Comparison of Capacitors by Means of a Double-Channel Digitizer

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Abstract- In the paper, a comparative method of measurement of the capacitors by measuring the voltages across them and phase angle between is presented. In the analysis of errors the systematic loading effects of the connection and their elimination has been described. The reductions of the systematic and the random errors of sampling and quantization have been performed in the frequency domain. Application of the method is possible with support of the modern digital signal processors and with interpolated DFT to improve the accuracy and speed of the measurement.

Keywords- measurement of capacitors, comparative method, error analysis, loading effect, leakage effect

I. Introduction

In measurement practice and especially in the industrial environment the comparative methods on series of the measurement objects are frequently used. The parameters of the measurement objects are compared with the parameters of the reference one. This could be a reference etalon or it could be taken from the series of the objects and the suitable measurand is precisely measured. Because the values of the particular quantities are almost the same in the comparative measurements, the systematic effects of influence quantities can be well reduced [1], [2].

The base electrical parameters of capacitors are the capacitance C and the quality of a dielectric expressed in terms of its 'loss tangent' or dissipation factor

$$d = \tan\delta = \frac{P}{Q} = \frac{U_R I}{U_C I} = \omega RC \,, \qquad (1)$$

where R is an associated resistance in serial with capacitance C. In the complete equivalent scheme we must take into consideration also the associated inductance L and the impedances of the terminals [3]. All mentioned quantities are frequency depended and particularly R and $\tan\delta$ are closely related with the analysis of the frequency characteristic of the impedance $\underline{Z}$, which represents the unknown capacitor.

In most cases of the estimation of the frequency characteristics the bridge methods of measurement are used, where come to the front the active AC bridges [4], [5] and their derivation [6]. The measurements of the amplitude and the phase characteristics of the capacitors with the 'method of two voltmeters' are based on the measurement of the voltages across two almost equal capacitors with the intention of using the benefits of the comparative method also in the AC circumstances (Fig. 1.).

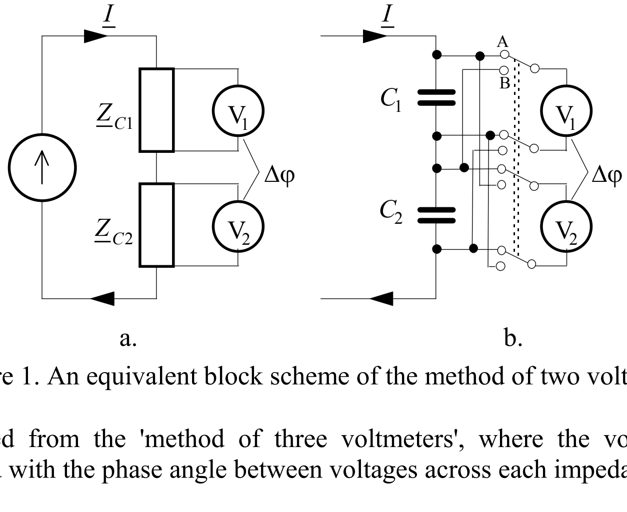


Figure 1. An equivalent block scheme of the method of two voltmeters

The method is derived from the 'method of three voltmeters', where the voltage (fasor) over both impedances is replaced with the phase angle between voltages across each impedance.

II. Systematic errors of loading and their correction

In the serial equivalent scheme for capacitor, the amplitude part of impedance can be written as:

$$Z_C = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} \approx \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \,. \tag{2}$$

In the approximation, the lower excitation frequency than $f < 1/2\pi \cdot \sqrt{1/LC}$ is considered (for example: $L < 10\text{nH}$, $C < 100\mu\text{F}$, $f < 160\text{kHz}$).

The ratio of the amplitude values of the voltages across two capacitors with the same sinusoidal excitation current is inversely proportional to the ratio of the capacitances if we consider the value for loss tangent lower than 10^{-2} ($\tan\delta = \omega RC << 1 \Rightarrow R << 1/\omega C$).

$$r = \frac{U_1}{U_2} = \frac{\underline{I}Z_{C1}}{\underline{I}Z_{C2}} = \frac{\sqrt{R_1^2 + \left(\frac{1}{\omega C_1}\right)^2}}{\sqrt{R_2^2 + \left(\frac{1}{\omega C_2}\right)^2}} \approx \frac{C_2}{C_1} \tag{3}$$

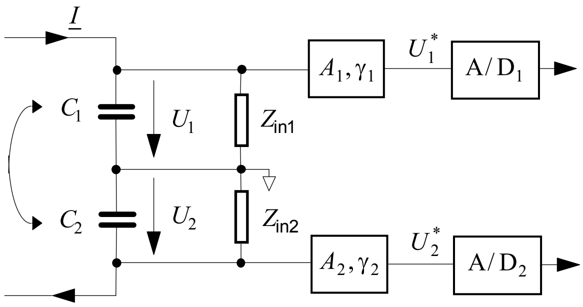


Figure 2. An equivalent scheme of the inputs

The voltmeters, which have been realized with the two-channel A/D converter (BB: DSP102) and the suitable front signal conditioning stages, have different gains (A_1, A_2). They also have different input impedances ($Z_{\text{in}1}$, $Z_{\text{in}2}$) including stray impedances (capacitances) of connection terminals to common ground (Fig. 2.).

Voltages U_1^*, U_2^* at inputs of A/D converters are systematically distorted with the loading effects of connection $\delta_1 = -Z_{C1}/(Z_{C1} + Z_{\text{in}1}) \approx -Z_{C2}/(Z_{C2} + Z_{\text{in}1})$; $\delta_2 = -Z_{C1}/(Z_{C1} + Z_{\text{in}2}) \approx -Z_{C2}/(Z_{C2} + Z_{\text{in}2})$

Input voltages U_1, U_2 are also different amplified. The gains of the input buffers, the nominal amplification factors, and the gains of A/D converters are included in the gain factors A_1 and A_2. For voltages U_1^* and U_2^*, it can be easily deduced

$$U_1^* = A_1 \underline{I} Z_1 (1 + \delta_1) \qquad U_2^* = A_2 \underline{I} Z_2 (1 + \delta_2). \tag{4}$$

Because the capacitances C_1 and C_2 are closely in values and considering that the input impedances are also very close, if we construct the input stages with equal components ($\delta_1 \approx \delta_2$), we could satisfactory eliminate the systematic errors with the ratio of voltages.

$$r_A = \frac{U_{1A}^*}{U_{2A}^*} = \frac{A_1 Z_1 (1 + \delta_1)}{A_2 Z_2 (1 + \delta_2)} \approx \frac{A_1 (1 + \delta_1) C_2}{A_2 (1 + \delta_2) C_1} = \frac{A_1 (1 + \delta_1)}{A_2 (1 + \delta_2)} r_C \tag{5}$$

If we want to eliminate the different gain factors A_1 and A_2, we change the positions of the capacitors (switch position A to B of the commutator on Fig. 1b.).

$$r_B = \frac{U_{1B}^*}{U_{2B}^*} = \frac{A_1 Z_2 (1+\delta_1)}{A_2 Z_1 (1+\delta_2)} \approx \frac{A_1 (1+\delta_1) C_1}{A_2 (1+\delta_2) C_2} = \frac{A_1 (1+\delta_1)}{A_2 (1+\delta_2)} \frac{1}{r_C} \tag{6}$$

We can deduce the desired ratio of two capacitances from (5) and (6), if their values are close enough ($|C_1 - C_2| / C_1 < 10^{-2}$).

$$\frac{r_A}{r_B} = r_C^2 \quad \Rightarrow \quad \frac{C_2}{C_1} = \sqrt{\frac{r_A}{r_B}} \tag{7}$$

Like amplitude, the phase angle is also distorted. The problem of estimation of the difference of the loss tangents ($\tan\delta_1 = 1/\tan\varphi_1 << 1$, $\tan\delta_2 = 1/\tan\varphi_2 << 1$; $\varphi_1 = \angle(\underline{I}, \underline{U}_1)$, $\varphi_2 = \angle(\underline{I}, \underline{U}_2)$) can be translated into the problem of the estimation of a small phase difference between sinus signals ($\Delta\delta = \delta_1 - \delta_2 = \varphi_2 - \varphi_1 = -\Delta\varphi$).

$$\tan(\delta_1 \pm \Delta\delta) \approx \tan(\delta_1) \pm \tan(\Delta\delta) \tag{8}$$

Systematic phase deformations (γ_1, γ_2) could be eliminated with the subtraction in the scheme of changing the positions of the impedances.

$$\begin{aligned} \varphi_{1A}^* &= \varphi_1 + \gamma_1 \qquad \varphi_{2A}^* = \varphi_2 + \gamma_2 \\ \varphi_{1B}^* &= \varphi_2 + \gamma_1 \qquad \varphi_{2B}^* = \varphi_1 + \gamma_2 \end{aligned} \tag{9}$$

The additional phase angles between voltages and current γ_1 and γ_2 are the contributions of the input stages and the A/D converters. After exchange of the capacitor positions the phase difference can be easily obtained as

$$\begin{aligned} \Delta\varphi_A^* &= \varphi_{1A}^* - \varphi_{2A}^* = \varphi_1 - \varphi_2 + \gamma_1 - \gamma_2 \\ \Delta\varphi_B^* &= \varphi_{1B}^* - \varphi_{2B}^* = \varphi_2 - \varphi_1 + \gamma_1 - \gamma_2 \end{aligned} \tag{10}$$

$$\Delta\varphi = \varphi_1 - \varphi_2 = \frac{\Delta\varphi_A^* - \Delta\varphi_B^*}{2} = -\Delta\delta \tag{11}$$

IV. Systematic and random errors of sampling

The first step in the digital measurement procedure is a uniform sampling of the time dependent signal. The sampled analog signal $g(t)$, in our case is sinusoid with excitation frequency f_0 and the additional disturbing frequencies f_m, can be written as follows:

$$g(k\Delta t)_N = w(k) \qquad g(k\Delta t)_\infty = \sum_{m=0}^{M} A_m \sin(2\pi f_m k\Delta t + \varphi_m) \tag{12}$$

From the infinite line of sampling points only a finite number N ($k = 0,1,\ldots,N-1$) is taken with a multiplication of the theoretically infinite sampled signal with a window of finite duration $w(k)$. A_m, f_m, and φ_m are the amplitude, frequency, and phase relevant to the measured component of the signal. Sampling frequency $f_S = 1/\Delta t$ is supposed to fulfil the Shannon theorem $f_S > 2f_M$ (f_M - the highest frequency component).

Continuing the measurement procedure, we make DFT on the obtained sequence of samples. Multiplication of the sampled signal $g(k\Delta t)$ by the finite duration window $w(k)$ in the time domain causes convolution of the frequency transform of the signal (for the single component that is Dirac impulse with amplitude A_m and phase φ_m) with the frequency transform of the window $W(\theta)$. The DFT of the signal on N sampled points (12) at the spectral line i is given by [7]:

$$G(i) = -\frac{j}{2} \sum_{m=0}^{M} A_m \left[W(i-\theta_m) e^{j\varphi_m} - W(i+\theta_m) e^{-j\varphi_m} \right] \tag{13}$$

θ_m is the component frequency related to base frequency resolution depending on the window span $\Delta f = 1/N\Delta t$. Spectral lines of the signal do not coincide with the DFT coefficients in general, unless the measurement interval $N\Delta t$ is the integer multiple of the period of the frequency signal component $T_m = 1/f_m$. For this reason the relative frequency can be written in two parts:

$$\theta_m = \frac{f_m}{\Delta f} = i_m + x_m \ ; \quad -0{,}5 < x_m < 0{,}5 \tag{14}$$

i_m is an integer value, determining approximately the position of the signal component. The displacement term x_m is caused by nonsinhronization.

For displacement x_m estimation of the single component, at first, we idealize circumstances and in the equation (13) neglect the second part and contributions of other signal components. This means in practice that the measurement lasts long enough, or, the frequency components are sufficiently interspaced [8]. For a two-point interpolation the local maximum in the amplitude part of DFT with the largest coefficients $|G(i)|$ and $|G(i+1)|$, surrounding the position of component m, must be found. Considering the equation (14), we can write as follows:

$$\alpha_m = \frac{|G(i_m \pm 1)|}{|G(i_m)|} = \frac{|W(1-x_m)|}{|W(x_m)|} \quad \Rightarrow \quad x_m = f(\alpha_m) \tag{15}$$

The component amplitude is obtained if the function $W(\theta)$ of the window used is analytically known.

- rectangular window: $A_m = \frac{2}{N}|G_m(i)|\frac{\pi x_m}{|\sin(\pi x_m)|}$

- Hanning window: $A_m = \frac{2}{N}|G_m(i)|\frac{\pi x_m(1-x_m^2)}{|\sin(\pi x_m)|}$ (16)

For the component phase can be written [10]:

$$\varphi_m = \arg G_m(i) - \pi\frac{N-1}{N}x_m + \frac{\pi}{2} \tag{17}$$

The nonsinhronization of the measurement interval $T = N\Delta t$ with multiple of the period of the frequency signal component T_m, that exhibits as shifting of the component frequency f_m from the largest local amplitude coefficient of the DFT, cause the short-range and the long-range leakage influences on the DFT coefficients. The short-range leakage effects on the amplitude and phase estimation can be well corrected by interpolations as in (16) and (17). The long-range leakage effects are reduced with the suitable window and the multipoint interpolation [9]. Both tasks could be very difficult if analytically not well-defined window is used.

The leakage effects can be very reduced with the comparative measurement, if the simultaneousness of the sampling on both channels is assumed and the measurement time of signals is the same. The assurance of these conditions gives equal displacements x_1 and x_2, if the measurement frequency is the same.

The comparative method easily eliminates the displacement contributions in (16) and (17), if $A_{m1} = U_1^*$ and $A_{m2} = U_2^*$ are considered.

$$r = \frac{U_1^*}{U_2^*} \approx \frac{|G_1(i)|}{|G_2(i)|} \ ; \quad \varphi_1 - \varphi_2 \approx \arg G_1(i) - \arg G_2(i) \tag{18}$$

It is reasonable to take the amplitude and the phase of the largest local DFT coefficient, where the signal to noise ratio is the largest. The quantization error is a minimum that must be taken into consideration in the measurement uncertainty of the final result. The error is assumed to be random with the uniform density distribution between $-\Delta/2$ and $\Delta/2$ ($\Delta = G_{\text{Range}}/(2^n - 1)$) in the time space and with the 'white' spectrum in the frequency space. The probability density distribution of DFT coefficients is Gaussian owing to the great number of summations in DFT procedure. For the rectangular window, the SNR is given by [11]:

$$\mathrm{SNR}_{\mathrm{DFT}}(i) = \frac{A}{\Delta}\sqrt{\frac{12N}{\pi}} \quad \text{or} \quad \mathrm{SNR}_{\mathrm{DFT}}(i) = \frac{A}{\Delta}\sqrt{\frac{12N}{\pi \mathrm{ENBW}}} \tag{19}$$

for an arbitrary window. The S/N ratio is improved with the square root of the number of points at given quantization and it decreases with the equivalent noise bandwidth ($ENBW$). The least one has the rectangular window ($ENBW = 1$). In our case Hanning window with larger noise bandwidth ($ENBW = 1{,}5$) has been used, but the systematic leakage errors are smaller.

In spite of good correction properties of the comparative method the measurement time must be close to the multiple of the component period. Even more important, the measurement time should be multiple of the period of the difference between excitation frequency and disturbing ones (In most cases $50/60\mathrm{Hz}$ in the supply systems. $f_m = i \cdot f_{50}$, $i = 1,2,...$ multiple of the supply frequency).

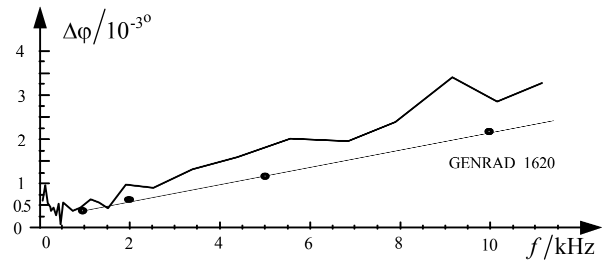
$$T = N\Delta t \approx p\frac{1}{f_0} \approx r\frac{1}{|f_0 - f_{50}|} \tag{20}$$

For Hanning window the constants must be $p, r \geq 3,4,...$ ($p = r(2i+1)$). The excitation frequency should be in the middle of the two successive multiples of the supply frequency ($f_0 = (i + 1/2)f_{50}$). The measurement time is given by:

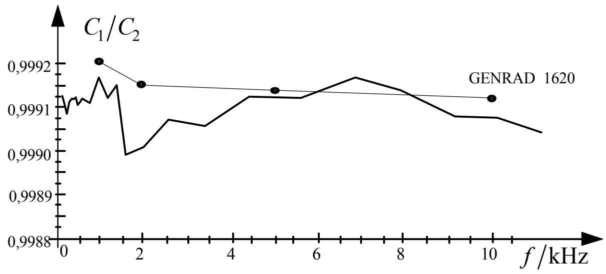
$$T = \frac{p}{(i+1/2)f_{50}} = \frac{r(2i+1)}{i+1/2}T_{50}\,. \tag{21}$$

IV. Experimental results

To check the proposed method a laboratory prototype was constructed. The two-channel A/D converter ($f_S < 200\,\mathrm{kHz}$) with the resolution of 13 effective bits has been used. For signal processing the DSP module with TMS 320C30 has been added.



a.



b.

Figure 3. Phase difference $\Delta\varphi$ (a.) and the ratio of capacitances C_1/C_2 (b.)
($C_1 = 180{,}78\mathrm{nF}$; $C_2 = 180{,}92\mathrm{nF}$)

The distribution of $\Delta\varphi$ measurements has been near the level of the theoretical 12-bit A/D converter ($s(_{15\mathrm{measurements}}) = 0{,}6 \cdot 10^{-3^\circ}$). The results have been compared (Fig. 3.) with the reference measurement instrument GENRAD 1620 (measurement accuracy: $ma_C = 10^{-4}$; $ma_{\tan\delta} = 5 \cdot 10^{-4}$; $ma_{\varphi} = 1{,}7 \cdot 10^{-3^{\circ}}$) and confirm the success of the method and the possibility of its use in the industrial environment.

IV. Conclusions

A comparative method - method of two voltmeters - of measurement of the capacitors by measuring the voltages across them and phase angle between has been presented to show the effectiveness of the measurement system. In the analysis of errors the systematic loading effects of the connection and their elimination has been described.

References

[1] K. B. Klaassen, *Electronic Measurement and Instrumentation*, Cambridge Uni. Press, 1996.

[2] R. Pallas-Areny, J. G. Webster, *Sensors and signal conditioning*, John Wiley & Sons, Inc, 2001.

[3] G. Ramm and H. Moser, "New multifrequency method for the determination of the dissipation factor of capacitors and of the time constant of resistors", *IEEE Transactions on Instrumentation and Measurement*, vol. 54, pp. 521-524, April 2005.

[4] H. Bachmair and R. Vollmert, "Comparison of admittances by means of a digital double-sinewave generator", *IEEE Transactions on Instrumentation and Measurement*, vol. IM-29, pp. 370-372, December 1980.

[5] S. S. Award, N. Narasimhamurthi and W. H. Ward, "Analysis, design and implementation of an AC bridge for impedance measurements', *IEEE Transactions on Instrumentation and Measurement*, vol. 43, pp. 894-899, December 1994.

[6] M. A. Atmanand, V. J. Kumar and V. G. K. Murti, "A novel method of measurement of L and C", *IEEE Transactions on Instrumentation and Measurement*, vol. 44, pp. 898-903, August 1995.

[7] C. Narduzzi and C. Offeli, "Real-time high accuracy measurement of multifrequency waveforms", *IEEE Transactions on instrumentation and measurement*, vol. 36, pp. 964-970, December 1987.

[8] V. K. Jain, W. L. Collins and D. C. Davis, "High-accuracy analog measurements via interpolated FFT", *IEEE Transactions on Instrumentation and Measurement*, vol. IM-28, pp. 113-122, June 1979.

[9] D. Agrež, "Weighted multi-point interpolated DFT to improve amplitude estimation of multi-frequency signal", *IEEE Transactions on Instrumentation and Measurement*, vol. 51, pp. 287-292, 2002.

[10] D. Agrež, "Improving Phase Estimation with Leakage Minimization", *IEEE Transactions on Instrumentation and Measurement*, vol. 54, August 2005.

[11] O. M. Solomon, "The effects of windowing and quantization error on the amplitude of frequency-domain functions", *IEEE Trans. on Instrumentation and Measurement*, vol. 41, pp. 932-937, December 1992.