Uncertainty In Measuring The Noise Factor Of Cascade-Connected Linear Two-Ports

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Abstract- The paper presents the derived dependence on the total noise factor of cascade-connected linear two-ports in the conditions of energy mismatching. The method of assessing the uncertainty in measuring this factor, which takes into account propagation of standard partial uncertainties of the measurement of the set of four noise parameters, disposable power gains and input and output impedance of particular two-ports, is determined. The uncertainty analysis takes into account the correlation among these parameters. A strict dependence between the mismatching stages of two-ports in the cascade and the uncertainty in measuring the total noise factor is shown. Sensitivity coefficients determining the participation of particular standard partial uncertainties are defined. The results of the analyses and simulations of particular uncertainties components, which were carried out for a typical communications receiver, are presented.

***Keywords*:** linear two-ports, noise factor, combined standard uncertainty

I. Introduction

The degradation in the ratio of signal power to noise power, when the signal passes through a linear two-port, is expressed by the noise factor. It can be expressed as the ratio of total noise power on the output of a two-port to the part of noise power on the output, which comes only from the signal source. For the cascade of *n* linear two-ports the internal noise of each cascade stage can be transferred to the input circuit and presented in the form of noise sources of disposable power $\left(F_k - 1\right)kT_0 \Delta f$ (fig. 1) [1, 2]. The total noise factor of the set of cascade-connected linear two-ports depends on noise and functional parameters of particular stages, their mutual matching and signal source impedance.

II. Noise factor of cascade-connected linear two-ports

Assuming that all two-ports of the set are unilateral, it can be easily shown that actual power gain of k-th stage of the cascade K_{pk} expressing the ratio of actual power on two-port load (input impedance of the next two-port) to actual power on its input is described by the dependence:

$$K_{pk} = K_{prk} \frac{R_{ok} R_{i(k+1)}}{R_{ik}^2} \frac{\left|Z_{o(k-1)} + Z_{ik}\right|^2}{\left|Z_{ok} + Z_{i(k+1)}\right|^2} \qquad (1)$$

where Z_{ik}, R_{ik} are input impedance and resistance respectively, Z_{ok}, R_{ok} are output impedance and resistance respectively and K_{prk} is disposable power gain of k-th stage, however effective power gain of k-th stage of the cascade K_{pek} is determined by the dependence:

$$K_{pek} = K_{prk} \frac{4 R_{ok} R_{i(k+1)}}{\left|Z_{ok} + Z_{i(k+1)}\right|^2} \qquad (2)$$

In the dependence (1), it results from such accepted denotations that $Z_{o(1-1)} = Z_s$, while $Z_{i(n+1)} = Z_L$.

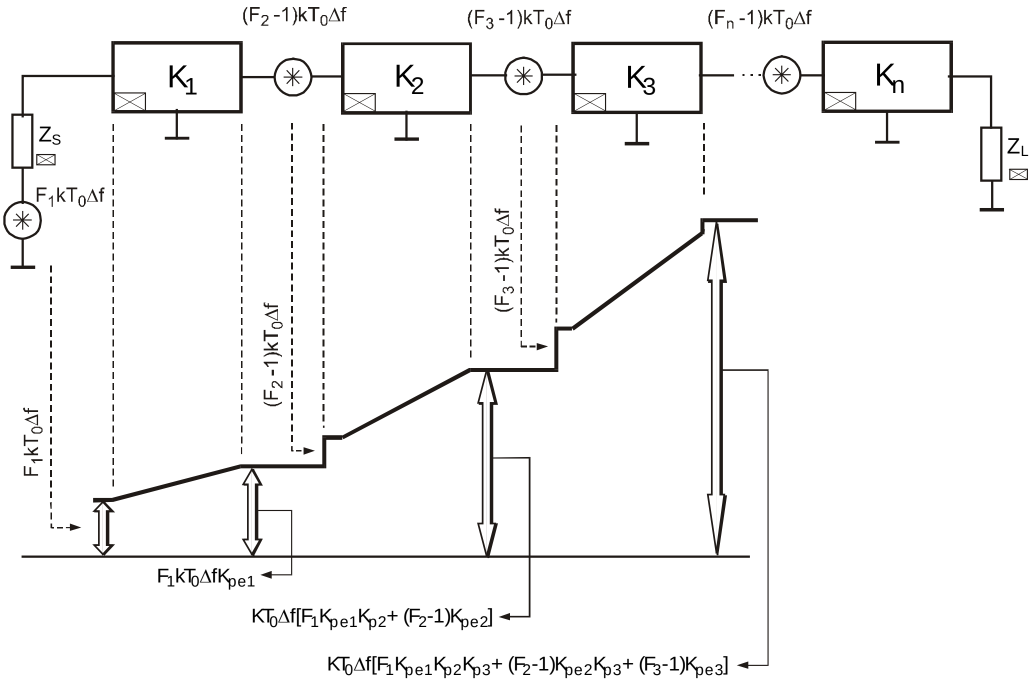

Fig. 1. The cascade connection of n noise two-ports; F_k – noise factor of k-th stage, Δf – energy noise pass-band, K_k – a general symbol of power gain of k-th stage, K_{pek} – effective power gain, K_{pk} – actual power gain, Z_s – internal impedance of signal source, Z_L – cascade load impedance, $k = 1{,}38 \cdot 10^{-23}$ VAs/K – Boltzman constant, $T_0 = 290$ K

The noise factor of k-th stage of the cascade F_k can be calculated on the basis of the dependence [1]:

$$F_k = F_{0k} + \frac{R_{nk}\left(R_{o(k-1)}^2 + X_{o(k-1)}^2\right)}{R_{o(k-1)}}\left\{\left[\frac{R_{o(k-1)}}{R_{o(k-1)}^2 + X_{o(k-1)}^2} - G_{0k}\right]^2 - \left[\frac{X_{o(k-1)}}{R_{o(k-1)}^2 + X_{o(k-1)}^2} + B_{0k}\right]^2\right\} \quad (3)$$

where $X_{o(k-1)}$ is output reactance of (k-1)-th-stage of the cascade, and $F_{0k}, R_{nk}, G_{0k}, B_{0k}$ constitute the set of four noise parameters fully characterising the two-port in relation to noise.

It was shown that when the values of disposable power gain K_{prk} and the sets of four noise parameters $F_{0k}, R_{nk}, G_{0k}, B_{0k}$ of all n cascade-connected two-ports (fig.2) are known, the total noise factor F_w is described by the following dependence:

$$F_w = F_1 + \frac{(F_2 - 1)R_{i2}}{K_{pr1}R_{o1}} + \frac{(F_3 - 1)R_{i2}R_{i3}}{K_{pr1}K_{pr2}R_{o1}R_{o2}} + \ldots + \frac{(F_n - 1)R_{i2}\ldots R_{in}}{K_{pr1}\ldots K_{pr(n-1)}R_{o1}\ldots R_{o(n-1)}} \quad (4)$$

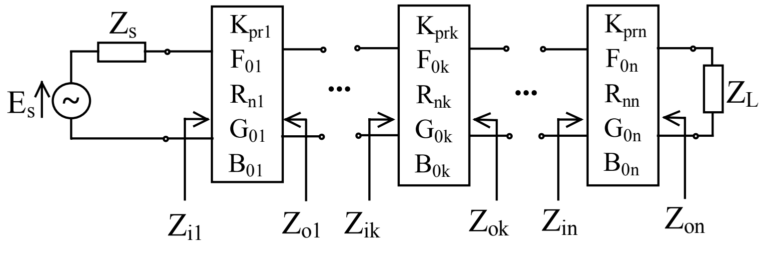

Fig. 2. The set of cascade-connected n linear two-ports with the values marked for the measurement of the total noise factor F_w

Marking in the equation (4) the appropriate ratios of resistance with wn_k equal:

$$wn_k = \frac{R_{ok}}{R_{i(k+1)}} \qquad (5)$$

and, which the authors suggest, should be named an effective mismatching index of k-th stage, the dependence (4) assumes the form of:

$$F_w = F_1 + \sum_{k=2}^{n} \frac{(F_k - 1)}{\prod_{m=1}^{k-1} K_{prm} wn_m} \qquad (6)$$

III. Combined standard uncertainty

In the considerations it was assumed that the values of the sets of four noise parameters $F_{0k}, R_{nk}, G_{0k}, B_{0k}$ and disposable power gains K_{prk} are measured with standard uncertainty $u(F_{0k})$, $u(R_{nk})$, $u(G_{0k})$, $u(B_{0k})$, $u(K_{prk})$ respectively. To assess the combined standard uncertainty $u_c(F_w)$ of the total noise factor F_w of the cascade-connection of n linear two-ports, the measurement function (6) was approximated with a first-order Taylor series [3] assuming as the input parameters of the function: noise factors F_k, effective mismatching indexes wn_k and disposable power gains K_{prk}. The noise factors F_k (3) and effective mismatching indexes $wn_{(k-1)}$ (5) are correlated variables because they depend on the same output resistances $R_{o(k-1)}$.
Partial derivatives in the dependence describing the combined standard uncertainty (tab. 1), known as sensitivity coefficients express the participation of the particular uncertainty components in the combined uncertainty. Their values are affected by mismatching indexes between the successive stages of the cascade and disposable power gains of particular stages. In case of sensitivity coefficients for effective mismatching indexes, their values depend also on noise factors of two-ports (tab. 1).
Standard partial uncertainty in measuring the noise factor of k-th stage of the cascade $u(F_k)$ is described by assuming as input variables: the set of four noise parameters $F_{0k}, R_{nk}, G_{0k}, B_{0k}$, the output resistance $R_{o(k-1)}$ and the reactance $X_{o(k-1)}$ of (k-1)- th two-port (tab. 1). The correlation between the resistance and the reactance, which depends on the use of the method in measuring the output impedance of the two-port, was taken into account.
The standard uncertainties in measuring the input resistance $u(R_{i(k+1)})$ and the output resistance $(X_{o(k-1)})$ (tab. 1) influence the partial uncertainty in measuring the effective mismatching index of k-th stage of the cascade $u(wn_k)$. Their values, similarly to the value of the standard uncertainty in measuring the output reactance $u(X_{o(k-1)})$, depend on the used method of measurement of these values and the accuracy of the used measurement tools. As the techniques of their measurement are similar, it was assumed that these uncertainties are equal to each other.
The presented dependencies (tab. 1) constitute the basis for the measurements and simulations determining the influence of particular parameters of the set of four noise parameters, input resistances and output impedances, disposable power gains of particular two-ports on the value of the uncertainty of the total noise factor of the cascade-connected linear two-ports.

IV. Simulation examinations

The paper presents the results of the simulations carried out for a typical communications receiver, in which the noise properties are determined by its three first stages. The cascade of two-ports consisting of: input filter, amplifier and mixer is analysed (tab. 2).

Table 2. Basic parameters of the input stages of a communications receiver

Parameters	Relative uncertainty of the measurement [%]	The stage of the two-port k		
		1-st	2-nd	3-rd
F_{0k} [kT$_0$]	10	1,25	1,25	1,25
R_{nk} [Ω]	10	0	100	100
G_{0k} [mS]	10	0	0,15	0,15
B_{0k} [mS]	10	0	-2	-2
K_{prk}	0,5	0,8	10	0,16

Table 1. Formulae determining the combined standard uncertainty in measuring the total noise factor

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Combined standard uncertainty

$$u_c^2(F_w) = \sum_{k=1}^{n} \left(\frac{\partial F_w}{\partial F_k}\right)^2 u^2(F_k) + \sum_{k=1}^{n-1} \left(\frac{\partial F_w}{\partial wn_k}\right)^2 u^2(wn_k) + \sum_{k=1}^{n-1} \left(\frac{\partial F_w}{\partial K_{prk}}\right)^2 u^2(K_{prk}) + 2\sum_{k=2}^{n} \frac{\partial F_w}{\partial F_k}\frac{\partial F_w}{\partial wn_{k-1}} u(F_k, wn_{k-1})$$

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Sensitivity coefficients

$$\frac{\partial F_w}{\partial F_1} = 1, \quad \frac{\partial F_w}{\partial F_k} = \frac{1}{\prod_{m=1}^{k-1} K_{prm} wn_m},$$

$$\frac{\partial F_w}{\partial wn_k} = -\sum_{l=k+1}^{n} \frac{(F_l - 1)}{wn_k^2 \prod_{m=1}^{l-1} K_{prm} \prod_{\substack{i=1 \\ i \neq k}}^{l-1} wn_i},$$

$$\frac{\partial F_w}{\partial K_{prk}} = -\sum_{l=k+1}^{n} \frac{(F_l - 1)}{K_{prk}^2 \prod_{m=1}^{l-1} wn_m \prod_{\substack{i=1 \\ i \neq k}}^{l-1} K_{pri}}$$

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Covariance

$$u(F_k, wn_{k-1}) = \frac{\partial F_k}{\partial R_{o(k-1)}} \frac{\partial wn_{k-1}}{\partial R_{o(k-1)}} u^2(R_{o(k-1)})$$

$$\frac{\partial F_k}{\partial R_{o(k-1)}} = \frac{(F_k - F_{0k})(R_{o(k-1)}^2 - X_{o(k-1)}^2)}{(R_{o(k-1)}^2 + X_{o(k-1)}^2) R_{o(k-1)}} +$$

$$+ \frac{2R_{0k}}{R_{o(k-1)}^2 + X_{o(k-1)}^2}\left[\left(\frac{R_{o(k-1)}}{R_{o(k-1)}^2 + X_{o(k-1)}^2} - G_{0k}\right)\frac{X_{o(k-1)}^2 - R_{o(k-1)}^2}{R_{o(k-1)}} + \left(\frac{X_{o(k-1)}}{R_{o(k-1)}^2 + X_{o(k-1)}^2} + B_{0k}\right) 2X_{o(k-1)}\right]$$

$$\frac{\partial wn_{k-1}}{\partial R_{o(k-1)}} = \frac{1}{R_{ik}}$$

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Standard partial uncertainties

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$$u^2(F_k) = \left(\frac{\partial F_k}{\partial R_{o(k-1)}}\right)^2 u^2(R_{o(k-1)}) + \left(\frac{\partial F_k}{\partial X_{o(k-1)}}\right)^2 u^2(X_{o(k-1)}) + \left(\frac{\partial F_k}{\partial F_{0k}}\right)^2 u^2(F_{0k}) + \left(\frac{\partial F_k}{\partial R_{nk}}\right)^2 u^2(R_{nk}) + \left(\frac{\partial F_k}{\partial G_{0k}}\right)^2 u^2(G_{0k}) + \left(\frac{\partial F_k}{\partial B_{0k}}\right)^2 u^2(B_{0k}) + 2\frac{\partial F_k}{\partial R_{o(k-1)}}\frac{\partial F_k}{\partial X_{o(k-1)}} u(R_{o(k-1)}, X_{o(k-1)})$$

$$\frac{\partial F_k}{\partial X_{o(k-1)}} = \frac{(F_k - F_{0k}) 2X_{o(k-1)}}{(R_{o(k-1)}^2 + X_{o(k-1)}^2)} + \frac{2R_{0k}}{R_{o(k-1)}^2 + X_{o(k-1)}^2}\left[\left(\frac{R_{o(k-1)}}{R_{o(k-1)}^2 + X_{o(k-1)}^2} - G_{0k}\right)\frac{2X_{o(k-1)}}{R_{o(k-1)}} + \left(\frac{X_{o(k-1)}}{R_{o(k-1)}^2 + X_{o(k-1)}^2} + B_{0k}\right)\frac{X_{o(k-1)}^2 - R_{o(k-1)}^2}{R_{o(k-1)}}\right],$$

$$\frac{\delta F_k}{\delta F_{0k}} = 1, \quad \frac{\delta F_k}{\delta R_{nk}} = \frac{F_k - F_{0k}}{R_{nk}}, \quad \frac{\delta F_k}{\delta G_{0k}} = -\frac{2R_{nk}(R_{o(k-1)}^2 + X_{o(k-1)}^2)}{R_{o(k-1)}}\left[\frac{R_{o(k-1)}}{R_{o(k-1)}^2 + X_{o(k-1)}^2} - G_{0k}\right], \quad \frac{\delta F_k}{\delta B_{0k}} = -\frac{2R_{nk}(R_{o(k-1)}^2 + X_{o(k-1)}^2)}{R_{o(k-1)}}\left[\frac{X_{o(k-1)}}{R_{o(k-1)}^2 + X_{o(k-1)}^2} + B_{0k}\right]$$

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$$u^2(wn_k) = \sum_{k=1}^{n}\left(\frac{\partial wn_k}{\partial R_{ok}}\right)^2 u^2(R_{ok}) + \sum_{k=1}^{n-1}\left(\frac{\partial wn_k}{\partial R_{i(k+1)}}\right)^2 u^2(R_{i(k+1)}),$$

$$\frac{\partial wn_k}{\partial R_{ok}} = \frac{1}{R_{i(k+1)}}, \quad \frac{\partial wn_k}{\partial R_{i(k+1)}} = -\frac{R_{ok}}{R_{i(k+1)}^2}$$

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The influence of effective mismatching indexes of the 1st and the 2nd stage of the receiver on the standard partial uncertainties and the combined standard uncertainty was analysed with the assumption that the output and input impedances of two-ports and the impedance of the signal source are actual. The relative combined standard uncertainty in measuring the total noise factor $u_c(F_w)$ depends on the mismatching stage of the two-port. For matching on the output of the 1st stage of the receiver (wn_1=1), together with an increase in the value of the effective mismatching index of the 2nd stage (wn_2), its value decreases, whereas for matching on the output of the 2nd stage, it increases (fig. 3).

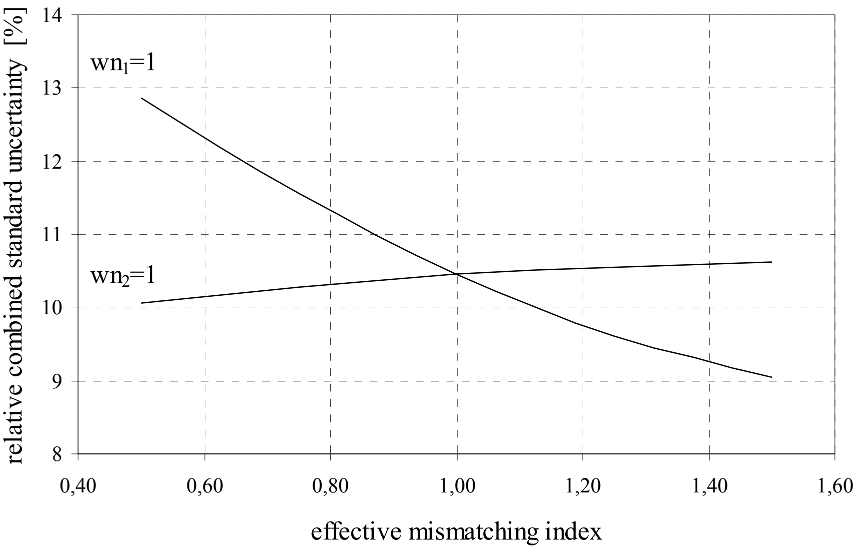


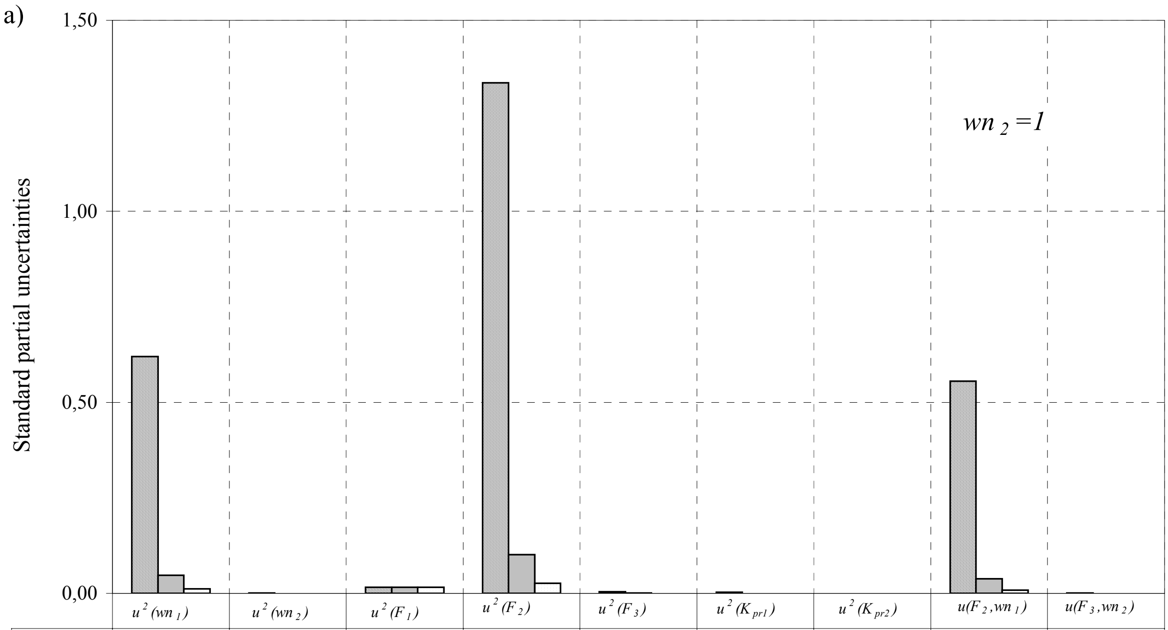
Fig. 3. Dependence of the relative combined standard uncertainty of the total noise factor of effective mismatching indexes of two-ports; the reference resistance is assumed to be 50 Ω

The biggest participation in the combined uncertainty in measuring F_w have the components connected with the noise factor of the amplifier F_2, the effective mismatching index on the filters output wn_1 and the covariance between these parameters (fig. 4). This is a result of the smaller than 1 value of the disposable power gain of passive input filters K_{pr1}. Therefore, also partial uncertainties of the noise factor of the mixer $u(F_3)$ in the considered cascade as well as the effective mismatching index on the amplifier output $u(wn_2)$ are to be neglected. In the conditions of matching on the output of the amplifier (wn_2=1) at the filters load with the resistance higher than the output resistance of this stage (wn_1<1), the values of the uncertainty components increase considerably (fig. 4).

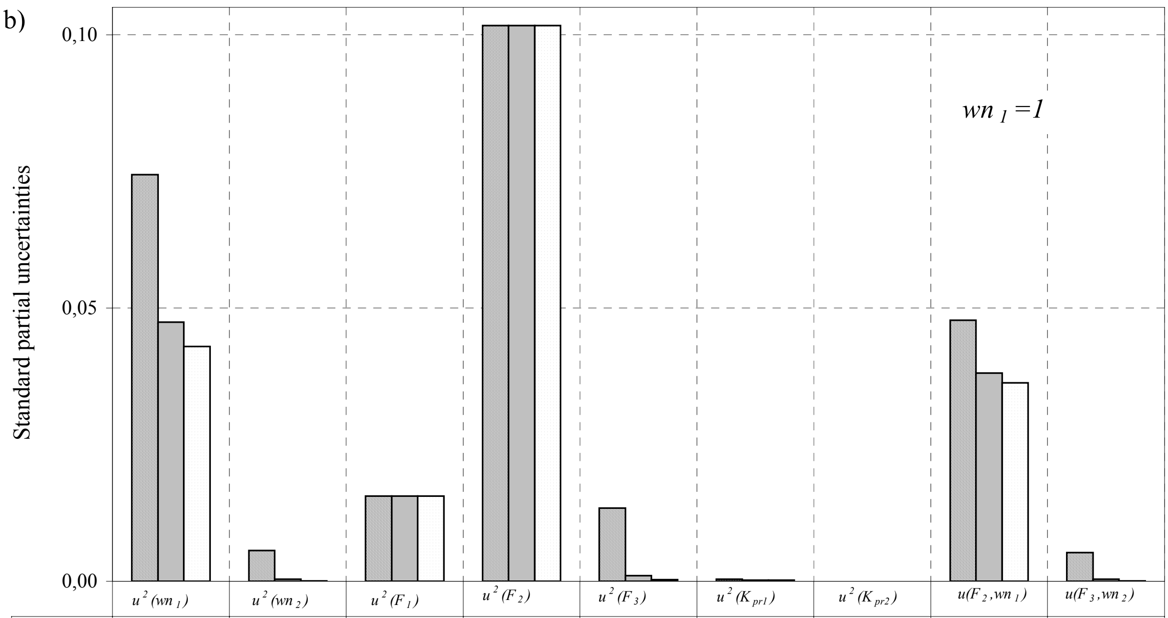
V. Conclusion

The derived dependences on the combined standard uncertainty $u_c(F_w)$ in measuring the total noise factor of the cascade of linear two-ports in the conditions of energy mismatching result from the propagation of partial uncertainties of the measurement of the set of four noise parameters, input and output impedances, disposable power gains of particular two-ports and the internal impedance of the signal source. On the basis of the literature and the authors' experience it was assumed that the values of the disposable power gains of two-ports can be measured with negligible uncertainty in comparison to other values characterising particular two-ports, obtained by measurements. The strict dependence between the stage of two-ports mismatching in the cascade and the uncertainty in measuring the total noise factor was shown. The participation of particular components in the combined standard uncertainty depends on sensitivity coefficients defined in the paper. The levels of these uncertainties obtained in the simulations given in the paper result from the assumed values of relative uncertainties of the measurements of the parameters describing the analysed cascade of two-ports.

Theoretically, it was proved that always when in the cascade of the connected linear two-ports on the input there is a passive set, the second stage does not only determine the noise properties of the cascade, but it has a fundamental influence on the combined uncertainty in measuring the total noise factor.



	$u^2(wn_1)$	$u^2(wn_2)$	$u^2(F_1)$	$u^2(F_2)$	$u^2(F_3)$	$u^2(K_{pr1})$	$u^2(K_{pr2})$	$u(F_2,wn_1)$	$u(F_3,wn_2)$
wn1=0,5	0,61994113	0,00156800	0,01562500	1,33651364	0,00406574	0,00309971	0,00000784	0,55535813	0,00138600
wn1=1	0,04743200	0,00039200	0,01562500	0,10164354	0,00101644	0,00023716	0,00000196	0,03811500	0,00034650
wn1=1,5	0,01130006	0,00017422	0,01562500	0,02613003	0,00045175	0,00005650	0,00000087	0,00818481	0,00015400



	$u^2(wn_1)$	$u^2(wn_2)$	$u^2(F_1)$	$u^2(F_2)$	$u^2(F_3)$	$u^2(K_{pr1})$	$u^2(K_{pr2})$	$u(F_2,wn_1)$	$u(F_3,wn_2)$
wn1=0,5	0,07440153	0,00559153	0,01562500	0,10164354	0,01336514	0,00037201	0,00002796	0,04773656	0,00527428
wn1=1	0,04743200	0,00039200	0,01562500	0,10164354	0,00101644	0,00023716	0,00000196	0,03811500	0,00034650
wn1=1,5	0,04297335	0,00008668	0,01562500	0,10164354	0,00026130	0,00021487	0,00000043	0,03627938	0,00007169

Fig. 4. Standard partial uncertainties for various effective mismatching indexes at (a) $wn_2=1$ (b) $wn_1=1$

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