Noise Figure Measurement in Highly Mismatched Systems

Vaclav Papez[1], Stanislava Papezova[2]

[1]*Faculty of Electrical Engineering, CTU in Prague, Technicka 2, Prague 6, Czech Republic,*
phone: 420 224 352 165, fax: 420 224 313 949, papez@feld.cvut.cz
[2]*Faculty of Mechanical Engineering, CTU in Prague, Technicka 4, Prague 6, Czech Republic,*
phone: 420 224 352 401, fax: 420 224 310 292, papezova@fsid.cvut.czT

Abstract- This paper describes problems of measuring the noise figure when the noise figures are low and there is high mismatching of measurement system. Measurement errors can be identified using this method. Methods limiting the influence of a measurement system with real parameters are presented. The available accuracy in dependence on the parameters of the measurement system is described.

I. Introduction

Noise is a very basic property of all circuits which process low power signals. The noise figure characterizes the quality of the components and equipment used in electronics and telecommunications. The basic method for measuring the noise figure were worked out more than 50 years ago, and they remain fundamentally in use today. However, due to the continually improving parameters of evaluated components and equipment, the measurement quality requirements have also increased. Therefore it is now necessary to measure very low noise figures (0.2 - 0.3 dB) accurately. Effects whose influence have not been evident in measurement previously must now be taken into account. Mismatches in a measuring circuit have a large influence, i. e. mismatches on the side of the measured component and also on the side of the noise generator.

II. Noise figure measurement

The noise figure characterizes the degradation of the signal by the intrinsic noise of the two-port, and it is defined by the relation (1).

$$F = \frac{S_i / N_i}{S_o / N_o} \tag{1}$$

where S_i and S_0 are signal power levels available at the input and the output of the two-port
N_i and N_0 are the available noise power at the input and the output of the two-port

Relation (1) is inconvenient for measuring the noise figure, because noise power is dependent on the noise bandwidth of the power meter. As a result, the Y-factor method is used to measure the noise figure. The noise power has to be measured at the output of the DUT for two different (hot and cold, on and off) temperatures of the noise source. The ratio of these two noise output power levels, N_{cout} and N_{hout}, is known as the Y-factor (2).

$$Y = \frac{N_{hout}}{N_{cout}} = \frac{kT_h B + kT_i B}{kT_c B + kT_i B} = \frac{T_h + T_i}{T_c + T_i} \tag{2}$$

where T_i is the noise temperature which determines the noise power on the input of DUT
T_h and T_c are the noise temperatures of the noise generator switched on and switched off

Using the formula $T_i = (F\text{-}1)T_o$ (T_o is the ambient temperature), F can be expressed with by relation (3).

$$F = \frac{(T_h / T_0 - 1) - Y(T_c / T_0 - 1)}{Y - 1} \tag{3}$$

Simplified with T_{cold}=T_o=290 K and ENR= (T_{hot}/T_{cold}) -1 we get (4)

$$F = \frac{\text{ENR}}{Y - 1} \tag{4}$$

III. Measuring errors

The following error sources commonly occur:

1. Influence of some noise from the power meter in the output of the measured two-port
2. Power loss between the noise generator and an input of a measured two-port
3. Noise source mismatching
4. Noise source temperature dependence

1. The influence of the noise power meter can be corrected according to the Friis formula for a two-stage cascade; this process is called second-stage correction [1].

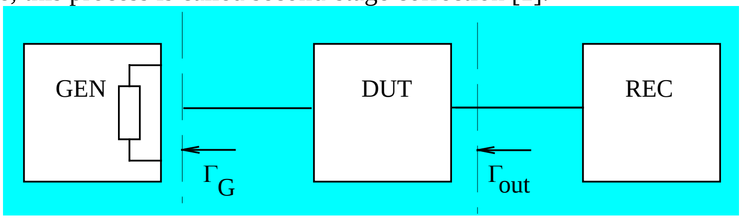

Fig. 1 Noise figure measurement

$$F_{DUT(\Gamma G)} = F_{\Gamma(G)} - \frac{F_{REC(\Gamma out)} - 1}{G_T} \tag{5}$$

where:

Γ_G	reflection coefficient of the noise generator
Γ_{out}	reflection coefficient of the output DUT
G_T	available gain of DUT
F_{REC}	noise figure of the receiver
F	indicated noise figure of the system
F_{DUT}	noise figure of the measured DUT

The secondary correction error due to the dependence of F_{rec} on Γ_{out} and G_T on Γ_G is not great, even for minimal noise figures, if F_{rec} is small (e.g 2) and G_T achieves an order of hundreds. In other cases, the best corrections to place a further low noise amplifier behind the measured two-port network.

2. The power loss between the noise generator and the input has a direct influence on ENR. A measurement error can be made by damping the cables and connectors, which we have to keep in the range of several hundredths till tenths dB. The matching network, which must also have minimum losses, is also a source of such error during component measurement. The inductors and capacitors must have the Q factor Qo approximately 1000, and the operating Q factor of the loaded networks Qz should be minimal. Ratio Qo/Qz =20 makes an error of 0,2 dB.
3. It is suitable to analyse the influence on the Y factor directly according to the definition relation for assessing of the influence of a mismatched noise generator on the value of the Y factor.

$$Y = \frac{N_{h\,out}}{N_{c\,out}} = \frac{G_{ON}\left(N_{ON} - N_{DON}\right)}{G_{OFF}\left(N_{OFF} + N_{DOFF}\right)} \tag{6}$$

where:

G_{ON}	is the transducer gain with the noise generator switched on
G_{OFF}	the transducer gain with the noise generator switched off
N_{DON}	intrinsic noise power of the amplifier on the input with the noise generator switched on
N_{DOFF}	intrinsic noise power of the amplifier on the input with the noise generator switched off
N_{ON}	noise power on the noise generator output with the noise generator switched on
N_{OFF}	noise power on the noise generator output with the noise generator switched off

Further evaluations are available, with the following simplifying conditions

$$G_{ON} = G_{OFF}\ , \qquad N_{DON} = N_{DOFF} = N_D\ , \qquad \Gamma_{ON} = \Gamma_{OFF} = 0$$

i.e., the available power output of the noise generator is equal to the power supplied into the matching load.

Then is valid [3] equivavalent to (4)

$$F = \frac{\frac{N_{ON}}{N_{OF}} - 1}{Y - 1} \qquad \text{or} \qquad F = \frac{ENR}{Y - 1} \tag{7}$$

Measurement errors occur as a consequence of not-keeping to the simplification conditions.
State $G_{ON} \neq G_{OFF}$ requires the correction

$$Y_{TRUE} = \frac{G_{ON}}{G_{OF}}.Y \tag{8}$$

The correction coefficient G_{ON}/G_{OFF} can be expressed according to the transducer power gain dependence of the amplifier on the reflection coefficient of the noise generator.

$$G_T = |s_{21}|^2 \frac{\left(1 + |\Gamma_G|^2\right)\left(1 - |\Gamma_L|^2\right)}{\left|(1 + s_{11}\Gamma_G)(1 - s_{22}\Gamma_L) - s_{12}s_{21}\Gamma_G\Gamma_L\right|^2} \tag{9}$$

Expression [2]

$$\frac{G_{ON}}{G_{OFF}} = \frac{1 - 2|s_{11}||\Gamma_{OFF}|\cos(\Phi_{11} + \Phi_{OFF})}{1 - 2|s_{11}||\Gamma_{ON}|\cos(\Phi_{11} + \Phi_{ON})} \tag{10}$$

Where:

- s_{11} to s_{22} are amplifier scattering parameters
- Γ_{ON} output reflection coefficient with the noise generator switched on
- Γ_{OFF} output reflection coefficient with the noise generator switched off
- Φ_{11} argument s_{11}
- Φ_{ON} argument Γ_{ON}
- Φ_{OFF} argument Γ_{OFF}

The expression applies to a stable two-port network and small values of the reflection coefficient ($|\Gamma| < 0{,}05$)

The error value due to changes of the gain can be in the range of tenths dB (e.g. 0,1 dB for HP 346A, 0,5 dB for HP346B). This error depends on the reflection angle coefficient of the generator, or on the electrical length between the generator and the measured two-port network. It can be positive or negative.

In practice, we indicate these errors if we detect dependence of the measured noise figure on the phase (length) of the lossless variable phase line connected between the generator and the measured two-port network. The intrinsic noise dependence of a measured two-port network on the impedance of the noise generator usually also has an influence on the detected dependence.

An exact analysis can be made according to the term (10), if we know the other necessary values. This error can be minimized by using a noise generator with a minimum reflection coefficient, by inserting a precise attenuator between the noise generator and a measured two-port network, and by inserting an insulator between the noise generator and a measured two-port network, with which can reduce the reflection coefficient of the given generator.

The possibility of receiving a higher output power on the noise generator output than it supplies to a matched load is another source of errors. The ENR value (excess noise ratio) is given for the reflectionless load, the ENR_I value for suitable mismatched load will be different. The term (11) represents the corrected value of ENR_I. The inaccuracy is determined by first fraction in this relation.

$$ENR_I = \frac{1 - |\Gamma_{OFF}|^2}{1 - |\Gamma_{ON}|^2} \cdot \frac{N_{ON}}{N_{OF}} - 1 \tag{11}$$

This error can reach a level of about 0,05 dB, but it is not significant for small values of ENR(3dB) and reflection coefficients of the order of units per cent .

The intrinsic noise dependence of the measured two-port network on the noise generator impedance and these impedance changes lead to various values of the noise temperature, which determine the noise power on the input of the measured two-port network in the noise generator on and off state.

The change in the noise power brought about by noise generator impedance modification can be evaluated by using the term

$$F = F_{\min} + \frac{R_n}{G_s}\left[(G_s - G_0)^2 + (B_s - B_0)^2\right] \qquad (12)$$

where G_s, B_s are complex admittance components on the transistor input, and G_o, B_o are complex admittance components of noise matched input admittance on the transistor.

An example of these dependences is illustrated in the Fig. 2. Near the optimum $G_s \cong G_o$, $B_s \cong B_o$, the noise power depends only slightly on the impedance.

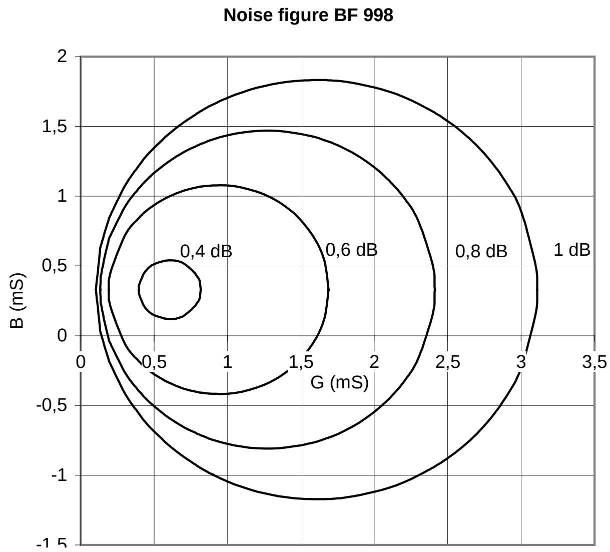

Fig. 2 Constant noise figure contours in the input admittance plane

The intrinsic noise power dependence of generator impedance is more evident in generator state off.

This means that a given noise figure applies to the input impedance which is equal to the noise generator impedance in the OFF state, not the defined impedance Z_o.

Evaluation of these effects points to the following simplified quantitative analysis, which was processed for a substitute lumped-parameter circuit and a MOSFET transistor.

We assume a equivalent circuit of the noise generator with a current source and load as shown in Fig. 3. For the noise power in the real load G_Z is valid:

$$N = I^2 \frac{G_Z}{|Y + G_Z|^2} \qquad (13)$$

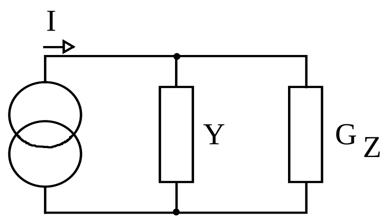

Fig. 3 Equivalent circuit

Then the current ratio I_{ON}/I_{OFF} can be expressed by the relation:

$$\frac{I_{ON}}{I_{OFF}} = \sqrt{\frac{N_{ON}}{N_{OFF}}}\frac{|Y_{ON} + G_Z|}{|Y_{OFF} + G_Z|} \qquad (14)$$

According to the measurement results, Im[Y] << Re[Y], see Fig. 4 and Fig. 5, and this relationship can be simplified. If we further assume $G_{ON} = G_{OFF} + \Delta G$, $G_{ON} \cong G_z$ and ENR>>1, then:

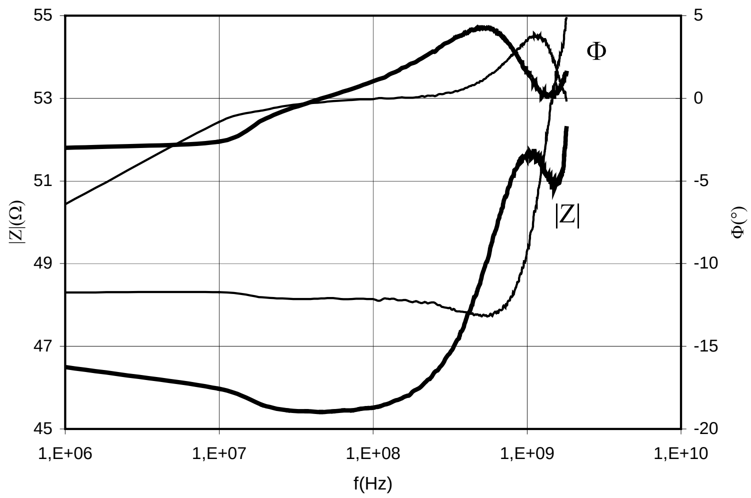

Fig. 4 Complex output impedance of noise generator NC 346B, ENR=15 dB (thick line – ON, thin line – OFF)

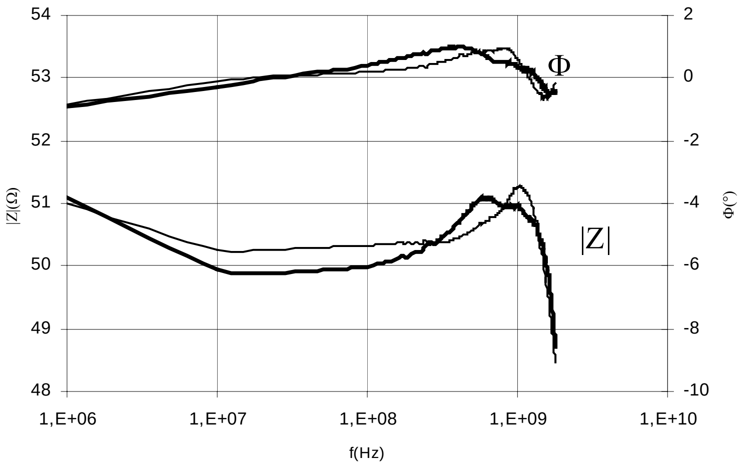

Fig. 5 Complex output impedance of noise generator NC 346A, ENR=6 dB
(thick line – ON, thin line – OFF)

$$\frac{I_{ON}}{I_{OFF}} = \sqrt{\frac{N_{ON}}{N_{OFF}}}.\left(\frac{\Delta G}{2\Delta G_Z} + 1\right) \tag{15}$$

or, alternatively:

$$ENR_I = ENR_P\left(\frac{\Delta G}{\Delta G_Z} + 1\right) \tag{16}$$

We also can express the noise power and coefficient Y for a equivalent circuit of the amplifier according to Fig. 5 with the help of noise currents.

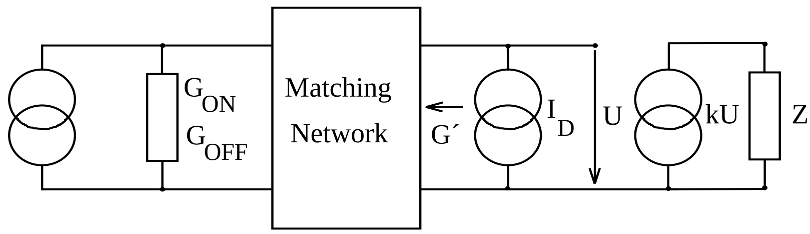

Fig. 6 Equivalent circuit of the MOSFET amplifier

The relation (17) is valid for an indicated value Y_I

$$Y_I = \frac{P_{0ON}}{P_{0OFF}} = \frac{I_{ON}^2 + I_D^2}{I_{OFF}^2 + I_D^2}\left[\frac{G_Z \pm \Delta G}{G_Z}\right]^2 \tag{17}$$

The + sign in this relation is valid for a matching circuit with which G' is directly proportional G_Z. The - sign is valid for a matching circuit with which G' is indirectly proportional G_Z. If we compare value Y_I with ideal value Y, which we obtain for ΔG = 0, we find out:

$$Y = Y_I\left[1 \pm 2\frac{\Delta G}{G_Z}\right] \tag{18}$$

Then by analogy with (10) the next term is valid for F:

$$F = \frac{\frac{I_{ON}^2}{I_{OFF}^2} - 1}{Y - 1} = \frac{ENR_I}{Y - 1} \approx \frac{ENR}{Y_{ind} - 1}\left(1 \pm \frac{\Delta G}{G_Z}\right)\left(1 \pm \frac{2\Delta G}{G_Z}\right) \tag{19}$$

The influence of a change in generator impedance is clearly evident if we compare this measurement with the next measurement. In this case a low-loss quarter-wave line is inserted between the generator and the tested circuit. A change in the indicated value shows the influence of the impedance change.

If $\Delta G>0$, then the minimum relative error F_{min} is approximately equal to $-\Delta G/G_Z$, and the maximum relative error $F_{max} = 3\Delta G/G_Z$. If $\Delta G<0$, F_{min} is approximately equal to $3\Delta G/G_Z$, $F_{max} = -\Delta G/G_Z$.

4. The temperature dependence of every noise source is expressed minimally as the temperature dependence of the noise power that is generated by every real resistance. Calibration of the noise generators by the company NOISE COM series NC 346 is provided at a temperature of 25°C according to the Certificate of Calibration. It is valid in the temperature range 21-27°C. The temperature dependence of the generators is not shown in the documentation.

According to information provided by Rhode Schwarz, a company associated with NOISE COM, the temperature coefficient of the generators is approximately 0,01 dB/C.

In the process of measuring, the generator is heated by the electrical power, and its temperature is thereby changed by a maximum of about 5°C. Thus the incurred measurement error is not great. A change in the indicated value is problematical, but it is not due to a real change in the noise figure. This error may lead to wrong results when tuning the tested circuit.

IV. Experimental Data

The results of noise figure measurement on an experimental amplifier with a double-gate FET BF998, which were worked out using Rhode Schwarz ESPI a noise figure meter with option FSK-3 and a ZFL-500LNA Low Noise Amplifier and NC346A and NC346B noise generators are shown in fig.7. The dependence of the measured values on the length of the connecting line and the measurement error when using the 346B generator are evident. No similar definite dependence was detected with the 346A generator (the deviation is greater than 0.05), and the error was probably also small (in the order of hundredths).

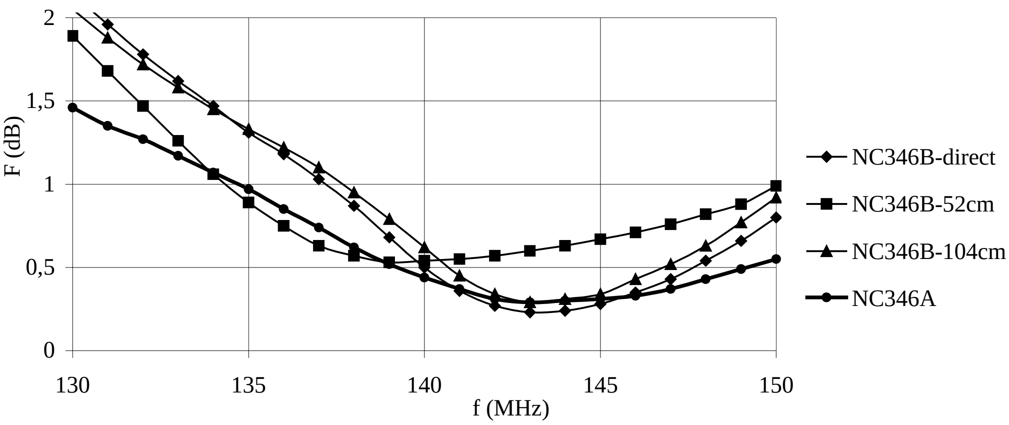

Fig.7: The frequency dependence of noise figures with different noise generators and cable length

V. Conclusions

The noise figure measurement is high effected by a noise generator mismatching. It is possible to determine this influence by comparing some measuring, which are provided at the reflection factor reduction (using of the attenuator) or at the reflection phase change (using of phasing line). The dependence of a measured noise figure value on an attenuation or a phase indicates the measurement error. The errorless measured value is phase and attenuation independent. This research work is supported by research projects CEZ J04: 210000012 and 99212 Diagnostics of materials

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