From Errors to Uncertainties in Basic Course in Electrical Measurements

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Abstract **-** Expressing accuracy of measurement by means of uncertainties becomes step by step common practice both in research and industry. It is therefore highly desirable that university graduates in electrical engineering know how to use uncertainties. That is why we started teaching practical use of uncertainties in basic measurement methods in our basic course in Electrical Measurement and Instrumentation, an obligatory course for all students of Faculty of Electrical Engineering of the Czech Technical University in Prague in the 3rd semester. The used approach completed with some examples of both the demonstration of uncertainty theory implementation in the lectures and the practical application in laboratory exercises are presented in the paper.

I. Introduction

Using measurement uncertainties to characterize measurement accuracy has become required practice in metrological publications in the last years [1], [2]. At present, this practice is being in increasing range used also in common laboratories and in industry [3]. It is therefore highly desirable that all technical university graduates know basics of measurement uncertainty theory and be able to use it at least in simple practical cases. That is why theory of measurement uncertainties has been recently started to be explained and used also in the basic course in Electrical Measurement and Instrumentation at the Faculty of Electrical Engineering of the Czech Technical University in Prague.

The first lesson of course concerns measurement accuracy. The basic principle of uncertainty evaluation is mentioned here. The content and the used approach are presented in par. 2.

The implementation of theory of uncertainty by electrical quantities measurement is mentioned in the lessons, where the relevant methods are explained. It concerns the voltage and current measurement using operational amplifiers, power measurement in 1-phase and 3-phase net, frequency measurement using counters, resistance measurement etc. An example will be presented in par. 3.

As for laboratory exercises, concept of errors has been (apart from systematic (methodical) error correction and instrument errors serving as the base for instrument standard uncertainty estimation) abandoned at present and students are requested to evaluate only uncertainties. The uncertainty is evaluated in about one third of tasks, two typical examples will be presented in par. 4.

II. Explanation of basic rules of evaluating uncertainty of measurement

The measurement accuracy evaluation is the main topic of the first lesson in the basic course in Electrical Measurement and Instrumentation. It starts with classical error definition, which is applied to the measurement using simple instruments. The problem of true value is presented and the definition of type B uncertainty and its meaning follows. The similarity of random error and type A uncertainty is also mentioned and the meaning of expanded uncertainty as well as combined uncertainty is explained. The practical advantage of uncertainty application instead of usage of maximum possible error is underlined mainly for indirect measurement using complex measuring systems. This strategy was applied because according to our opinion the explanation of this topic without understanding the error conception is impossible.

The basic principle of theory of uncertainty is explained, the type A and type B of uncertainties are mentioned and the meaning of expanded uncertainty as well as combined uncertainty is illustrated. The problem of influencing quantities is also briefly mentioned. The evaluation of type B uncertainty is described in more detailed way, because it is more frequently used in practice and its evaluation is asked also in laboratory exercises in the course. The overview of content of the first lesson (a part concerning uncertainties) follows.

The basic quantitative characteristic of uncertainty of measurement is the standard uncertainty. It is the standard deviation of the quantity the uncertainty of which we look for. It is denoted as u. According to the way of their evaluation, the standard uncertainties can be divided into

- Standard deviations of the type (of the category) A (denoted as u_A), which are found from the results of repeated measurements. Their sources are considered to be unknown, and their value decrease with number of measurements.
- Standard deviations of the type (of the category) B (denoted as u_B), which are found by other means than by statistical processing of results of repeated measurements. They are evaluated for individual sources of uncertainty identified for given measurement, and their values do not depend on number of repeated measurements (similarly like systematic errors of measurement). They stem from different sources and their common influence is expressed in resulting uncertainty of type B.

Only one of the types of uncertainties defined above is not sufficient to characterize measurement uncertainty in number of cases. Then it is necessary to find the resulting effect of the combination of both types of uncertainties, A and B. Combined standard uncertainty u_C is used in these cases. It expresses resulting effect of both type A and type B uncertainties and can be found from the formula

$$u_C(x) = \sqrt{u_A^2(x) + u_B^2(x)} \qquad (1)$$

The evaluation of the type A uncertainty is mentioned only in brief. The inference of the type A uncertainty from statistical analysis of a series of repeated measurements is described. (If there are n independent and equally accurate observations, the estimate of resulting value x of the measured quantity X is the value of sample average - arithmetic mean. The uncertainty ascribed to the estimation x can be found as experimental standard deviation of this value, i.e. of arithmetic mean of all observations.)

The evaluation of the type B uncertainty is mentioned in more detail, because instruments used in industrial measurement have usually limited resolution and readings of repeated measurements are mostly practically identical. The situation when the measuring instrument is not used in prescribed working conditions is also mentioned, but the influence of the rather complicated topic of mutual correlation among influencing quantities was omitted from the explained theory.

A. Uncertainty estimation using analogue and digital instruments

The maximum possible measurement error was in classical measurement accuracy evaluation found as the maximum possible deviation between measured and true value. It is determined using the accuracy class AC for analogue instruments and it can be calculated using formula

$$\Delta_I = \frac{AC}{100} M \, , \qquad (2)$$

where M is the used measuring range.

To change over from errors to uncertainties, we suppose that the interval $<-\Delta z_{max}, +\Delta z_{max}>$, in which the value of measured quantity lies with great probability is equal $<-\Delta_I, +\Delta_I>$ and that the observations probability distribution is uniform. The standard uncertainty of the value measured using analogue instruments is then estimated as

$$u_B = \sigma = \frac{\Delta z_{max}}{\sqrt{3}} = \frac{AC/100}{\sqrt{3}} M \qquad (3)$$

Using similar procedure, the change over from "errors" to "uncertainties" can be used also for digital instruments. The maximum possible error (tolerance) $|\Delta_X| = \frac{\delta_1}{100} X + \frac{\delta_2}{100} M$ of the measured value X computed using errors in percent of reading δ_1 and of range δ_2 defines the limits of interval $<-\Delta z_{max}, +\Delta z_{max}>$. Then the standard uncertainty of the measured value is estimated by

$$u_B = \sigma = \frac{\Delta z_{max}}{\sqrt{3}} = \frac{\frac{\delta_1}{100} X + \frac{\delta_2}{100} M}{\sqrt{3}} \qquad (4)$$

In connection with possible application of precise digital multimeters, the type A uncertainty is mentioned ($u_A = s$, where s is an experimental standard deviation) and its meaning is explained together with the combined uncertainty $u_C(x) = \sqrt{u_A^2(x) + u_B^2(x)}$.

The situation when the measuring instrument is not used in prescribed working conditions is explained only in brief. In this case the influencing quantities *Z* reach values outside the range specified by the manufacturer for the validity of error specifications. The sequence of steps to evaluate the uncertainty of meter reading by type B method is described for the case that the influence of these quantities on the meter reading is known.

B. Uncertainty esimation by indirect measurement

Concerning indirect measurement, the much spread method of systematic error estimation, based on replacement of absolute values of absolute errors by differentials and using Taylor series expansion limited to the first-order terms in total differential computation in the neighbourhood of the point (X_1, X_2, …, X_n) was used until now. It means that if the measured quantity *Y* is a function of several quantities X_i measured directly

$$Y = f(X_1, X_2, X_3, ...X_n), \tag{5}$$

the maximum possible absolute error is approximately

$$\Delta Y = \sum_{i=1}^{m} \left| \frac{\partial f}{\partial x_i} \Delta X_i \right| \tag{6}$$

This approach enables easy error estimation for broad class of simple functions, because using this rule, maximum possible absolute errors of sum and of difference of two quantities measured directly can be found as sum of absolute values of partial absolute errors, and relative errors of product of ratio of two directly measured quantities can be found as sum of absolute values of relative errors of these quantities. Students usually preferred using these rules to total differential calculating. However, the error estimation using (6) is too pessimistic. (Random errors were mentioned only in brief in the basic course.)

In theory of uncertainties of measurement ([1], [2]), the law of uncertainties propagation (used also for indirect measurement) is defined for non-correlated input quantities by formula

$$u_y = \sqrt{\sum_{i=1}^{m} \left(\frac{\partial f}{\partial x_i} u_{xi} \right)^2} \tag{7}$$

where u_y is combined standard uncertainty of the measured value *y*
u_{xi} is combined standard uncertainty of the i-th measured value x_i.

The rules mentioned above avoiding necessity of calculating partial derivatives for some simple mathematical relations of input quantities (sum, difference, product and ratio) are applicable also for evaluating uncertainties. However, arithmetic sums must be replaced by geometric sums, and absolute values of absolute or relative errors should be replaced by absolute or relative uncertainties.

III. The implementation of theory of uncertainty by electrical quantities measurement

Parts of lessons describing methods used for electrical quantity measurement, which concerned an error estimation were replaced by application of uncertainty determination. The relevant part of chapter concerning resistance measurement using R $\rightarrow$ U converter can be presented here as a typical example:

The influence of tolerances of used components, parasitic parameters of operational amplifier and the uncertainty of output voltage measurement are taken into account by uncertainty estimation.

The connection presented in the Figure 1 can be used for resistance to voltage conversion. This

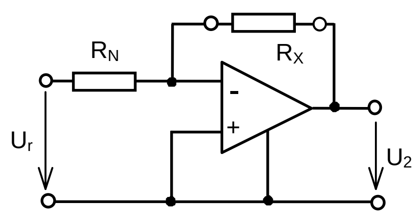

Figure 1. R/U converter with OA

connection is suitable for measurement of resistance of several kΩ or higher. Supposing an ideal operational amplifier, the value of the measured resistance is calculated using the formula

$$R_X = -\frac{U_2}{U_r} R_N \tag{8}$$

In this case, the standard uncertainty of the measured resistance R_X can be estimated applying the uncertainties propagation law:

$$u_{Rx(id)} = \sqrt{\left(\frac{\partial R_X}{\partial R_N} u_{RN}\right)^2 + \left(\frac{\partial R_X}{\partial U_2} u_{U2}\right)^2 + \left(\frac{\partial R_X}{\partial U_r} u_{Ur}\right)^2} = \sqrt{\left(-\frac{U_2}{U_r} u_{RN}\right)^2 + \left(-\frac{R_N}{U_r} u_{U2}\right)^2 + \left(\frac{U_2 R_N}{U_r^2} u_{Ur}\right)^2} \tag{9}$$

where u_{U2} is the standard uncertainty of measurement of output voltage U_2,

$u_{RN} = \Delta R_N / \sqrt{3} = \frac{\delta R_N}{100\sqrt{3}} R_N$ is the standard uncertainty of a value of the resistor R_N; δR_N is the given tolerance of R_N in %,

$u_{Ur} = \Delta U_r / \sqrt{3} = \frac{\delta U_r}{100\sqrt{3}} U_r$ is the standard uncertainty of the reference voltage U_r; δU_r is the given tolerance of U_r in %.

Using a real operational amplifier, the offset voltage U_{OS} and input bias currents I_{1N} and I_{1P} should be taken into account. The Figure 2 shows the influence of these parasitic parameters. I_{1P} is short circuited by the voltage source U_{OS}. The influence of U_{OS} can be neglected because the value of U_r is usually several volts. Then the measured resistance can be calculated according to the formula

$$R_X = -\frac{U_2}{U_r} R_N \pm I_{1N} \frac{R_N^2}{U_r^2} U_2 \tag{10}$$

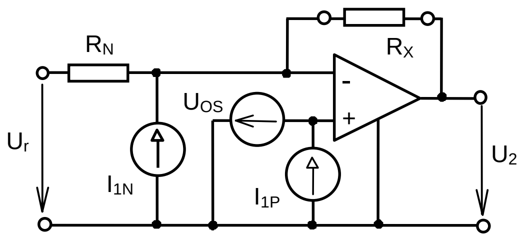

Figure 2. Equivalent circuit of R/U converter with real OA

The maximum value of I_{1N} is given in the OA data sheet allowing the tolerance range definition (the rectangular distribution is supposed). Applying these conditions, the standard uncertainty caused by I_{1N} can be calculated as

$$u_{OA(I_{1N})} = \frac{I_{1N}}{\sqrt{3}}, \tag{11}$$

The standard uncertainty of resistance R_X measurement respecting the input bias current I_{1N} of the OA can be estimated using formula

$$u_{Rx(OA)} = \sqrt{u_{Rx(id)}^2 + \left(\frac{I_{1N}}{\sqrt{3}} \frac{R_N^2}{U_r^2} U_2\right)^2} \tag{12}$$

IV. Typical examples of laboratory tasks containing an uncertainty evaluation

Altogether 17 laboratory tasks are solved by students during laboratory exercises in the basic course in Electrical Measurements, in the following 7 of which the uncertainty of measurement has to be evaluated:

- Measurement of output voltage of a resistance divider using analogue and digital voltmeter
- Low current measurement using analogue and digital mikroammeter and current-voltage converter
- Low voltage measurement using operational amplifier
- Power measurement of one-phase load
- Power measurement of three-phase unbalanced load
- Measurement of frequency and period time using counter
- Low resistance measurement

The tasks are chosen so that the uncertainty of both the direct measurements and the indirect ones are to be evaluated. In the case of direct measurement, the uncertainty of for both analogue (e.g. for power measurement) and digital instruments are determined. In the case of indirect measurements using amplifiers the tolerance of used components (resistors, voltage references) and parasitic parameters of operational amplifiers (offset, input current) are taken into account. Two of the tasks mentioned above were selected and they are presented here as an example:

Low voltage measurement using operational amplifier

- Measure the voltage of the given thermocouple (Fe-Ko) using a DVM and find the expanded measurement uncertainty of this measurement using DVM error specifications (coverage factor 2).
- Using operational amplifier OP7 design the circuit diagram of an inverting voltage amplifier with voltage gain 100 and input resistance 1 kΩ,
- Use the amplifier mentioned above to amplify the thermocouple voltage. The amplifier output voltage should be measured by the same DVM, which was used for direct measurement. Make correction of finite input resistance of the used amplifier (if necessary), the resistance of thermocouple is given. Find the expanded measurement uncertainty (coverage factor $k = 2$) of the thermocouple voltage. Take into account not only error of the DVM and tolerances of the resistors used but also input offset voltage of the operational amplifier.
- Calculate the temperature measured by the thermocouple, if its constant is $K = 54$ μV/°C. Suppose that the temperature of the reference end of the thermocouple is 20 °C (temperature of the laboratory).
- Verify that the real input voltage offset is lower than the value (maximum or even typical) declared in the data sheet.

Power measurement of three-phase unbalanced load

Find the active power and the reactive power of the unbalanced three-phase load in a three wire net (suppose an ideal symmetry of voltage system).

- Measure the active power
  a) using three wattmeters,
  b) using two wattmeters (verify the Blondel theorem by three measurements).
- Measure the reactive power using three wattmeters.
- Make correction of methodical error where appropriate. Estimate the type B expanded uncertainty of all measurements (coverage factor $k = 2$).

V. Conclusions

There was a misgiving, how students will accept the new approach to the measurement accuracy evaluation if uncertainty instead of maximum error will be used already in the basic course. It concerned especially students coming from electrical engineering high schools, who used the maximum error for accuracy evaluation there. To make it easier to change from errors to uncertainties, an appendix to the textbook of the basic course in Czech devoted to uncertainties was added in the last edition [5], and a separate supplementary textbook with similar content was published also in English [4], because in original textbooks [6] the errors were used for accuracy evaluation and the theory of uncertainties was mentioned only in brief. A special web side containing relevant lecture slides was also prepared.

Fortunately, the original doubts concerning the introduction of theory of uncertainties of measurements into the basic course in electrical measurements proved themselves to be false possibly also thanks to a good preparation and an appropriate emphasis devoted to this topic.

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