New Digital Signal-Processing Approaches for Measuring Power CCDF Curves

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Abstract- Signals peculiar to modern telecommunication systems are characterized by higher peak-to-average power ratios than in the past and can potentially lead to higher distortion in amplification stages if signal power statistics are not accounted for properly. CCDF (Complementary Cumulative Distribution Function) curves are an effective way of fully specifying signal power characteristics. Some high performance analysers, currently available on the market, carry out CCDF measurements through a demodulation of the radiofrequency signal and the application of custom digital signal-processing algorithms to the samples of the baseband components thus retrieved. Two alternative digital signal-processing approaches for measuring CCDF curves are presented in the paper. The major advantage they offer consists in the absence of demodulation; they are, in fact, based on original algorithms that are directly applied to the samples of the radiofrequency signal.

I. Introduction

CCDF (Complementary Cumulative Distribution Function) curves provide a statistical description of power levels of a radiofrequency (RF) signal. A CCDF curve is, in fact, a plot of relative power level, expressed in decibel (dB), versus probability. Specifically, $CCDF(x)$ represents the probability that the signal envelope power is at least x dB above the average power; the envelope power is defined as:

$$P(t) = I^2(t) + Q^2(t) \qquad , \qquad (1)$$

where I and Q are the baseband in-phase and quadrature components of the RF signal. As it is suggested by the definition, a CCDF curve is a strictly decreasing function and its value in the origin of the x-axis represents the percentage of time the signal spends above its average power level [1].
CCDF curves are very important for designing, testing and troubleshooting telecommunication components and apparatuses [1],[2]. With regard to design, they give a valid help in preventing signal compression due to the non-linearity of some components such as power amplifiers. The information provided by CCDF curves, in fact, allows designing the amplifier on the basis of the particular signal expected in input (a QPSK-modulated input signal will impose different design requirements than a 64-QAM one); vice versa, given the amplifier gain versus input power, the CCDF curve can be useful to determine the optimal input signal level. Concerning testing and troubleshooting, CCDF curves are an excellent tool for quantifying compression effects; if a signal is linearly amplified, in fact, its CCDF does not vary, while compression due to non-linearity would result in a decrease of the CCDF. For the sake of clarity, Figure 1 shows the envelope power of a W-CDMA (Wideband-Code Domain Multiple Access) signal, peculiar to UMTS (Universal Mobile Telecommunication System), the European third generation (3G) communication system. Its CCDF curve is given in Figure 2: the percentage of time the signal spends above each power level specified by the x-axis estimates the probability for that particular power level.
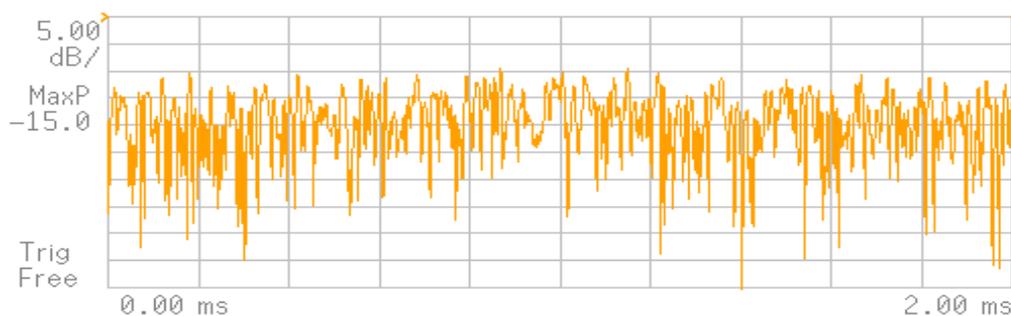

Figure 1. Envelope power of a W-CDMA signal; its high variability
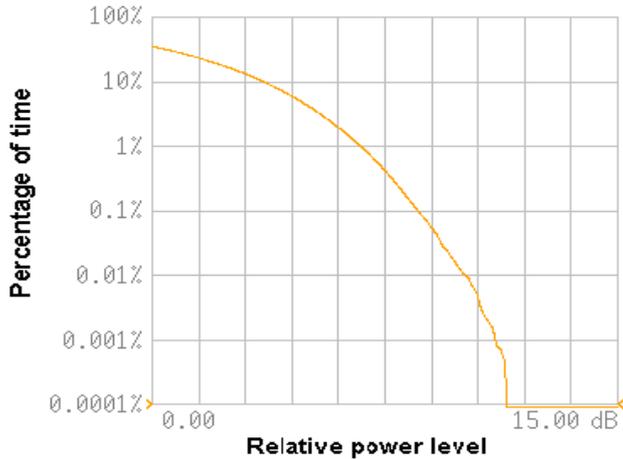is due to the noise-like nature of the signal.

Figure 2. Power CCDF curve of the signal referred to in Figure 1.

Instruments that perform CCDF measurements, currently available on the market, are high performance spectrum analysers, such as vector signal analysers and performance signal analysers, which demodulate the RF signal under test and then evaluate its CCDF curve by processing the samples of the baseband components.

Two alternative digital signal-processing approaches for CCDF measurements, based on original algorithms that are directly applied to the samples of the RF signal, are presented hereinafter. Both approaches (i) take advantage of proper sampling strategies also presented by the authors [3], which grant alias-free sampling and a digital downconversion of the RF signal and (ii) exploit its instantaneous power trajectory, suitably evaluated. In particular, to gain the instantaneous power trajectory, the first approach (time-domain approach) averages the quadratic values of the samples over a carrier period, while the second (time-frequency approach) applies Time-Frequency Representations (TFR) [4], whose employment has already been suggested with success by the authors for equipment testing in several telecommunication systems [5],[6].

The two approaches do not require any demodulation of the signal under test and do not need special purpose high-performance instrumentation: only a processing unit and a data acquisition system (DAS) are, in fact, needed.

The paper is organized as follows. Analytical and implementation issues of the two proposed approaches are described in Section II, the results of a wide experimental activity, carried out on 3G signals, with the aim of tuning TFR parameters and assessing the performance of the two approaches are presented in Section III, and, finally, conclusions and possible enhancements of the approaches, as the target of future research activity, are given in Section IV.

II. Proposed Approaches

As stated above, the two proposed approaches gain a measure of the CCDF from the instantaneous power trajectory. In particular, no analog downconversion and/or demodulation of the signal under test are needed; both approaches, in fact, simply process the samples of the RF signal. Specifically, they consist of three steps: (i) signal digitisation, (ii) instantaneous power trajectory evaluation and (iii) CCDF curve determination.

A. Signal digitisation

First, the RF signal, $x(t)$, is digitised by a DAS at the sampling frequency provided by one of the algorithms presented in [3]. The role played by such algorithm is quite important: it lets the user choose the integer ratio, p, between the sampling frequency, f_s, and the frequency at which the spectrum of the digitised signal will be centered, and then outputs the minimum value of the sampling frequency that satisfies such requirement. It is so possible to digitally downconvert the input signal thanks to a sampling frequency much lower than the carrier frequency, with consequent benefits in terms of frequency resolution. Equivalently, it is possible to analyse a larger time interval, given the number of samples. Moreover, as it will be clear soon with regard to the time-domain approach, it grants the advantage of acquiring an integer number of samples per carrier period.

B. Instantaneous power trajectory evaluation

➢ *Time-domain approach*

The first proposed approach moves from the following consideration: the carrier frequency is higher than the symbol rate (or *chip rate*, with regard to 3G signals) and consequently one symbol modulates several carrier periods. It therefore makes sense to average the square voltage values over one carrier period, which represents the minimum averaging interval for gaining a non trivial instantaneous power

value.

Thanks to the application of Algorithm I presented in [3], the same number of samples, p, are acquired per each carrier period. Then, the N acquired samples are divided into groups of p and each point of the power trajectory $P(k)$ is determined as the mean square value over p consecutive samples:

$$P(k) = \frac{1}{p} \sum_{n=1}^{p} x^2 \left(\frac{n + (k-1)p}{f_s} \right) \qquad . \qquad (2)$$

➢ *Time-frequency approach*

The second approach is based on the application of a particular TFR, the STFT (Short-Time Fourier Transform) [4]. The instantaneous power of the signal under analysis is, in fact, evaluated by applying the STFT, which is defined for, continuous-time signals, as

$$X_{STFT}(\tau, f; w) = \int_{-\infty}^{\infty} x(t) w^*(t - \tau) e^{-j2\pi ft} dt \qquad . \qquad (3)$$

The signal $x(t)$ is divided into a number of segments, each of which, weighted by the window function $w(t)$, is treated separately in order to establish its spectral content. STFT results are then taken in modulus and squared in order to attain the so-called spectrogram. Through the spectrogram, the evolution versus time of the power spectral density of the analysed signal can be highlighted. In the presence of discrete-time signals, the spectrogram is represented by means of a matrix whose values are dependent of time (column indexes) and frequency (row indexes). By summing the values along each column, the instantaneous power of the signal as a function of time can be calculated.

It is worth noting that spectrogram results depend on the particular windowing function adopted. The results of the experimental activity aimed at singling out the optimal tuning of the window parameters are given in the next Section.

C. CCDF curve determination

Finally, the mean and peak signal power levels are evaluated and the CCDF is measured by determining a set of levels, distanced one to another of 0.01 dB, between the mean and the peak signal power level, and calculating the percentage of time during which the signal is above each level. For the sake of clarity, Figure 3 shows the evolution of the instantaneous power of a 3G signal versus time. The bold line represents the mean power level, while the thin solid one represents a power level of 1.0 dB above the mean. The value of $CCDF(x)$, with $x=1.0$ dB is evaluated as the ratio of the time during which the signal is above the thin line to the observation interval.
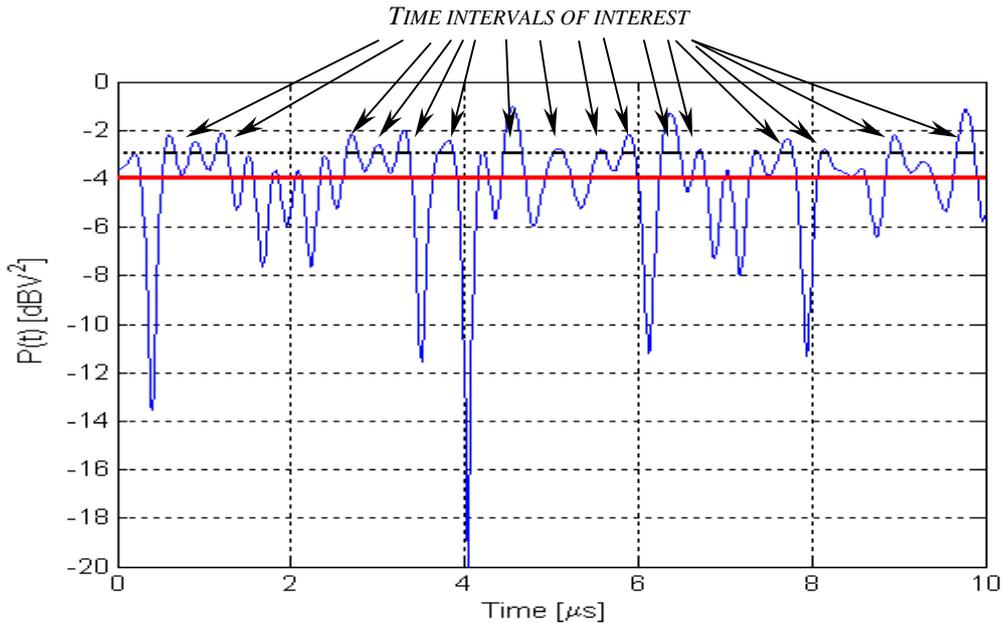

Figure 3. Evaluation of the CCDF from the instantaneous power trajectory: the bold line is the mean power level, while the thin dotted line is 1 dB above. The value of $CCDF(x)$, with $x=1$ dB, is computed as the ratio between the duration of all the time intervals indicated by the arrows and the overall duration of the signal.
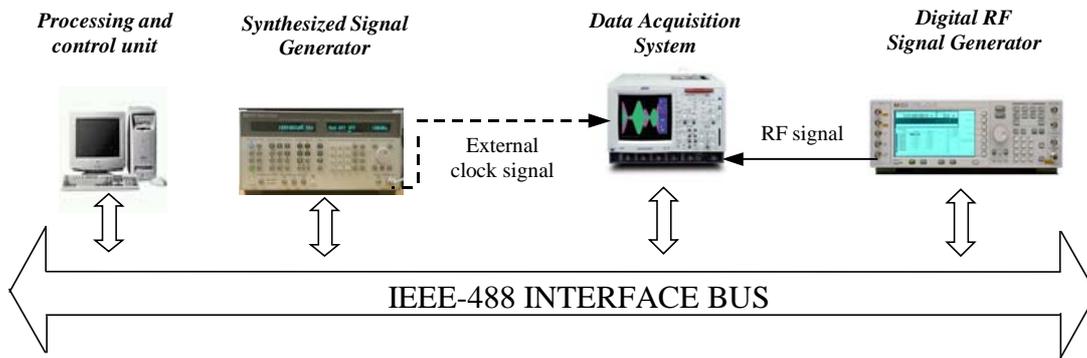
Figure 4. Measurement station.

III. Performance assessment

A suitable measurement station has been set up in order to validate the approaches with regard to actual 3G signals characterized by different carrier frequencies and average power levels. In particular, the performance of the two approaches has been evaluated as a function of the number of samples per carrier period, p. To this aim, CCDF measures gained through the application of the proposed approaches have been compared to those provided by a high performance Vector Signal Analyser (VSA).

A. Measurement station

The measurement station adopted for the experimental tests is shown in Figure 4. It consists of (i) a processing and control unit, namely a personal computer; (ii) a digital RF signal generator (250 kHz-3 GHz output frequency) equipped with 3G standard-compliant signal generation capability; (iii) a data acquisition system (8 bit vertical resolution, 1 GHz bandwidth, 8 GSample/s maximum sample rate, 8 MSample memory depth) and (iv) a synthesized arbitrary waveform generator (0.26-1030 MHz output frequency). They are all interconnected by means of a IEEE-488 standard interface bus.

The RF signal, provided by the digital signal generator and complying with the 3G specifications, is digitised according to the following procedure: the optimal sampling frequency, f_s, is provided by the algorithms presented in [3], and a sinusoidal signal at frequency f_s is generated by the synthesized signal generator to drive the DAS sampling clock (external clock). Subsequently, the CCDF is measured through the application of the proposed approaches to the acquired samples.

B. Optimal choice of time-frequency parameters

As stated before, spectrogram results depend on the particular window function $w(t)$ adopted for STFT in (3); an optimal choice of the type and size of the window has to be made. To this aim, preliminary measurements on test signals have been carried out considering rectangular, Hanning and Blackman windows, which are some of the most common ones; different window sizes have been taken into account, too. The optimal window type and size have been singled out as those that minimise the *rmse* (root mean square error) between the CCDF measured by applying the proposed approach and that given by the VSA, which has been taken as the reference curve. Even though both curves to be compared have been evaluated at power levels distanced of 0.01 dB one to another, the value of the peak-to-average power ratio (PAR) measured by the proposed approach and that furnished by the VSA have sometimes been not equal. In such cases, P and Q, the number of points constituting, respectively, the measured CCDF curve and the nominal one, were different; consequently, the *rmse*

Table I. Average values of *rmse* between measured and reference CCDF curves.

	WINDOW SIZE				
WINDOW TYPE	T_c	$2T_c$	$3T_c$	$4\,T_c$	$5\,T_c$
Rectangular	1.20	1.07	2.08	2.42	2.89
Hanning	0.89	0.81	0.99	1.28	2.02
Blackman	0.85	0.83	0.86	1.27	1.98

has been evaluated as

$$rmse = \sqrt{\frac{1}{M} \sum_{i=1}^{M} [y(i) - y_r(i)]^2} \qquad , \qquad (4)$$

where $y(i)$, $i = 1, \ldots, P$ are the values of the CCDF measured through the proposed approach, $y_r(i)$, $i = 1, \ldots, Q$, are the values of the CCDF given by the VSA and $M = \min(P, Q)$.

Table I shows the average *rmse* values achieved for the considered window type and size, as resulting from measurements carried out on several 3G signals characterized by different carrier frequencies and average power levels. Note that window size is expressed in terms of T_c, the carrier period. The experimental results generally show that shorter windows provide lower values of *rmse*. This is not unexpected because the better time resolution granted by shorter windows, at the expenses of frequency resolution, is preferable for our purpose of gaining the instantaneous power trajectory. In particular, the $2T_c$-long Hanning window, which grants the lowest *rmse,* turns out to be the most suitable windowing function.

C. Results

A second set of experimental tests have then been carried out on 3G signals, in order to assess the performance of the two proposed approaches. Each generated signal has been digitised at different sampling frequencies, with the aim of investigating the role of the parameter p. For each signal and each given sampling frequency, several acquisitions have been made and the related CCDF curves have been evaluated. Then, the measurement result for the considered signal has been expressed in terms of the average CCDF curve, which has been obtained by calculating the average value for each abscissa (i.e. for each power level).

For the sake of brevity, Figure 5 shows the outcomes of the application of the two approaches to a 3G signal modulating a carrier at 449.4 MHz; in particular, Figure 5*a* is related to the time-domain approach, while Figure 5*b* refers to the time-frequency approach. The two figures compare the CCDF curves gained through the proposed approaches, when different sampling frequencies have been utilized, to the CCDF curve provided by the VSA. In all the cases, the digitised signal has been downconverted to 21.4 MHz with different values of the integer p. Specifically, in each figure, (i) the dashed line is the CCDF curve achieved when the RF signal is sampled at $f_s = 428$ MS/s ($p = 20$), (ii) the dotted line is related to a sampling frequency equal to 214 MS/s ($p = 10$), (iii) the solid line refers to a sampling frequency equal to 107 MS/s ($p = 5$) and, finally, (iv) the bold line is the CCDF curve provided by the VSA.

The results show that the correspondence between the CCDF curve gained through the application of the proposed approaches and that provided by the VSA is greater than that assured by analogous attempts recently made [7]. In particular, as the magnified subfigures prove, the point-to-point difference is particularly small for power levels in proximity to the mean value and is substantially independent from p. With regard to higher power values, lower values of p ($p = 5$ in Figure 4) seem to provide better results. It is worth noting that the y-axis is logarithmically scaled and, therefore, the point-to-point difference that seems to be quite evident above 3 dBm is, on the contrary, irrelevant. In other words, the discrepancy between the two curves that appears to be significant over 3 dBm is referred to power levels that are seldom reached by the RF signal. In conclusion, low values of p ($p \in \{4, 5\}$) are generally preferable, even though higher values have also provided good results.

Similar outcomes have been experienced with regard to other considered signals.

IV. Conclusions

Two new digital signal-processing approaches for measuring power CCDF curves have been proposed, based respectively on a time domain and a time-frequency approach. They both measure the CCDF curve from the instantaneous power trajectory of the RF signal under test and do not require any demodulation, nor any special purpose high performance instrumentation.

A wide experimental activity has been carried out to assess the performance of the approaches. The results have shown that the CCDF curve provided by the proposed approaches and that furnished by a VSA do not differ significantly, the point-to-point difference between them, Δ, always being within few percents.

The future research activity will be mainly focused on the optimisation of the time-frequency approach, with the aim of overcoming the limitations imposed by its computational burden and making it capable of taking into account larger time intervals.
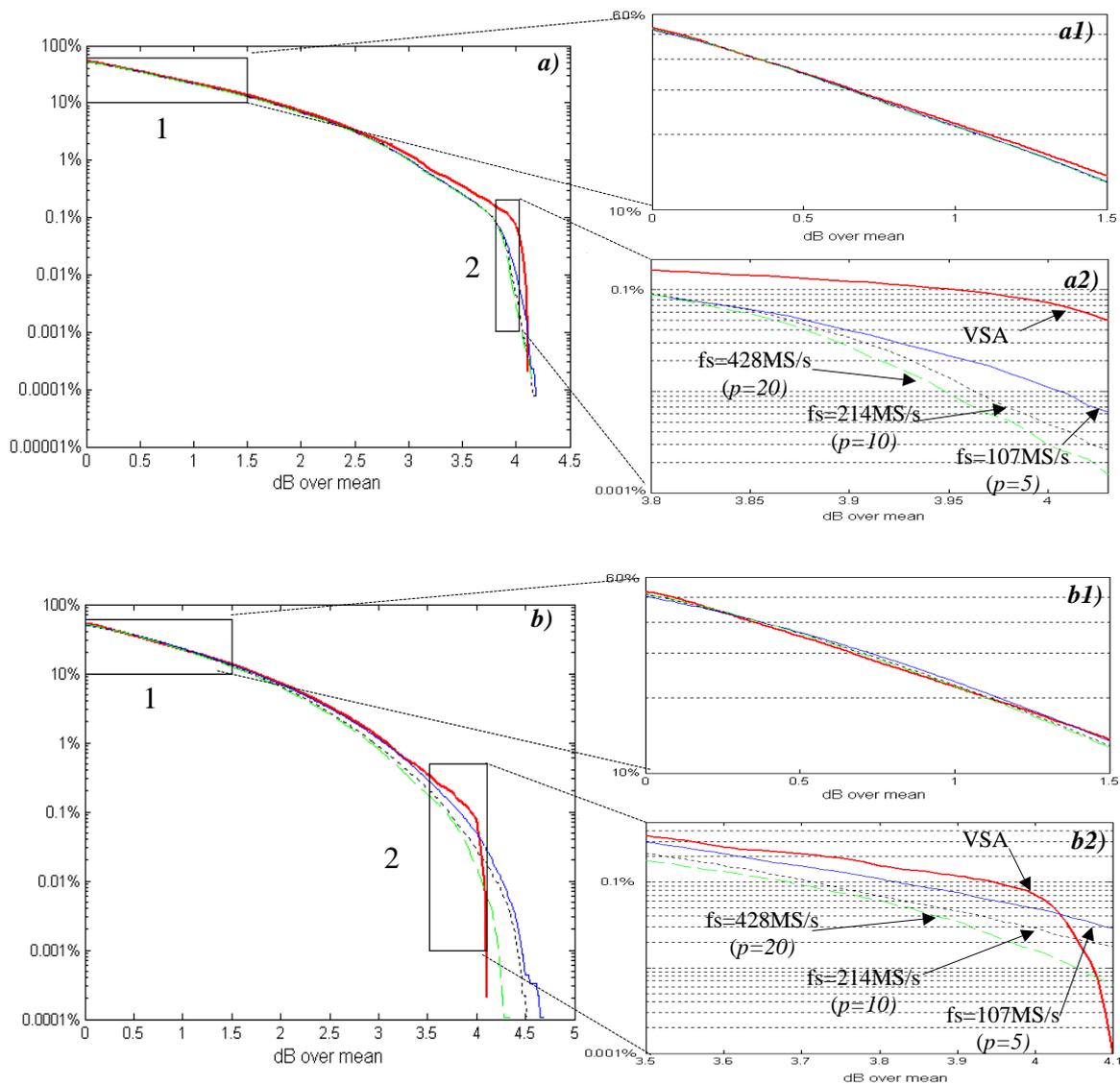
Figure 5. Comparison between the CCDF provided by the VSA and those provided by *a*) the time-domain approach, and *b*) the time-frequency approach, when different values of the sampling frequency, f_s, and therefore of the ratio *p*, are taken into account. Details of low and high power levels are given in *a*1), *a*2), *b*1), *b*2).

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