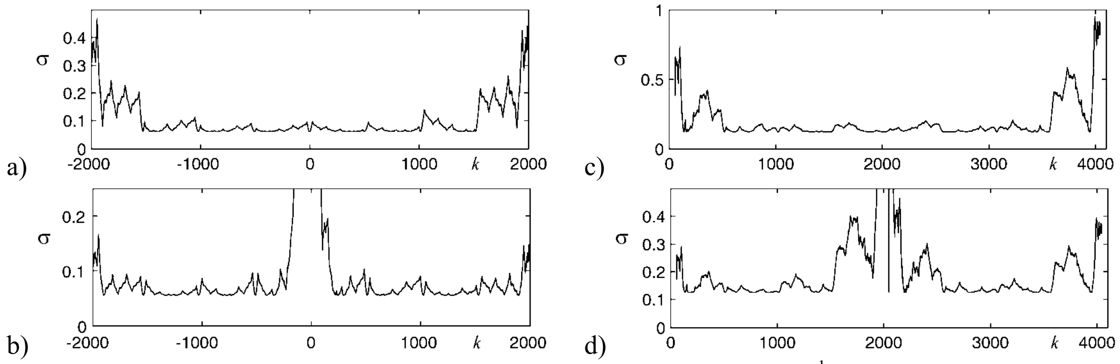

Figure 6. Standard deviation of LCF component as function of the a) c) 3rd node point position, L=2
b) d) 4th node point position; L=3; a) b) simulation with INL of Lab-PC-1200,
c) d) simulation with the INL of ADuC812, P=50.

The optimal peak-to-peak value $X_{pp} = P \cdot Q'$ has been analysed in the last simulation using the same simulation conditions as in the previous ones. The node points were set equidistantly from the beginning to the end of the ADC FS. For various numbers of the node points and various values P, the values of node points were calculated. Than the LCF component has been determined by the LMS algorithm and the standard deviation was calculated. As shown in Fig. 7a and 7b, the standard deviation σ decreases with increasing value of the parameter P. The simulation shows that the optimal peak-to-peak value X_{pp} covers about 50-70 code bins of the ADCs under test. The optimal number of node points for our simulations was 9, but also other numbers of node points, except 8, allow to approximate the LCF component with an acceptable accuracy. The results of LCF component modelling with optimal parameters are in Fig. 8.

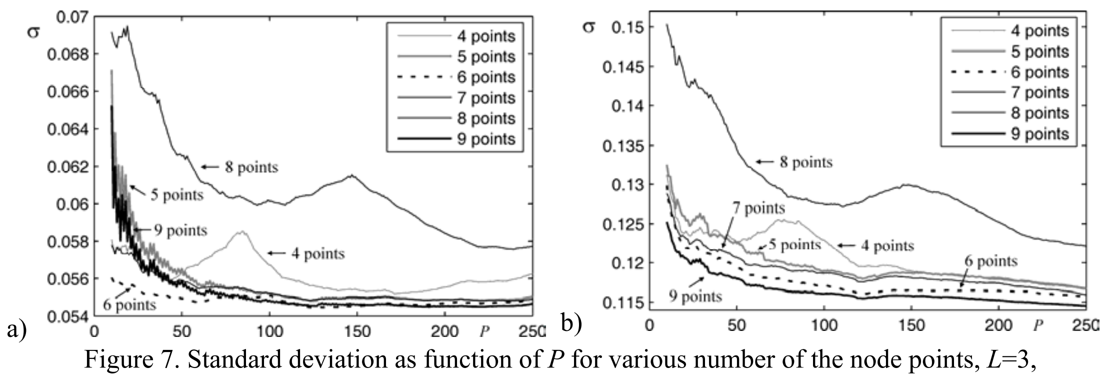

Figure 7. Standard deviation as function of P for various number of the node points, L=3,
a) simulation with Lab-PC-1200 INL, b) simulation with the INL of ADuC812.

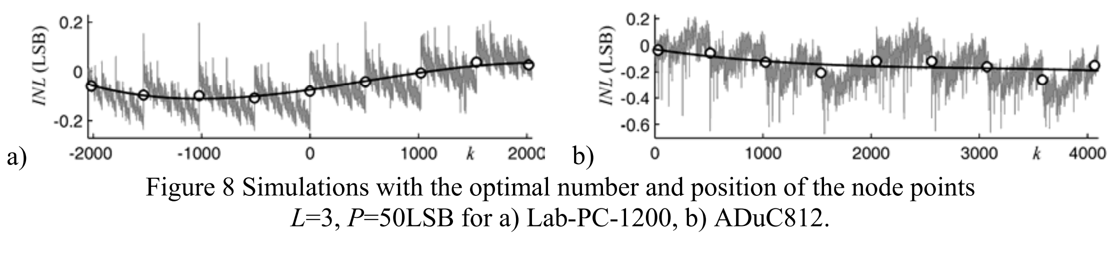

Figure 8 Simulations with the optimal number and position of the node points
L=3, P=50LSB for a) Lab-PC-1200, b) ADuC812.

Coming from the simulations the experiments were performed with the experimental setup mentioned above. The offset X_0 of the triangular testing signal was measured by the Agilent 34970A with its resolution equal to 25 bit. Integration time of the voltmeter was equal to 20 power line cycles (50Hz) and the testing signal frequency was 100Hz. The acquired results for 5 and 9 node points and ${}^{LCF}INL_m(k)$ estimation are shown in Fig. 9. The number of averaged samples varied from 100 to 10kS. Increasing the number of averaged samples for one node point calculation did not have noticeable impact on the uncertainty. If the gain and offset compensation is done according to the end-points straight line definition, the value of both end node points are equal to zero, and it has no sense to measure them. Otherwise the values of all node points must be determined by measurement, and they carry information about all: offset, gain and nonlinearity. Fig. 9 shows results of testing after the offset and gain correction using the least squares fit definition [5].
The final results of Lab-PC-1200 testing using proposed method composed of the LCF and HCF component, in comparison with the known INL, are presented in Fig. 10.

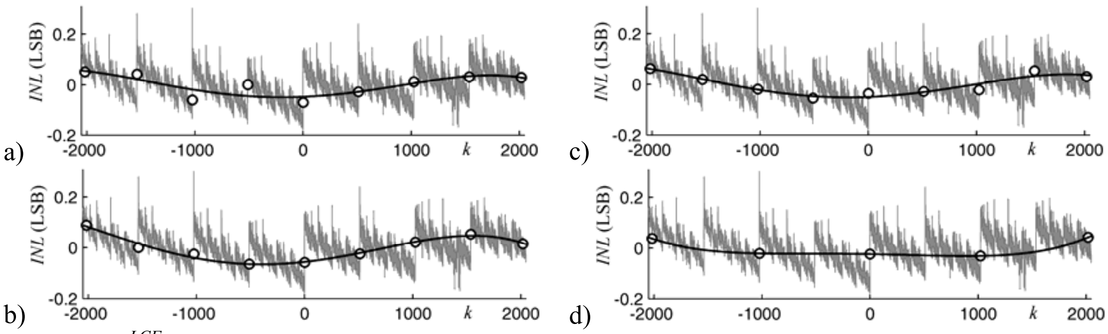

Figure 9 ${}^{LCF}INL(k)$ determined from the measured node points on LAB-PC-1200 L=4 compared to reference INL (grey), triangular signal frequency 100Hz, X_{pp}=125mV, V_{FS}=10V, 9 node points, each node point was calculated from a) 100 b) 500 c) 5000 samples d) 4 node points and 10kS

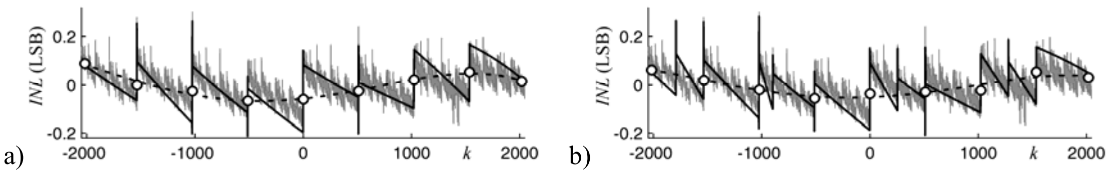

Figure 10 $INL(k)$ of LAB-PC-1200 modelled by the unified error model L=4
a) ${}^{HCF}INL(k)$ from 7 points histogram, 9 node points, each calculated from 500 samples,
b) ${}^{HCF}INL(k)$ from 11 points histogram, 9 node points, each calculated from 5000 samples

IV. Conclusions

This paper presents advantages of the new proposed method based on the identification of the unified error model parameters by the triangular stimulus signal with a DC value. The main advantages of the proposed method are (i) the possibility to use the same simple generator with only one metrological precisely known value X_0 - DC offset which can be simply measured by a precise voltmeter (ii). The third advantage (iii) is that the little number of measurements gives sufficiently precise information about the whole INL function composed from the LCF and HCF component. The performed simulations and experimental results confirmed the practicability of the proposed fast testing method.

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