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Binary Sequences for Test Signal Generation obtained by Evolutionary Optimization

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Abstract-This paper describes a novel method for the digital synthesis of high quality signals by means of one-bit Digital-to-Analog Converters. The aim of the work is the analysis of the best binary sequences for signal generation, obtained using evolutionary algorithms.

I. Introduction

Direct Digital Synthesis (DDS) based on ΣΔ techniques has been adopted in many applications for the generation of test signals [1]. The generated signal is obtained through low-pass or band-pass analog filtering of the output of a one-bit Digital-to-Analog Converter (DAC) fed with a suitably selected sequence. The bit stream output by the DAC is based on the periodical repetition of an appropriate sequence $x[.]$ stored in a Look-Up Table (LUT). The advantage of this technique is the capability of producing high quality sources, with low hardware complexity. For this reason the method has been used to generate analog and mixed signals for Built-In Self-Test (BIST) [2].

The same technique may be extended to multi-bit modulators and applied to standard Arbitrary Waveform Generators (AWG). In this case the performance of the AWG is improved by reducing the useful bandwidth [3]. It is important to stress that good performance can be obtained without changing memory space, sampling frequency F_s, and frequency resolution.

The binary digital sequence $x[.]$ to be reproduced can be generated by simulating the behavior of a ΣΔ Analog-to-Digital Converter (ADC) having as input a reference analog signal, e.g. a sinewave. The sequence of simulated binary samples, encoding the desired analog signal, can then be taken as the input to the DAC used for the signal reproduction. Integer-multiple periods of the tone are recorded and the bit stream is then repeated periodically. The resulting bit stream has the same noise shaping properties of the ΣΔ modulator. The noise shaping reduces the amount of quantization noise in the signal band, thus increasing the signal-to-noise ratio (SNR). Using a similar principle, a pure sinewave can be synthesized by a digital resonator that produces a binary representation of a sinusoidal signal encoded in a ΣΔ bit stream [4-6]. However, the design and the stability analysis of such oscillators are not straightforward. These techniques are very effective but they lead to sequences whose properties are related to the physical structure of ΣΔ modulators and there is no certainty that these methods provide the best available sequences. For example it will be demonstrated that the limit existing for the SNR of ΣΔ modulators can be overcome.

In this paper a novel approach is proposed: $x[.]$ is generated using evolutionary algorithms (EA) that maximize given optimization criteria defined in the frequency domain, independently of a physical implementation of the simulated ADC.

II. Fundamentals of signal generation

The basic hardware for this application is very simple: a set of one-bit shift registers with a feedback from the last to the first one, a one-bit DAC and an analog filter. In some cases the filter is not practically necessary due to the intrinsic low-pass behavior of the device under test (DUT). Only low-pass signals will be considered in the following, but the method can be extended to include the synthesis of band-pass tones. Therefore, it is assumed that, after the DAC, the generated sequence undergoes low-pass filtering.

The performance analysis requires processing in the frequency domain. Accordingly, the expression of the discrete Fourier transform of a sequence $x[.]$ is [7]:

$$X[k]=\sum_{n=0}^{N-1} x[n]e^{-j2\pi \frac{n}{N}k} \qquad 0 \le k \le N-1 \qquad (1)$$

where N represents the length of the sequence. If M is the useful signal bandwidth in bins, $N/(2M)$ approximately represents the oversampling ratio (OSR) when the ΣΔ approach is applied. For given values of N and F_s, the output signal can contain only discrete frequencies f_k:

$$f_k = \frac{k \cdot F_s}{N} \qquad 0 \le k \le M \qquad (2)$$

The sequence can be codified using values in [0,1] or in [-1,+1]. Several considerations suggest the latter choice. Since (2) includes f_k=0, also the dc component can be generated or included in the undesired noise. Obviously, if the binary sequence does not contain negative value, X[0] can not be equal to zero. This is valid also for all sequences with odd N.

III. Implementation of the evolutionary algorithm

The EA is a stochastic global search method that imitates natural biological evolution. It operates on a population of potential solutions applying the principle that best individuals tend to reproduce and survive thus improving successive generations. This provides an implicit as well as explicit parallelism in the optimization.

Individuals are encoded as chromosomes; the chromosome values (genotypes) are mapped onto the decision variable (phenotypic) domain. The chromosome data structure stores an entire population in a single matrix χ of size $N_{ind}\times N$, where N_{ind} is the number of individuals in the population. Each row corresponds to an individual represented as binary sequences. Since many EA procedures were developed for binary representation, the application of EA to one-bit sequences seems to be very suitable and effective.

The optimization starts by randomly generating an initial population P_0. Each element of the relative matrix is given by the sign of a random number in the interval [-0,5;0,5]. The convergence of EA is faster, if the search starts from P_0 derived other methods (analytical considerations or simulation of ΣΔ modulators). However this condition is not strictly necessary, because the algorithm converges to values that are weakly dependent on the starting conditions. Consequently, the implementation is more general, if the initial population is randomly generated.

The optimization algorithm iteratively chooses the sequence that exhibits the best value of a predefined fitness function Φ over the selected low-frequency band, which corresponds to the signal band when the ΣΔ approach is taken. As fitness function, several candidates are possible. In all cases the usual objective is the generation of a sinewave, whose spectral purity is measured against either the SNR, or the signal-to-noise and distortion (SINAD) or the spurious-free dynamic range (SFDR) [8]. The parameter considered in this paper is the SNR, that is defined:

$$SNR = \frac{|X(k_0)|^2}{\sum_{k=0,k\neq k_0}^{M} |X(k)|^2} \qquad 0 \le k_0 \le M \, , \qquad M < \frac{N}{2} \qquad (3)$$

When the criterion has been established, only the population members with a good value of the selected Φ are chosen for reproduction (crossover).

Many selection techniques employ a "roulette wheel" mechanism, namely the i-th individual is selected if a generated random number is in the interval Δ_i assigned to the individuals, so that its amplitude is proportional to its fitness. The selection may be applied on a transformed Φ (ranking) to prevent premature convergence or to adapt the numerical values. For example, since dB representation can be interpreted as a transformation Φ may become negative, so that an additive offset is necessary.

Like its counterpart in nature, crossover produces new individuals whose genetic material may belong to both parents, so the information is shared among individuals. All the individuals of the preceding population are replaced by new individuals (generation gap equal to 1). Moreover in order to avoid that an individual recombine with itself, the selected individuals was divided in two distinct groups (father matrix χ_f and mother matrix χ_m). The new chromosome matrix is based on the element-by-element sum $\chi_f+\chi_m$. Such sum produces ±2, if the elements are identical, or zero, if they are different. In the latter case the sign is selected at random with equal probabilities. This is accomplished by employing a given third random term as in the following expression:

$$\chi = sign\{\chi_f + \chi_m + r\} \qquad \mu \ge 0 \qquad (4)$$

where r is an $N_{ind}\times N$ matrix, whose entries are uniform random variables in the interval [-2-μ;2+μ]. If $\mu\neq 0$, also the sign of identical elements is influenced by the random term r. Depending on the value of

μ some chromosomes are probabilistically muted. Mutation is generally included in EA's because it ensures that the probability of searching a particular solution is never zero.

Often in binary EA's the bits are randomly shuffled to remove positional bias. In the proposed optimization the information is held in the sequence of the bits, therefore shuffle is not useful. On the contrary it may happen that two good parents generate a bad offspring. The reason is that the phase does not influence Φ, but it is significant in crossover. For example, the sequence obtained by juxtaposing bit streams representing $\sin(2\pi f_k t)$ and $\sin(2\pi f_k t+\pi)$ does not contain the frequency f_k at all. In order to compensate for this effect, at first the phase of both parents is estimated using (1); then one of the parents is circularly shifted by n bins depending on the estimated phase difference $\Delta\hat{\varphi}$:

$$n = \frac{N}{2\pi k_0}\Delta\hat{\varphi} \tag{5}$$

The effectiveness of (5) is limited because the second term is not integer. Moreover it clashes with the self-optimization of EA and actually when N_{ind} is suitably large, is not necessary.

The process is repeated during each generation. Since EA is a stochastic search method, it is difficult to formally specify convergence or termination. EA stops when the selected parameter is under (or over) a desired threshold. If the best possible value is not known the process is terminated after a number γ of generations. Only the best member of the population is considered a solution. If no acceptable solutions are found, the EA may be restarted or a fresh search initiated.

The performances of the applied algorithms for the optimization of the binary sequences was investigated and verified in order to individuate the best settings for the critical parameters.

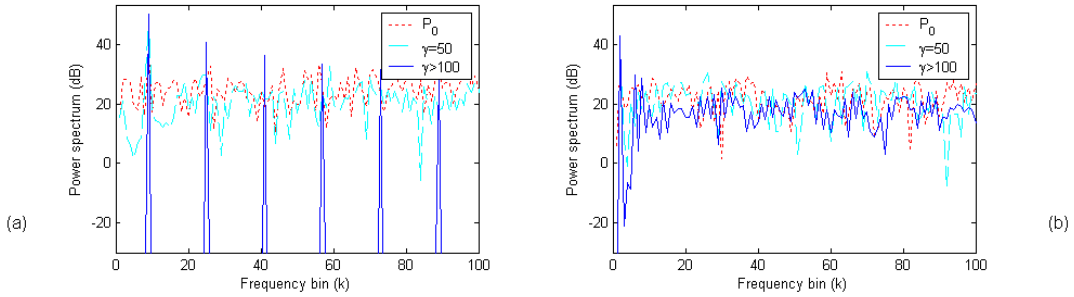

Figure 1. Power spectrum of the best individual, according to in-band SNR, of a population. Each graph refers to initial population (random), 50th and final generation.

IV. Results obtained by the evolutionary algorithm

In order to prove the effectiveness of the proposed procedure, the EA has been applied for several values of N, M and k_0. Fig. 1 shows the effects on the power spectrum of the application of an EA on a 2000-individuals population of sinusoidal sequences assuming N=512, M=23 (OSR≈11) and k_0=8, N=256, M=4 (OSR≈32) and k_0=1 respectively. The considered optimization criterion is the in-band SNR for a pure sinewave. P_0 and the associated SNR are chosen at random; afterwards the quality of the signal is improved. As can be seen from fig. 1, the optimization algorithm produces sequences whose spectral characteristics are iteratively improved by EA. The sequence obtained in fig. 1a is optimum in the sense that the in-band noise is exactly zero.

The quality of the bit stream is remarkably better than that obtained using ΣΔ-based techniques, even if compared to the theoretical SNR achievable by an L-th order ΣΔ modulator [9]:

$$SNR_{\max} = 10\log_{10}(\sigma_x^2/\sigma_e^2) - 9{,}94\cdot L + 10\log_{10}(2L+1) + 3(2L+1)\log_2 OSR \tag{6}$$

The SNR mainly depends on the OSR while σ_x and σ_e represent the standard deviations of the reference signal and of the quantization noise, respectively. Therefore the first term is less than the maximum wide-band SNR for b-bit DDS (6,02b+1,76 [7]). For example, [1] indicates that the maximum achievable SNR for fig. 1a (OSR=16) is 49 dB; [10] indicates 60 dB. It is interesting to stress that the quality of ΣΔ bit streams strongly depends on amplitude and phase of the modulated sinewave (even 40 dB differences for 50 µV or 3 rads variations [1] for a second order ΣΔ) and on the chosen beginning time of the sub-sequence extracted from the generated bit stream [1]. Practically, without the optimization of these factors, the SFDR can be as low as 50 dB, but can exceed 80 dB after optimization. The SFDR increases for higher orders but only up to the fifth order. Fig. 2 compares the results of EA with a fifth-order simulated ΣΔ modulator [11-12]. The latter bit stream is not optimized but it is very far from EA performance. The gap between the two methods is evident in this kind of situation when the OSR is low (OSR<8). Actually also for fifth-order modulators, the best theoretical SNR in (6) is equal to 100 dB when OSR=16 and to 66 dB when OSR=8.

These examples and the conditions presented in next sections suggest an important point: the SNR of ΣΔ-based methods can be improved because the frequency behavior of the best sequences is substantially different from the spectral profiles the noise shaping mechanism tries to impose.

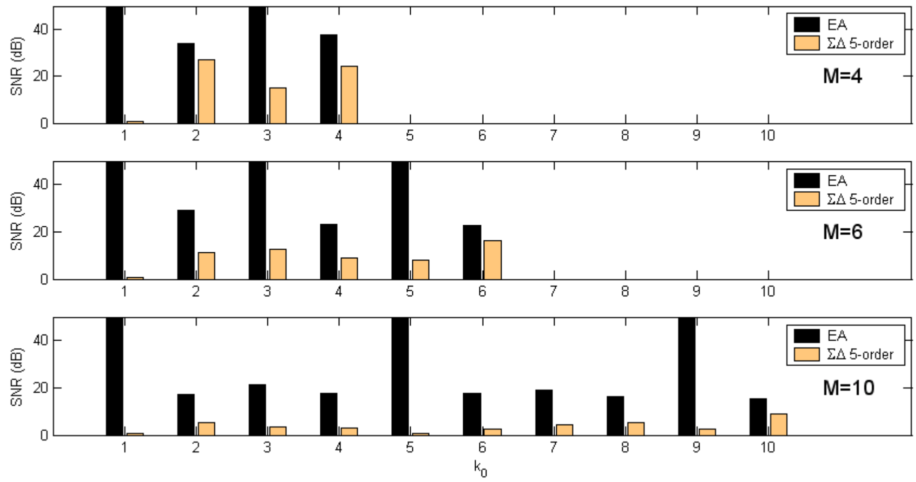

Figure 2. Best in-band SNR obtained by EA with N=64. It is compared with a fifth-order simulated ΣΔ modulator with no optimization. Values higher than 50 dB indicate infinite SNR.

The best way to carry out a verification of the proposed method is the comparison between its results and the optimum sequences individuated by the exhaustive analysis (EX) of all the possible sequences. It ensures to individuate without doubts the best solutions. The EX is very slow due to the amount of the considered sequences (2^N): for example, for N=32 it takes about 2 days on a 2.8 GHz Pentium IV, even for compiled C-code.

The EX was carried out for $N \le 32$ for every M and k_0. It demonstrated that, in these cases, EA always finds optimum sequences, spending less time than EX (few seconds instead of 2 days).

Although EA is able to find the optimal solutions, it is necessary to take in account the complexity of the search process. Experimental tests demonstrated that the value of N_{ind} ensuring good probability to find a suitable solution is related to N. If an appropriate N_{ind} is chosen, the influence of γ is negligible: EA converges for γ<200, otherwise it does not converge. As a rule of thumb, it was found:

$$N_{ind} > N \cdot 2^{\alpha} \qquad 1 < \alpha < 4 \qquad (7)$$

The relative complexity is very lower than EX. Anyway if a "good" $x[.]$ (not necessarily the best) is sufficient, EA always provides good results for N_{ind}<100 and γ<200, in few seconds.

V. Consideration about length of sequences and zero-noise sequences

The length of the sequence N is a fundamental parameter. From the above considerations and from the experimental tests, it is possible to deduce that the choice of N depends on the desired M and k_0, as well as on the analog frequency and amplitude.

The best available sequences are the so-called zero-noise sequences (ZNS), that is sequences exhibiting only the fundamental in the selected generated signal band. Among the ZNS there are basic ZNS (BZNS) featuring no noise but also the maximum concentration of power in the fundamental bin. By modifying two or more appropriate samples of BZNS, other ZNS are derived with a different (lower) $|X[k_0]|$. Therefore for a given N many ZNS exist.

It is important to stress out the characteristics of ZNS optimal sequences. In an N-length sequence the discrete period of the h-th harmonic of k_0 is:

$$T(h) = \frac{N}{k_0 h} \qquad h \le \frac{M}{k_0}, \qquad 0 \le k_0 \le M \qquad (8)$$

For values of N verified by EX, the in-band noise may be zero only if $T(1)$ is integer. In addition, T(h) must be integer for the first interesting H harmonics whose number depends on M: $h_2, h_3, \ldots, h_H$. Since it is sufficient to consider only harmonics whose order is a prime number, N must be multiple of $k_0 \cdot h_2 \cdot h_3 \ldots h_H$. It is simple to individuate the BZNS for H=1, namely for $M<2k_0$: every repetition of $T(1)$-length sub-sequences with no internal periodicity. Analogously, it is rather straightforward to build the

BZNS for H=2: a square wave with $T(2)$ samples equal to +1 and $T(2)$ to -1. The BZNS for H=3 requires the succession of three $T(6)$-length sub-sequences with their opposite; and so on for higher values of H.

Nevertheless a sequence not responding to the above requirements might theoretically be a ZNS, even if it was never experienced using EA and EX. So for non-integer T(h), the SNR is finite (fig. 1b). Its value increases as N increases and M decreases, even if its behavior is not monotone (fig.3). There is experimental evidence that, when N=64,128,256,512, EA always finds the considered ZNS (fig. 1a).

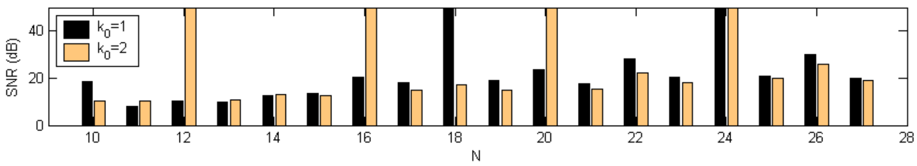

Figure 3. In-band SNR for the best sequence found by EX for two different k_0. The useful band is fixed: M=3. Values higher than 50 dB indicate infinite SNR (ZNS).

The techniques presented in next paragraph highlight the utility of choosing N as power of 2. In this case the FFT has the maximum efficiency. On the other hand, if for any reason a particular k_0 is needed, (8) and the experimental results (fig. 2 and fig. 3) suggest to choose a value of N that provides an integer period T(h). For example, N=60 allows the generation of 7 different values of f_k with zero noise; they become 11 for N=2520. In these cases, multitone signals can be ZNS too.

The choice of N should be mostly influenced by M. If low values of M are sufficient there is no reason to increase N beyond the minimum value allowing ZNS. The techniques based on a ΣΔ approach are forced to increase N because their performance is constrained by (6) and they do not appear to be able to find ZNS. However N may be long for four reasons: for multi-tone generation (with up-sampling); if a large frequency resolution is needed without changing F_s; to have the same register length for every test signal; to obtain a specific amplitude resolution.

As far as the amplitude resolution is concerned, actually it increases with N. In fact, let us consider (1) for a given k_0. Since x[.] may contain only -1 or +1, only discrete possible amplitudes $|X[k_0]|$ exist. The best SNR significantly depends on the desired amplitude. As reported above, it is very critical for [1]. Actually, the amplitude of the stimulus must be fixed in advance. EA automatically provides the amplitude that optimizes the SNR. It should be changed only if a specific value is required. By means of a self-test of the generator [2] and by using an active filter, it is simple to adapt the relative analog amplitude to compensate the ripple of the bit stream as well as of the filter.

VI. Techniques to improve the SNR by modifying the length of a sequence

The short sequences are useful not only for the opportunity to compare them to EX but also because they can be exploited to obtain longer sequences. Three methods were individuated to extend the length of a sequence: up-sampling, repetition and symmetry.

Up-sampling consists in inserting Z zeros between two samples of an N-length sequence x_1[.]. The obtained up-sampled sequence has length $(Z+1)N$. The same process is then applied to $Z-1$ sequences x_i[.] where i=2…Z, after that each x_i[.] was shifted by (i-1) samples. By adding or subtracting all the up-sampled x_i[.], a new $(Z+1)N$-length sequence is then obtained: it contains all the tones of the composing x_i[.]. It can be exploited to create multi-tone signals. If all the x_i[.] are identical, the result is a sinusoid with enhanced amplitude. The available amplitudes are related to the shorter sequences that have been summed.

The repetition of a sequence produces a new sequence with doubled N due to zeros created between frequency bins. It is important to stress that the ratio k/N is constant, so, according to (2), the synthesizable analog frequency f_k is the same.

The symmetry is an advanced analytical method to reduce the size of the population that has to be explored, risking losing some solutions. Let us assume that the k_0 bin is required with N bit. When the dc component (k_0=0) is not necessary, the number of +1's must be equal to the number of -1's. The k_0 component exists if the same sub-sequence x_1[.] is repeated k_0 times in the final sequence x[.]. If the sub-sequence x_1[.] has odd symmetry, x[.] has no even harmonics of k_0 (and no dc). Accordingly it is sufficient to search within x_1[.] a sub-sequence x_2[.]of length $T(2)$ with low odd harmonics. In practice, the following new fitness parameter Φ' can be minimized:

$$\Phi' = x[n]\left\{\sin\left(\frac{2\pi hk_0 n}{N}\right) + j\cos\left(\frac{2\pi hk_0 n}{N}\right)\right\} \qquad n < T(2)\,, \qquad h = 3,5,7,\ldots \qquad (9)$$

In this way a single harmonic component may be iteratively optimized: firstly the sequences for which (9) with *h*=3 is under a specific threshold (often exactly zero) can be individuated. Among these sequences the best ones minimize (9) for *h*=5, and so on. The real and the imaginary part of (9) may be separately analyzed. However if a further four quarters symmetry is forced, the imaginary part is zero. Also EX can be applied for (9): it runs on x_2[.] instead of x[.], thus reducing research complexity. Symmetry reduces the complexity of the problem and allows to find ZNS, but, using this approach, it is possible to neglect some good sequences that do not belong to the class of considered solutions.

VII. Conclusions

A novel method for the generation of test signals was presented. It is based on the generation of binary sequences through evolutionary algorithms that optimize given criteria in the frequency domain. The simulations demonstrate that the method is able to produce sequences with good spectral properties that were verified for multitone sinewaves too. This technique can easily be extended to a wide set of parameters and to multi-bit converters.

The proposed method is able to obtain good performances also for short sequences and low oversampling rates. In particular it is possible to generate zero-noise sequences. The approaches based on ΣΔ techniques appear not to be able to find this kind of sequences, therefore they are forced to increase the length of the bit stream.

Unluckily the complexity of evolutionary algorithms restricts their employment for very long sequences. Some theoretical processes can be exploited to overcome this limitation. The maximum effectiveness of the test signal generation is obtained provided that specific conditions are respected. Although through evolutionary algorithms it is difficult to obtain sequences as long as the sequence produced by other techniques, the useful band is adequate, as high oversampling is not strictly necessary.

References

[1] B. Dufort, G.W. Roberts, "On-Chip Analog Signal Generation for Mixed-Signal Built-In Self-Test," *IEEE Journal of Solid-State Circuits*, Volume: 34, Issue: 3, March 1999, pp. 318 – 330.

[2] M. Hafed, N. Abaskharoun, G.W. Roberts, "A Stand-Alone Integrated Test Core for Time and Frequency Domain Measurements," *Proceedings of IEEE International Test Conference ITC '01*, pp. 1190 – 1199.

[3] B. Dufort, G.W. Roberts, "Increasing the Performance of Arbitrary Waveform Generators Using Periodic Sigma-Delta Modulated Streams," *IEEE Transactions on Instrumentation and Measurement*, Volume: 49, Issue: 1, Feb. 2000, pp. 188 – 199.

[4] D. Macii, F. Pianegiani, P. Carbone, D. Petri, "A Stability Criterion for High-Accuracy Delta-Sigma Digital Resonators," *Proceedings of the 21st Instrumentation and Measurement Technology Conference IMTC '04*, Volume: 2, pp. 1160 – 1164.

[5] C. Rebai, D. Dallet, P. Marchegay "Signal Generation Using Single Bit Sigma Delta Techniques," *Proceedings of the 20th Instrumentation and Measurement Technology Conference IMTC '03*, Volume: 1, Pages: 619 – 623.

[6] A.K. Lu, G.W. Roberts, D.A Johns, "High-Quality Analog Oscillator Using Oversampling D/A Conversion Techniques," *IEEE Trans. on Circuits and Systems II*, vol. 41, July 1994, pp. 437-444.

[7] A.V. Oppenheim, R.W. Schafer, *Discrete-Time Signal Processing*, Prentice Hall, 1999.

[8] IEEE Standard for Terminology and Test Methods for Analog-to-Digital Converters, *IEEE Standard* 1241 – 2000.

[9] P.M. Aziz, H.V. Sorensen, J. Van Der Spiegel "An Overview of Sigma-Delta Converters," *IEEE Signal Processing Magazine*, vol. 13, pp. 61-84, Jan. 1996.

[8] X. Deng and P. Fan "New Binary Sequences with Good Aperiodic Autocorrelations Obtained by Evolutionary Algorithm," *IEEE Communications Letters*, vol. 3, No. 10, Oct. 1999, pp. 288-290.

[10] B. Dufort, G.W. Roberts, "Signal Generation Using Periodic Single and Multi-Bit Sigma-Delta Modulated Streams," *Proceedings IEEE International Test Conference ITC '97*, Pages: 396 – 405.

[11] R. Schreier, *The Delta-Sigma Toolbox*, Analog Devices Inc.

[12] S. Brigati, et al., "Modelling Sigma-Delta Modulator Non-Idealities in Simulink," *Proc. IEEE International Symposium on Circuits and Systems ISCAS '99*, vol. 2, pp. 384-387, May-June 1999.