Adapting Median Filtering to Measurement Signals

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Abstract - Filtering out disturbances from transmitted or recovered measurement signals is one of the most important tasks of analog and digital signal processing for many decades. Removing of specific disturbances, having form of separate pulses (impulsive disturbances), requires specific algorithms because of very wide spectra of such pulses. The use of median procedures, especially of weighted medians, is particularly effective in such filtering procedures. According to the applied algorithm, signal samples, which include disturbances, are eliminated and replaced by some values resulting from neighbouring nondisturbed samples. The optimum weights of median filter depend on signal shape of deterministic signals. An analysis of frequency responses of medians is given in the contribution.

I. Weighted Median Filtering

A. Introduction

A theoretical background of the filtering method was presented some years ago in [1]. It was shown there also that a simple combination of weighted medians, for instance a linear function of two weighted medians, has a definite frequency response.
Let us analyze a process x(n)

$$x(n) = s(n) + z(n) \qquad (1)$$

consisting of signal s(n) and impulsive disturbance z(n). If x denotes the vector of samples x(n), the following operator, mentioned in [1], can be used to remove the disturbance

$$\tilde{y}(n) = \alpha_1 y_1^{WM}(n) + \alpha_2 y_2^{WM}(n) \qquad (2)$$

where $y_1^{WM}(n)$ and $y_2^{WM}(n)$ are Weighted Median (WM) smoothers defined by

$$y_i(n) = MED\left[\mathbf{w_i} \bullet \mathbf{x}\right] \qquad (3)$$

where $\mathbf{w_i} \bullet \mathbf{x} = \left[\; w_{i1} \bullet x(n) + ... + w_{iN} \bullet x(n)\;\right],$

$\bullet$ - operator of repetition.

B. Adaptive Weighted Median

The vector $\mathbf{w_i}$ can be optimized of course with respect to the mean squared value of difference between the filtered signal $\tilde{y}(n)$ and its original shape *s(n)*, for the whole sequence **x**. On the other hand a simple simulation shows that it can be optimized and adapted locally to the shape of filtered signal. It can be seen for instance from Figs. 1 and 2 - where parameter values are identical for both examples in order to illustrate the necessity of optimization, especially in case II, Fig.2 - that a proper choice of $\mathbf{w_i}$ in a particular case results in significant distortions of signals having different shapes. In both figures the continuous line denotes disturbed signal, the dotted line corresponds to signal after filtering process.

In the text of contribution the problem of filtering was investigated for $s(n)$ being a periodic sequence and $z(n)$ being an exponentially decaying pulse for different values of α_1 and α_2. It must be noted that the exponential shape of impulsive disturbance results from practical situations where very short pulses are corrupted by some elements of the system, for instance by transmission lines.
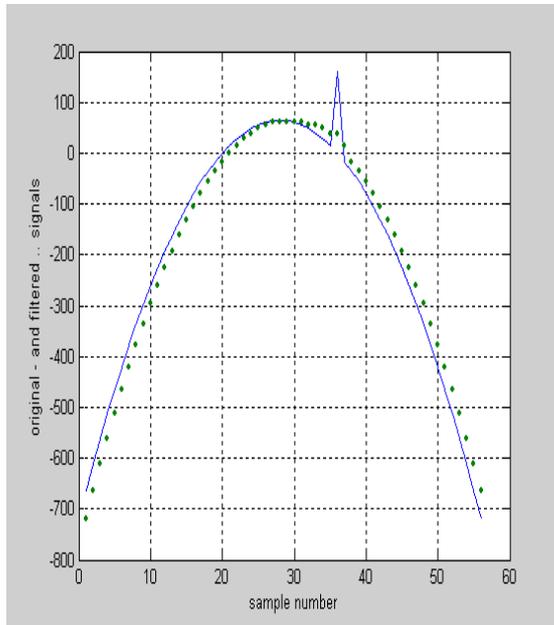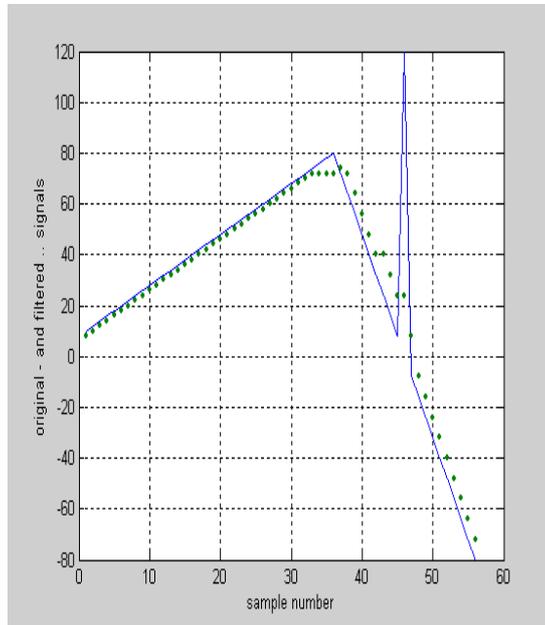

Figure 1. Filtering effects - case I.

Figure 2. Filtering effects - case II

A systematic sweeping of the $\mathbf{w_i}$ space, limited by certain practical bounds, for different local shapes of the signal seems to be the only practical approach to the optimization task. Such procedure was applied to a disturbed sine wave and the results were described in [4]. The frequency properties of the applied algorithm are interesting of course, from the point of view of any measurement signal. some simulation results are given in the next Section.

II. Results of Simulation Experiment

A simulation using MATLAB package was implemented (for $\alpha_2=0$). The $\mathbf{w_1}$ values were optimized for a sliding median window (N=5) and the frequency responses in the neighbourhood of optimum weights were calculated. The results are shown in Figs. 3-7. In all figures the optimum values of $\mathbf{w_1}$ components are indicated in order to illustrate the sensitivity of frequency characteristics with respect to weight vector parameters.

III. Conclusions

A possibility of simple adaptation of weighted medians used in filtering procedures was described in [1] as well as in previous contributions of the author. The presented frequency responses illustrate some dynamic properties of the weighted medians and their possible deviation for median weights differing from optimum values. It can be seen that in most cases the frequency response does not deviate from the optimum curve in a similar way, for weight increases and decreases.

References

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[4] J. Sawicki, "Adaptive Median Filtering", *Proceedings of the Workshop on Multimedia Communications and Services, May 5-6, 2004, Cracow, Poland, pp. 77-82*
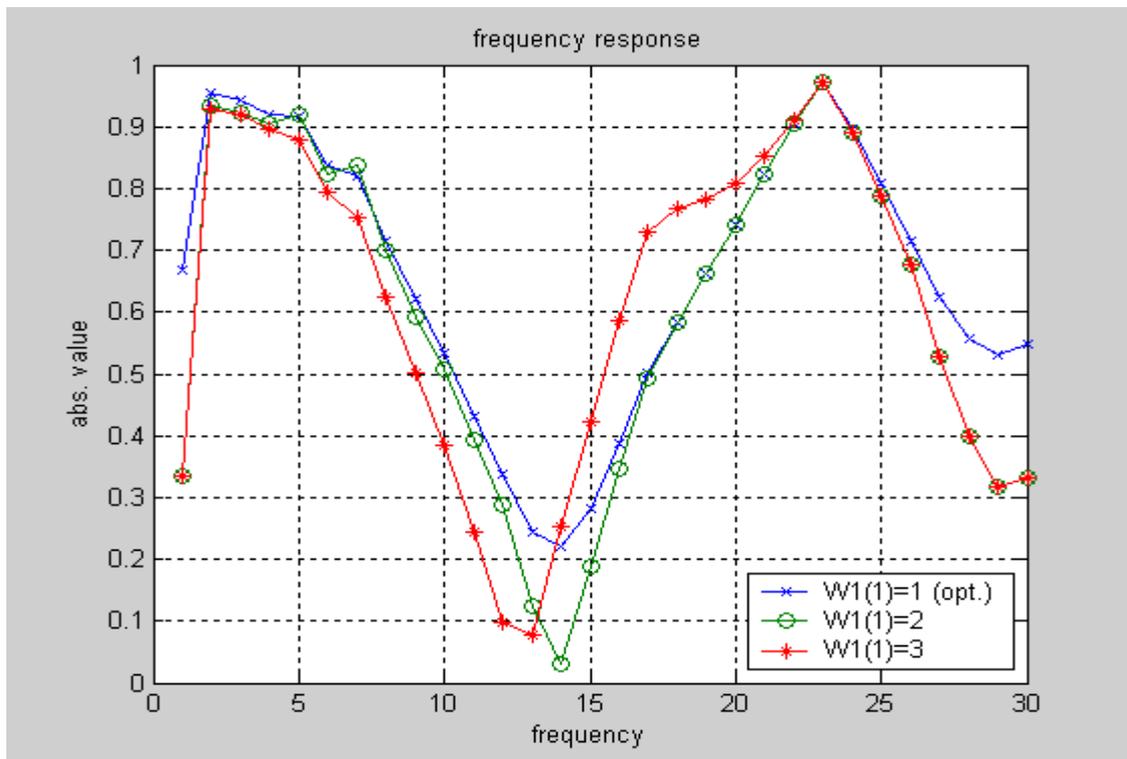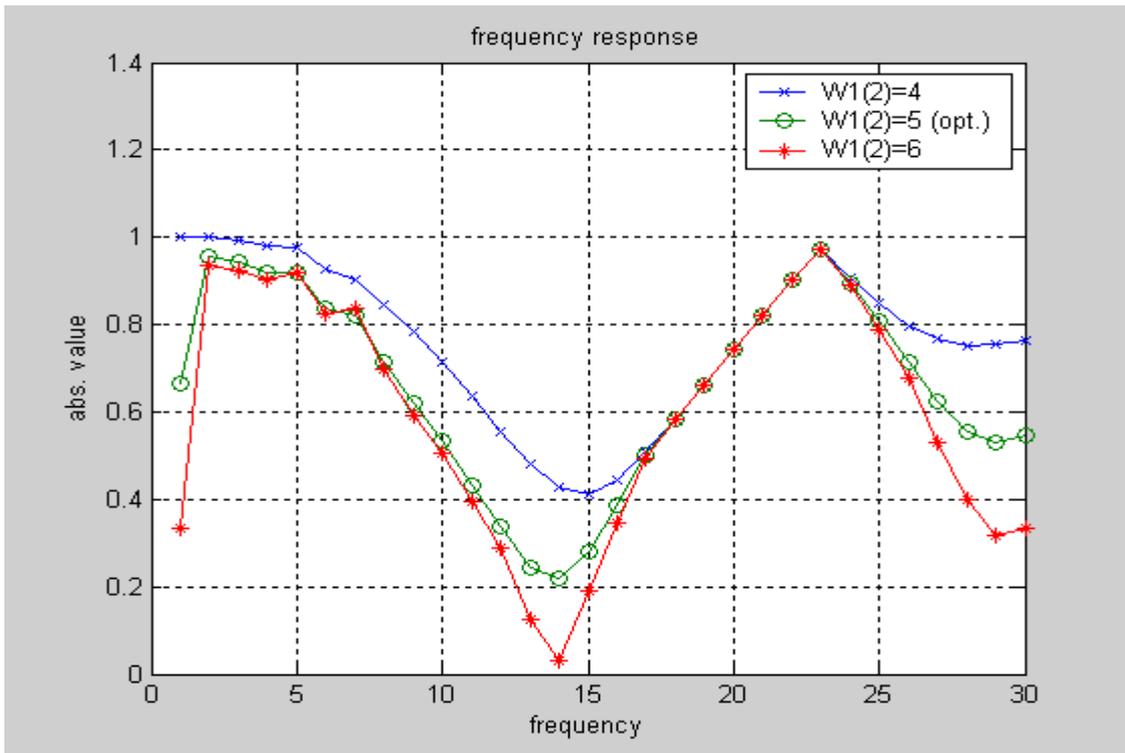
Figure 3. Frequency responses for different W1 values
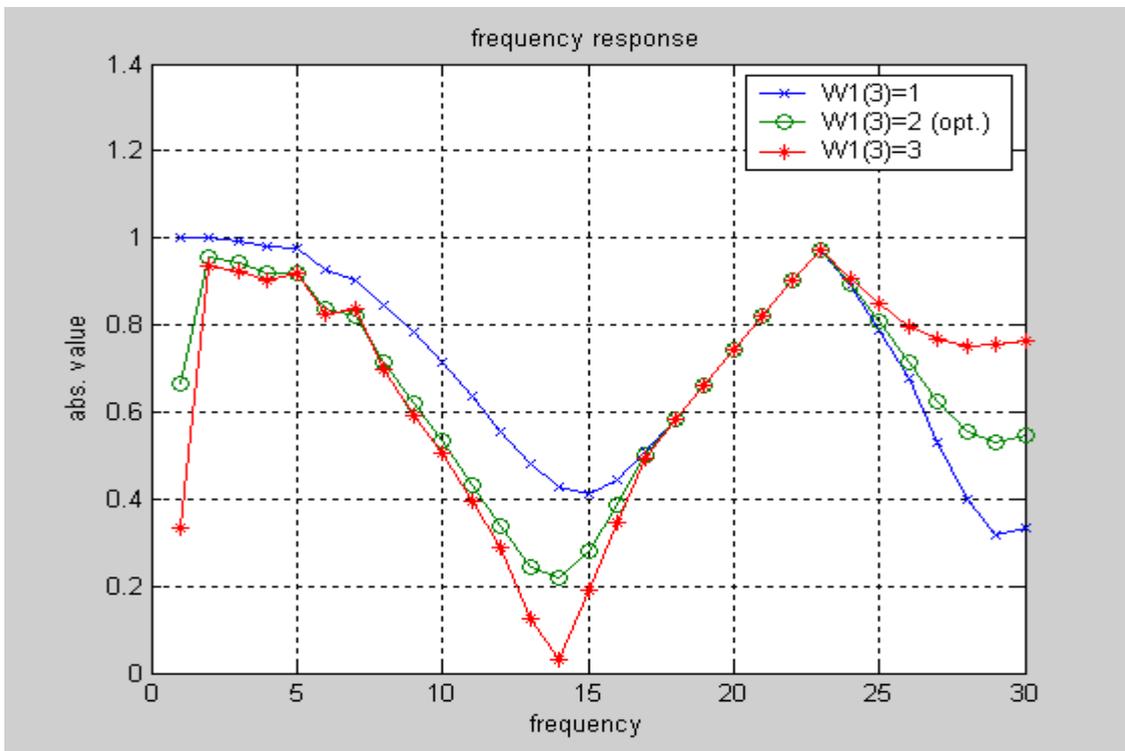
Figure 4. Frequency responses for different W1(2) values
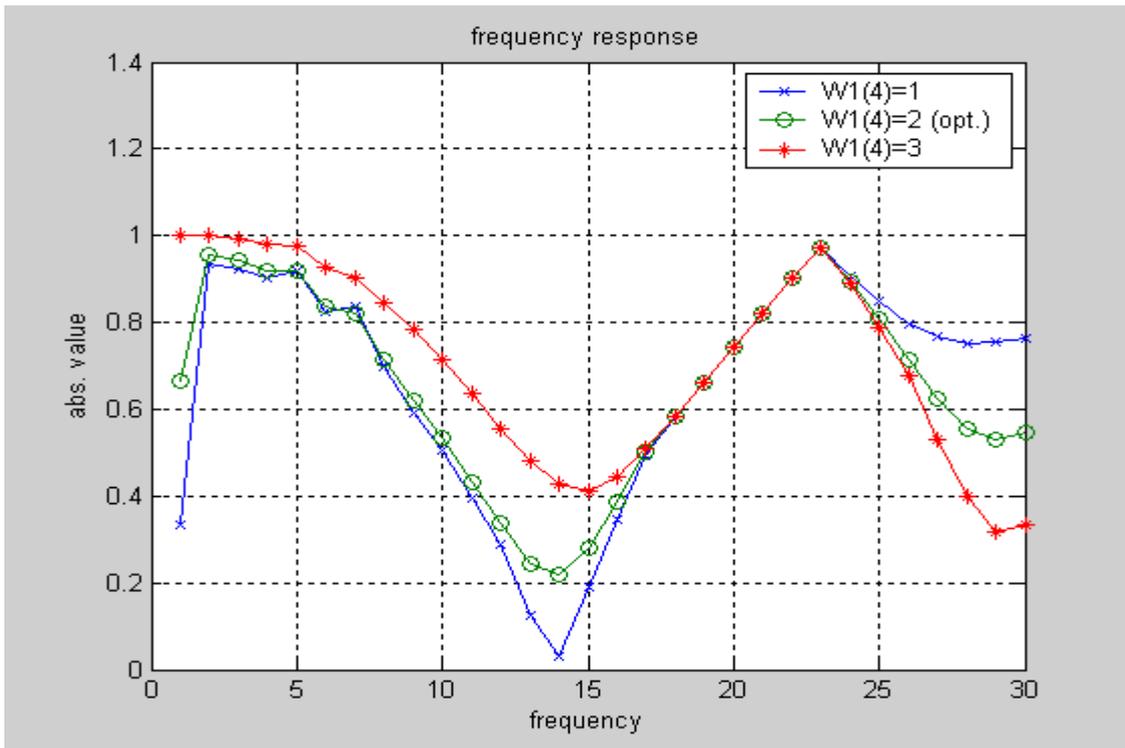

Figure 5. Frequency responses for different W1(3) values
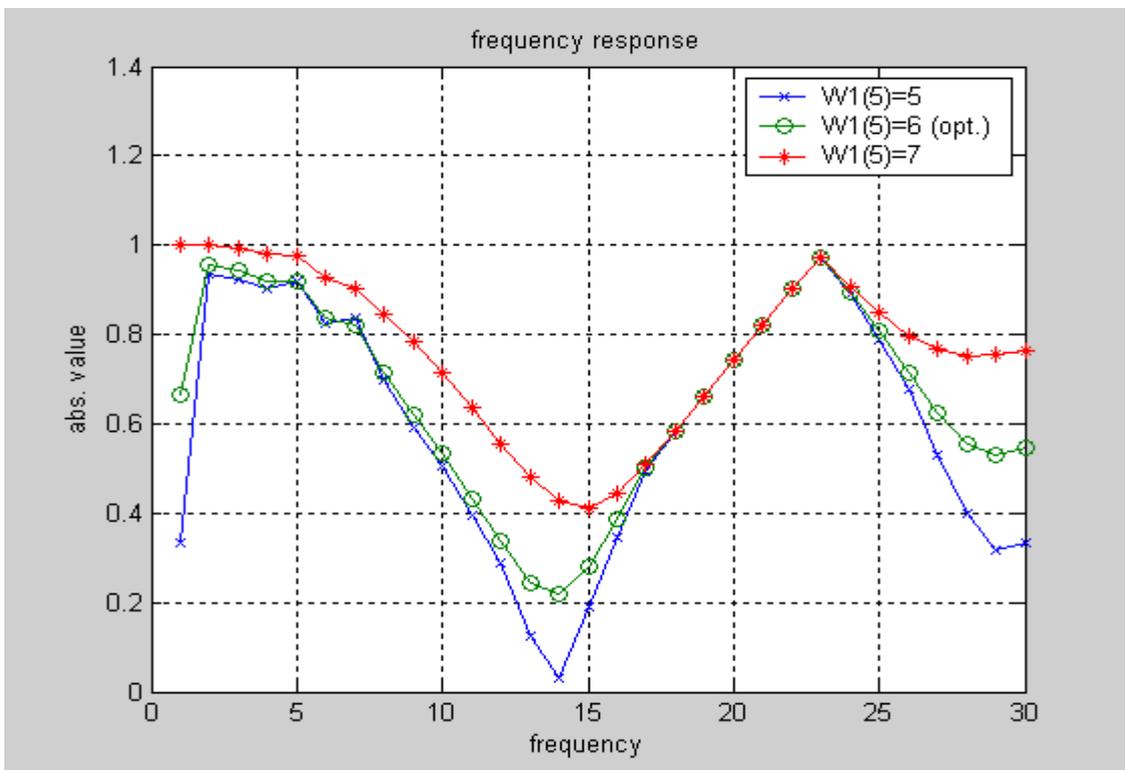
Figure 6. Frequency responses for different W1(4) values

Figure 7. Frequency responses for different W1(5) values