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Decoding of the FSK signal with noise and distortion with the use of coefficients of the time-frequency transform

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Abstract- The coefficients of wavelet packet decomposition of the signal represent the components of the instantaneous spectrum. For the FSK signal which consists of two instantaneous frequencies $f_0 + df$ and $f_0 - df$ thresholding and weighing of the absolute values of decomposition coefficients at properly selected nodes shows which of the two frequencies was present in the spectrum at a time interval. In the proposed algorithm more than one vector of coefficients represent each frequency for better distinguishing the instantaneous frequencies.

I. Introduction

Signal decomposition with wavelet packets gives, at the first level, two vectors of coefficients: approximation and detail coefficients which represents spectrum properties of the signal in the low and high frequency bands. The result of decomposition can be presented in the structure of a tree with three nodes: the parent node which is the vector of samples of the original signal and two child nodes – the vectors of approximation and detail coefficient. Each node can undergo further decomposition which splits its vector of coefficients in two. Time resolution in the child nodes is twice worse and frequency resolution is twice better than in the parent node. At each level the nodes for further decomposition can be chosen among all giving the possibility of adopting time and frequency resolution in the sub-band to best show the character of the signal. One of the ways of finding whether further decomposition of the node improves the representation of the signal is to compare the entropy of the parent node to the sum of entropies of the child nodes.

II. The idea of decoding the FSK signal by weighing the coefficients of the wavelet packet decomposition

The decomposition coefficients represent the components of the instantaneous spectrum of the signal. For the FSK signal which consists of two instantaneous frequencies $f_0 + df$ and $f_0 - df$ weighing the absolute values of decomposition coefficients at properly selected nodes shows which of the two frequencies was present in the spectrum at a time interval.

The ability to distinguish instantaneous frequencies in the decomposition coefficients of the FSK signal depends on the choice of the wavelet packet used for the decomposition.

The choice should be made between wavelet packets of compact time support and perfect frequency support. Wavelet packet of compact time support perfectly represent the properties of the signal in a certain time interval, but their frequency support is wider than the Heisenberg rectangle. Among the wavelet packets of compact support wavelet packet with smallest time support are of greatest interest. Daubechies1 (Haar) and Symlet1 wavelets have smallest width equal to [0 dt], where dt is the sampling interval (other Daubechies and Symlet wavelets have support width of $2N - 1$).

Shannon wavelet packets computed with perfect discrete low-pass and high-pass filters stand on the opposite side. Their frequency support is perfect – the coefficients represent the properties of the signal in the selected frequency sub-bands, while their time support is wider the Heisenberg rectangle (Shannon wavelet packet can be written as cosine modulated windows).

Matlab simulations show that compactly supported wavelets with small support width are best suited for the demodulation of the FSK signal. Big value coefficients point to the instantaneous frequencies $f_0 + df$ and $f_0 - df$, but due to the fact that sampling is not synchronized with the stream of modulated bits big coefficients appear in frequency bands neighboring the bands where they were expected in certain sampling intervals. In some other sampling intervals the selected coefficients are more or less equal and weighing them does not point to any instantaneous frequency.

In the proposed algorithm more than one vector of coefficients represent each frequency for better distinguishing the instantaneous frequencies. Vectors representing one frequency belong to the nodes lying side by side on the frequency axis. Thresholding is applied to the vectors of coefficients, coefficients under threshold become zero. In the first iteration the level of the threshold is equal to the rms value of the coefficients of the certain node and is different for each node. After the first iteration the thresholded coefficients are compared to zero at each sampling moment. If, at any sampling moment, all of them are equal to zero (which makes it impossible to read the bit) the threshold is lowered by $\sqrt{2}$ and the second iteration takes place. Next the non-zero coefficients become 1. Coefficients from the nodes representing the instantaneous frequencies $f_0 + df$ and $f_0 - df$ are added and the two sums are weighed to decode the original stream of bits.

III. Simulation results

The signal chosen for the simulations is of the type used in maritime radio-communication on channel 70: carrier $f_0 = 1700Hz$, modulation frequency $f_m = 600Hz$ (modulation speed $1200bps$), frequency deviation df=400Hz, linear phase. The modulation index of the signal is $\beta = df / f_m = 2/3$ which makes the conditions of distinguishing the instantaneous frequencies harder than with signals of big modulation index.

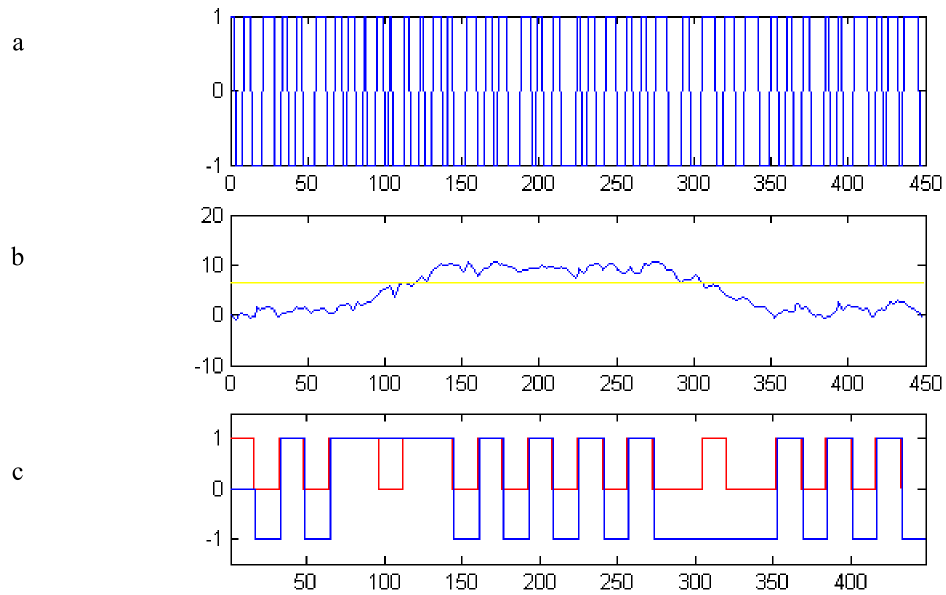

Fig.1. The results of the Matlab simulations of decoding the FSK signal with noise by weighing the coefficients of wavelet packet decomposition: a– the FSK signal with noise amplified and limited, b– the phase of the modulating stream of bit + the phase noise, c– modulating <-1 1> and decoded <0 1> stream of bits

The sampling frequency $f_p = \frac{1}{dt} = 2Kf_m$ where $K = 2^W$ is the number of samples taken per bit, the maximum spectrum frequency in the decomposition is $f_p / 2$. At the decomposition level W the time resolution is equal to the time duration of a single bit and the frequency resolution to fm, so W should be the maximum level in the decomposition of the FSK signal. The frequency resolution $f_m = 600Hz$ is enough to distinguish the two instantaneous frequencies: 2100Hz and 1300Hz. With W=4, $K = 2^W = 16$, 16 samples are taken per bit, sampling frequency is equal $f_p = 2Kf_m = 19200Hz$, the highest frequency in the spectrum is 9600Hz. At the decomposition level four 16 vectors of coefficients can be computed. Vectors [4,1] and [4,3] are chosen to reveal the presence the frequency 1300Hz in the instantaneous spectrum, vectors [4,2] and [4,6] will show 2100Hz. Vectors of coefficients are thresholded, coefficients under threshold become 0, over threshold 1. The sum S_1 of the thresholded vectors [4,1], [4,3] is compared to the sum S_2 of the thresholded vectors [4,2], [4,6]. If $S_1 > S_2$

the current bit is equal 1, if $S_1 < S_2$ the current bit is equal 0, if $S_1 = S_2$ the current bit is different from the previous bit.

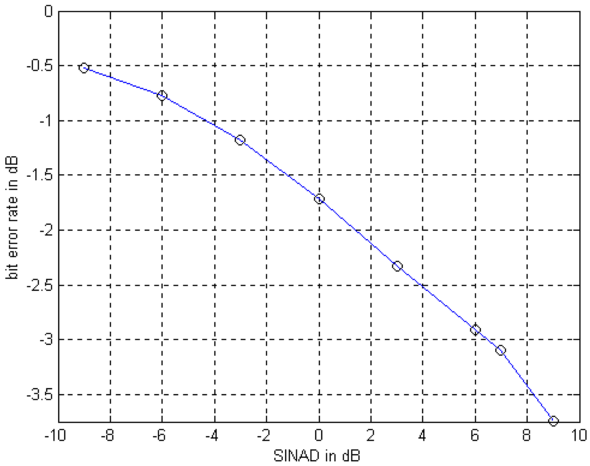

Fig.2. Estimated bit error rate as a function of the output S/N ratio in dB for the algorithm of decoding the FSK signal with noise by weighing the coefficients of the wavelet packet decomposition

The effectiveness of the algorithm was checked for noisy signals. Phase fluctuations of gaussian type with maximum frequency 3kHz were added to the signal. With the modulation index 2/3 the power *S* of the decoded stream of bits is equal to $10log(4/18) = -6.5dB$. The power *N* of the phase fluctuations was changed so that the S/N ratio varied from $-10dB$ to $10dB$ (fig. 2). Simulations show that decoding errors often occur when the phase passes the value of 2π which develops an impulse in the instantaneous frequency (fig.1).

IV. Conclusions

Demodulation of the FSK signal demands the usage of a microprocessor capable of computing the required node coefficients in real time. For the packet *P* bits long and the sampling rate 2^W per bit the length of the signal under test is $2^W P$. The length *L* of the Daubechies1 and Symlet1 filters is equal 2 ($L = 2$), so at the first level of decomposition the vector [1,0] is computed with $2^0 2^W PL$ additions and multiplications. Vectors [2,0], [2,1] are computed with $2*(2^{-1}2^W PL)$ operations, vectors [3,0], [3,1], [3,3] with $3*(2^{-2}2^W PL)$ operations and vectors [4,1], [4,], [4,3], [4,6] with $4*(2^{-3}2^W PL)$ operations. The total number of additions and multiplications while computing the coefficients, for $W = 4$ is equal 104*P*. On the 4-th level the rms value is computed separately for each vector of coefficients which demands 8*P* additions and multiplications, the vectors are thresholded (e.g. twice) – 8*P* operations and weighed – 2*P* additions and 2*P* weighing operations, which gives 20*P* operations all together. The total number of operations is equal to 124*P* and for the modulation speed 1200*bps* gives about 65*μs* per operation.

Some simplification of the algorithm for the FSK signal limited in amplitude at 2 comes from the fact that decomposition coefficients computed with Daubechies1 or Symlet1 wavelet packets at even levels are all integers and coefficients at odd levels are the multiple of $\sqrt{2}$ or $-\sqrt{2}$. Real time additions and multiplications can then be replaced by a loop-up table which considerably simplifies the complexity of microprocessor operations.

References

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