Power Measurement in the Non-coherent Sampling

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***Abstract* -** An algorithm to improve the estimation of the active power of electrical systems in the non-coherent sampling is proposed in this paper. It is based on smoothing sampled data by windowing, and then averaging of the DFT (discrete Fourier transform) coefficients in the frequency domain to reduce leakage effects. The simulation and experimental results are presented showing that averaging of DFT coefficients provides better estimation of the active power then only by windowing. The use of a suitable algorithm depends on positions of the frequency components, on a number of observed periods, and on signal to noise ratio.

I. Introduction

The paper proposes and discusses an algorithm to improve the estimation of the active power of electrical systems under non-coherent sampling conditions. It is based on smoothing sampled data by windowing, prior to their numeric integration, and then averaging of the DFT coefficients in the frequency domain to reduce the leakage effects. An introduction to the leakage problem and an overview of the properties of different windows can be found in [1-3]. Many among the estimation techniques of power components rely on a time-domain approach [4,5]. In the paper, estimation algorithms for the active power in the frequency domain are proposed by averaging of the amplitude DFT coefficients around zero component. The simulation and experimental results are presented showing their effectiveness in estimating the active power.

II. Power Estimation

The basic measuring principle in the numerical-based wattmeters is the equally spaced simultaneous sampling of voltage $u(k)$ and current $i(k)$. They usually estimate active power by averaging of the instantaneous power:

$$\overline{P} = \frac{1}{n}\sum_{k=0}^{n-1} u(k)\cdot i(k) \qquad k = 0,\ldots,n-1 \tag{1}$$

If an integer number of periods $T_{u,i}$ is observed in the measurement interval T_M ($\theta = T_\mathrm{M}/T_{u,i}$, θ - relative frequency), then (1) is a correct estimation of the active power in time space. $\overline{P}$ can be also estimated by the value of the DFT at null frequency.

In practice, the data acquisition is usually non-coherent ($\theta = i + \delta$; $\delta \neq 0$). The long-leakage tails of used windows ($w(k)$ in (3)) convolved around the power spectral components from the whole frequency axis disturb the real value in (1). At least the rectangular window has to be considered. The amplitude coefficients surrounding the component m are composed of the short-range leakage contribution of the window spectrum $W(\theta)$ weighted by the amplitude of the frequency component and the long-range leakage contribution.

$$|G(i)| = \left| -\frac{j}{2}A_m\left[W(i-\theta_m)e^{j\varphi_m} - W(i+\theta_m)e^{-j\varphi_m}\right]\right| = \frac{A_m}{2}|W(\delta_m)| \pm |\Delta(i_m)| \tag{2}$$

The leakage contributions can be reduced by relative enlarging of the measurement time $(\theta >> 1)$ and by weighting the signal samples with suitable windows $w(k)$.

$$\overline{P} = \frac{1}{W(0)}\sum_{k=0}^{n-1} w(k)\cdot u(k)\cdot i(k) \qquad W(0) = \sum_{k=0}^{n-1} w(k) \tag{3}$$

where $W(0)$ is the window gain and for rectangular window is equal $W(0) = n$ like in (1). In the power estimation different windows are used: Kaiser-Bessel window [5]; Rife-Vincent windows: class I [6];... For the sake of analytical simplicity cosine-class windows are frequently used:

$$w(k) = \sum_{l=0}^{p-1} a_l \cos\left(\frac{2\pi}{n}\, l \left(k - \frac{n-1}{2}\right)\right) \qquad (4)$$

When p is 1, the coefficient a_0 is 1 and the equation (4) gives rectangular shape. If p is 2 ($a_0 = 1/2$, $a_1 = 1/2$) we get Hanning window. Higher values of p ($p = 3$: $a_0 = 3/8$, $a_1 = 4/8$, $a_2 = 1/8$; $p = 4$: $a_0 = 10/32$, $a_1 = 15/32$, $a_2 = 6/32$, $a_3 = 1/32$) expand the window transform main-lobe and reduce the spectral leakage. This effect can be usefully applied in the active power estimation when the relative frequency is large enough $\theta > 2$,.. The effects of data truncation and windowing can be deduced from Fig. 1, in which the normalized deviation from the true power $|e| = \left|\overline{P}/P^* - 1\right|$ (P^* is the true value) has been plotted assuming cosine windows. In simulations the angle between u and i has been changed from $-1{,}569$ to $+1{,}569$ with step $\Delta\varphi = \pi/1800$. It can be deduced that when the number of voltage and current periods is great enough, the use of suitable windows can reduce the long-range leakage systematic contributions in the measurement results.

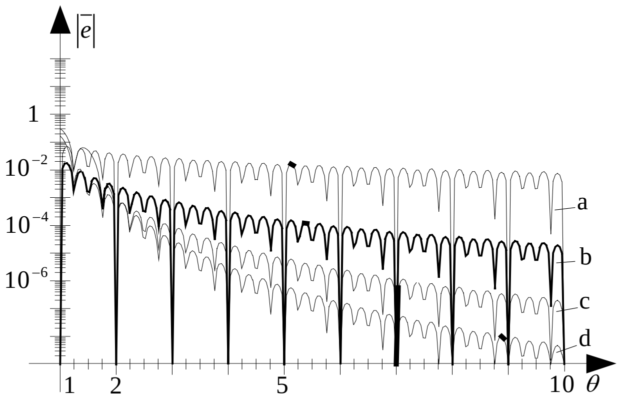


Figure 1. Mean values of errors of the active power estimation by windowing (n =1024; a: p =1, b: p =2, c: p =3, d: p =4)

With higher order cosine windows ($p \geq 2$) one or more DFT coefficients around the zero coefficient have non-zero values $|G(i \neq 0)| > 0$ due to the window main-lobe. We can use these coefficients to improve the estimation of the signal average value (in our case the active power).

The amplitude of any frequency component m can be estimated, when the displacement δ_m and the spectrum of used window are formally known (like Hanning):

$$A_m = 2\left|\frac{\pi\delta_m\left(1-\delta_m^2\right)}{\sin\left(\pi\delta_m\right)}\right| \left|G\left(i_m\right)\right| \qquad (5)$$

We can enlarge the number of interpolation coefficients in estimation by summing of the largest three local DFT coefficients around signal component [3]:

$${}_3A_m \approx \left|\frac{\pi\delta_m}{\sin(\pi\delta_m)}\right| \frac{\left(1-\delta_m^2\right)\left(4-\delta_m^2\right)}{6} \cdot \left(|G(i_m - 1)| + 2|G(i_m)| + |G(i_m + 1)|\right) \qquad (6)$$

The amplitude estimation of the zero DFT component has some particularities. The relative displacement δ_0 is zero and the DFT coefficients are known only on positive frequency axis. If we want to follow the equation (6), we can take double values of coefficients $2 \cdot |G(i \geq 1)|$. The one-point estimation with Hanning window (5) can be rewritten in $A_0 = 2 \cdot |G(0)|$ and the three-point estimation

(6) in:

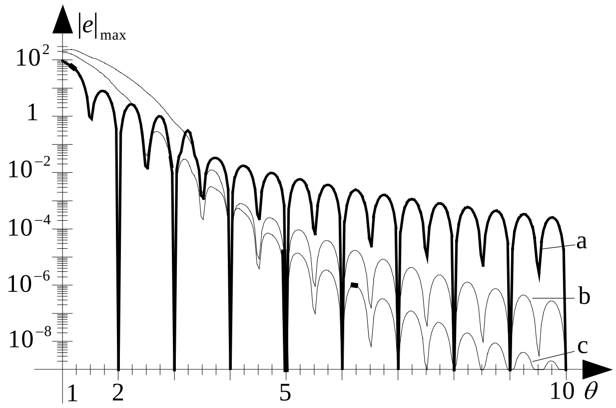
$$\overline{P} = A_0 = \frac{4}{6}[2|G(0)| + 2|G(1)|] = \frac{4}{3}[|G_0| + |G_1|] \tag{7}$$

In similar manner higher order cosine windows ($p = 3, 4, \dots$) can also be used. It can be easily found out that the weights of coefficients in averaging are in the same proportion as coefficients a_l in the expression for cosine-class windows (4).

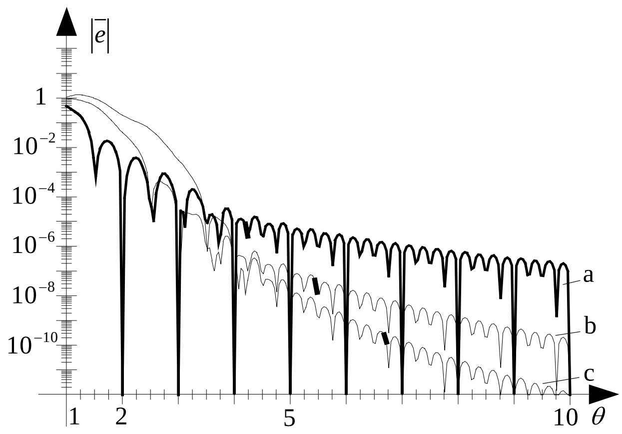
$$p = 3: \ \overline{P} = \frac{16}{35}[3 \cdot |G_0| + 4 \cdot |G_1| + 1 \cdot |G_2|] \tag{8}$$

$$p = 4: \ \overline{P} = \frac{32}{231}[10 \cdot |G_0| + 15 \cdot |G_1| + 6 \cdot |G_2| + 1 \cdot |G_3|] \tag{9}$$

In Fig. 2 we see the relative errors of the active power estimation by averaging of the DFT coefficients under same conditions of simulation as in Fig. 1.



a. Error limits



b. Mean values of errors

Figure 2. Relative errors of the active power estimation by averaging of the DFT coefficients (n =1024; a: p =2, b: p =3, c: p =4)

Comparison of proposed method with estimation of the active power by windowing shows great improvements at relative frequencies higher than 4 (Fig. 3.). Curves a ($p = 2$), b ($p = 3$) and c ($p = 4$) are the results of only windowing in the active power estimation (3), in curves b, d and e the averaging of the amplitude DFT coefficients are added.

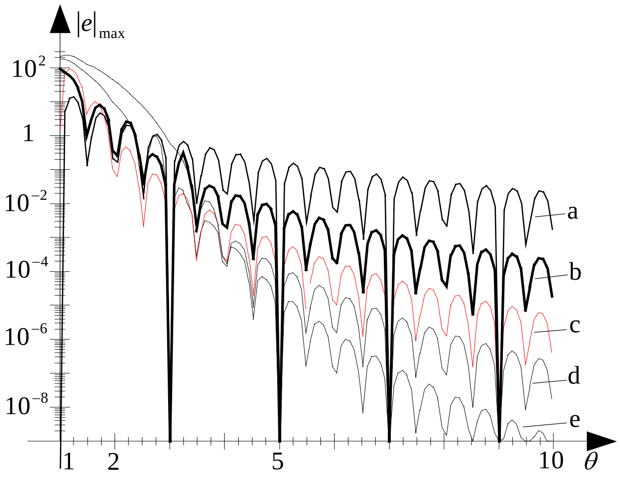


Figure 3. Relative errors of the active power estimation by cosines windowing (n =1024; a: p =2, b: p =3, c: p =4) and by averaging of the DFT coefficients (b: p =2, d: p =3, e: p =4)

III. Experimental Results

The proposed method has been validated on a real measurement system in noisy circumstances. For simulation of u and i the dual-channel arbitrary waveform generator has been used (HP3245A) and for the data acquisition two sampling voltmeters (HP3458A: $f_{\mathrm{S,\,max}} = 100\mathrm{kHz}$, $n_{\mathrm{resolution}} = 16\mathrm{bits}$, $n_{\mathrm{ef}} = 11\mathrm{bits}$) have been synchronized. The effectiveness of method is evident at lover values of $\cos\varphi$, when u and i are composed of a single tone (Fig. 4).

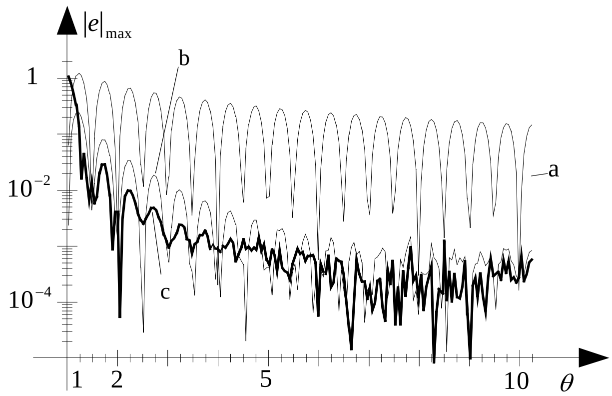


Figure 4. Maximal relative errors of the active power estimation by rectangular window (a), Hanning window (b) and averaging of DFT - Hanning (c) (n =1024; $n_{\mathrm{repetition}} = 30$; $\cos\varphi = 0{,}1$)

Averaging algorithms have been further validated assuming non-sinusoidal conditions, that are with signals composed of three components $m_i\left(f_i, A_i, \varphi_i\right)$: u: $m_1\left(f_1, 1, 0\right)$, $m_3\left(5f_1, 1/5, 1{,}396\right)$, $m_2\left(3f_1,\ 1/3, 1{,}047\right)$; u_i: $m_1\left(f_1, 1, -1{,}484\right)$, $m_2\left(3f_1, 1/3, -0{,}436\right)$, $m_3\left(5f_1, 1/5, -0{,}087\right)$ and rectified by the second signal u_i (signals differ from zero for half period and start at $\varphi_{\mathrm{start}} = 1{,}596$ - Fig. 5: $g/\hat{g}$ - normalized signals with the maximal value $\hat{g}$).

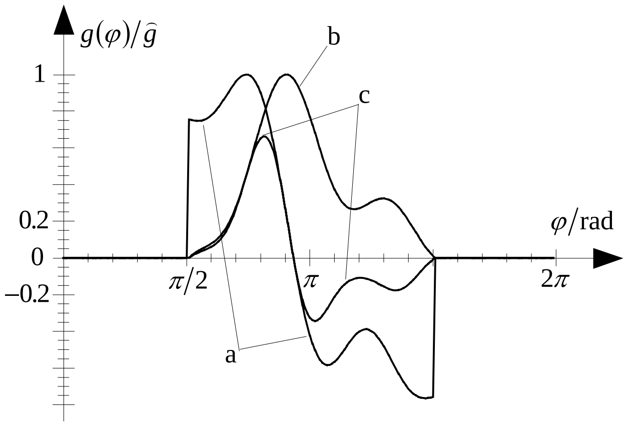


Figure 5. One period of testing signals u (a), u_i (b), and their product (c)

Fig. 6 confirms that the use of suitable window and averaging the DFT coefficients greatly reduces the estimation error.

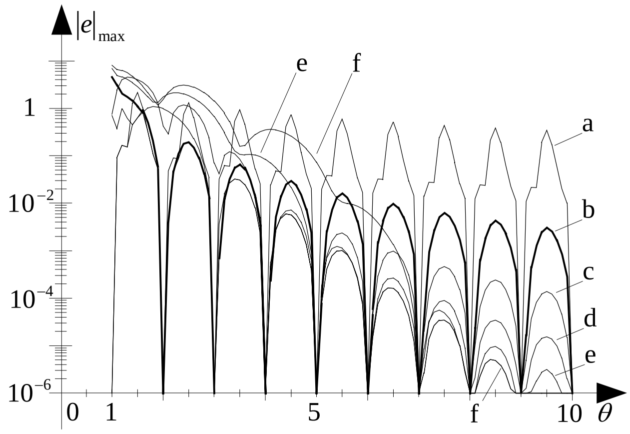


Figure 6. Maximal relative errors of the active power estimation by rectangular window (a), by Hanning window (b), by Hanning window and averaging (c), by cosine $p = 3$ window and averaging (d), by cosine $p = 4$ window (e), and by cosine $p = 4$ window and averaging (f)

The number of the DFT coefficients depends on signal cycles in T_{M} or value of the relative frequency. In interval $1 < \theta < 2$ the best results are attainable with rectangular window. For other values of the relative frequency, if the noise is negligible, we have: $2 < \theta < 3 \Rightarrow$ Hanning window without averaging; $3 < \theta < 4 \Rightarrow$ Hanning window with averaging; $4 < \theta < 6 \Rightarrow$ cosine ($p = 4$) window without averaging; $6 < \theta < 8 \Rightarrow$ cosine ($p = 3$) window with averaging; $8 < \theta < 10 \Rightarrow$ cosine ($p = 4$) window with averaging etc.

Increasing the order of cosine windows and the number of used DFT coefficients in averaging is reasonable until the systematic errors drop under noise errors [7]. As a consequence, that noise $n(*)$ is added before the multiplication of voltage and current

$$[u(*) + n_u(*)] \cdot [i(*) + n_i(*)] = u(*) \cdot i(*) + n_p(*), \tag{10}$$

the noise distortion of power estimation depends also on values of pure multiplicands as values of additional weights.

$$n_p(*) = u(*) \cdot n_i(*) + i(*) \cdot n_u(*) + n_u(*) \cdot n_i(*) \tag{11}$$

The third term is practically negligible. Thus, the noise influence on power estimation can be evaluated only for particular case of signals and their signal-to-noise ratios [6]. In Fig. 7 voltage and current signals from Fig. 5 are affected by white, zero-mean noise $A_{noise} = 10^{-4}$. In the interval $1 < \theta < 5$ the best results of the proposed active power estimation are attainable by Hanning window ($p = 2$), in $5 < \theta < 7$ by $p = 3$ cosine window, and in $7 < \theta < 10$ again by $p = 2$ cosine window, because it has the lowest noise distortion among them.

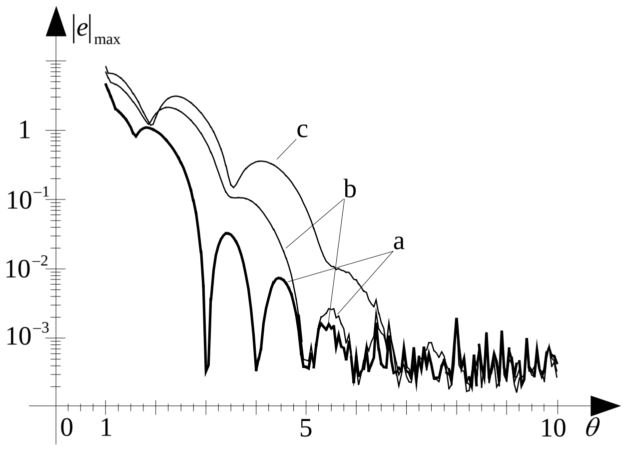


Fig. 7. Maximal relative errors of the active power estimation in presence of noise by Hanning window and averaging (a), by cosine p = 3 window and averaging (b), and by cosine p = 4 window and averaging (c)

IV. Conclusions

From analysis and experiments of the non-coherently sampled voltages and currents it can be deduced that averaging of DFT coefficients provides better estimation of the active power then only by windowing. Distortions of DFT coefficients and their number in interpolation have significant influence on the uncertainty. At the same time the systematic errors decrease with the increasing number of points. Increasing the number of used DFT coefficients is reasonable until the systematic error drops under the noise floor.

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