

Use of precise digital voltmeters for impedance measurements

Damir Ilić, Ivan Leniček

Department of Electrical Engineering Fundamentals and Measurements
Faculty of Electrical Engineering and Computing, Unska 3, HR-10000 Zagreb, Croatia
Phone +385 1 6129753, Fax +385 1 6129616, E-mail: damir.ilic@fer.hr, ivan.lenicek@fer.hr

Abstract – The traceability chain of the Primary Electromagnetic Laboratory (PEL) is presented, and the place of the resistance standard of 100 MΩ is analysed in details, as well as its construction and characteristics. Its one hundred 1-MΩ elements in parallel connection forms the 10-kΩ value, and can be easily compared to the reference 10 kΩ standard of PEL; in that way its long-term drift was found to be $\approx 0,067$ ppm/day (ppm means parts per million). Furthermore, the method of comparison of 100 MΩ resistance standard by means of 100 pF capacitance standard and high-resolution digital voltmeters (DVs) is analysed, where DVs measure ac voltage of frequency close to 16 Hz in dcV range, and the mean value of one-third cycle is measured. It was found that the voltage ratio could be stable within the limits of 0,1 ppm when the additional frequency stabilization of the ac calibrator is used.

I. Traceability chain of PEL

In the Primary Electromagnetic Laboratory (PEL), which is a part of the Faculty of Electrical Engineering and Computing of the University of Zagreb and a part of the Croatian metrology system, the basic quantities of its traceability chain are: voltage, capacitance, resistance, and frequency. The system of traceability is shown in Fig. 1, where the reference resistance standards are marked as RR1 (L&N 4210¹, 1 Ω), RR10k (L&N 4040-B¹, 10 kΩ) and RR10M (Fluke 742A¹, 10 MΩ), the Caesium standard is marked as RF (Oscilloquartz S.A. 3200¹), while the reference voltage and capacitance standards are marked as RU (Fluke 732A¹) and RC (GR 1408¹, 100 pF), respectively. Reference standards RR1 and RR10k have been maintaining into the oil ultrathermostat at a temperature of 23 °C, and their characteristics are presented in [1]. Some other resistance standards of PEL, as well as their maintenance and methods of comparison, were presented in [2].

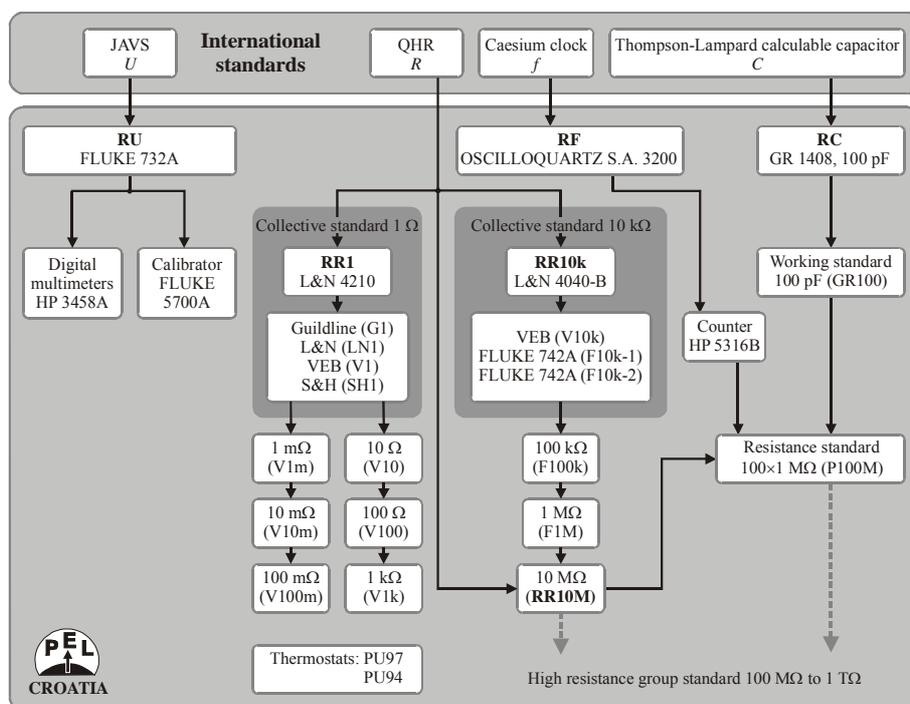


Fig. 1. A traceability chain of the PEL: reference standards of voltage, frequency, capacitance and resistance (1 Ω, 10 kΩ, 10 MΩ) are marked as RU, RF, RC, RR1, RR10k, and RR10M, respectively

I.A. Long-term drift of 100 M Ω resistance standard

The working standard of 100 M Ω (in Fig. 1, as well as in further text, marked as P100M, while its resistance as R_X) is self-developed (in 1989) standard of Hamon type that consists of 100 equally balanced 1 M Ω resistors connected in series, and placed in a sealed and very well temperature controlled casing, filled with dry air. Primary it was used in the experiments and investigation with the Voltage Balance ETF-84 [3], which have been done in PEL for many years. The detailed description of that standard is pointed out in [4]. The chosen elements are commercial metal-film resistors with tolerances $\pm 5\%$ having the temperature coefficients between $(-61$ to $+76)$ ppm/K, and the individual resistances between 0,996 M Ω and 1,01 M Ω . These elements were, at that time, available to us, and this was the reason why they were selected. In order to balance them it was necessary to connect in series appropriate additional resistances; therefore their total serial resistance is above 101 M Ω . The construction of the P100M is shown in Fig. 2: the elements $R_1 \div R_{100}$ have a nominal value of 1 M Ω , $S_1 \div S_{101}$ are short-circuit elements, $DK_1 \div DK_{51}$ and $GK_1 \div GK_{50}$ are contact elements, while R_{k1} and R_{k2} are resistances of the conducting wires between the outer connectors and short-circuit elements. Between the inner parts and the chassis the thermal isolation is inserted, which is not shown in Fig. 2.

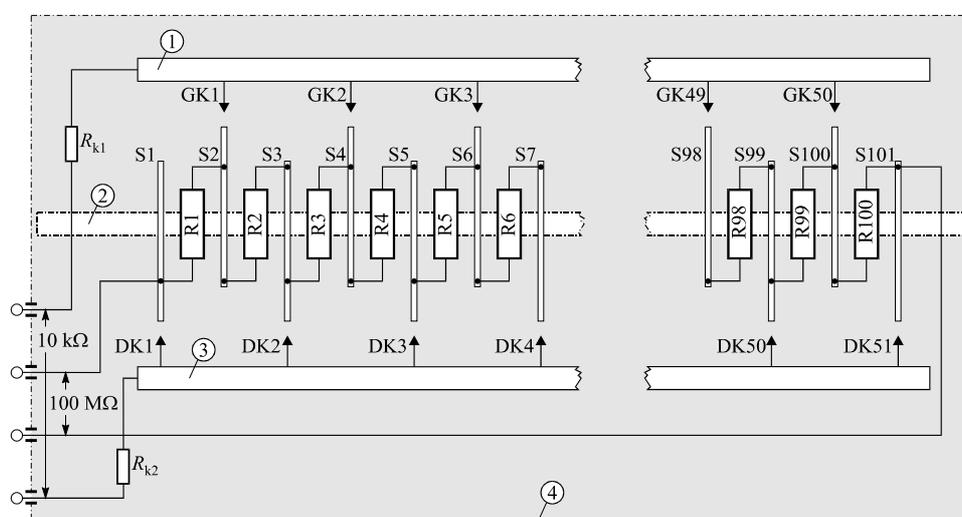


Fig. 2. Construction of the 100 M Ω standard (P100M): 1 and 3 – metal short-circuit elements, 2 – isolation bar, 4 – aluminium chassis

Using a special switching device the resistors can be circuited in parallel, and 100^2 times smaller resistance (i.e. 10 k Ω) can be obtained. If the resistors are balanced within 100 ppm, the error of the ratio do not exceed 0,01 ppm, and the simple calibration of the R_X with the 10 k Ω is possible. Following this possibility, the measurement of its long-term drift was performed in 1989, after its construction and composing has been finished, and gave out the relative value of 0,23 ppm/day.

In 1992 another method of comparison has been developed and tested in PEL [5] (a part of this is analysed in the following section II), in which the resistance standard P100M is compared to the capacitance standard of 100 pF (GR100, Fig. 1) by means of DVs which measure very precisely the ratios of two dc voltages, as well as of low-frequency ac voltages. These measurements showed that the long-term drift decreased to a relative value of 0,15 ppm/day.

Another measurement of its long-term drift was done in 1999 for a period of approximately seven months, during that the P100M (with parallel connection of its elements) has been compared to the reference 10 k Ω standard of PEL (RR10k) using the self-developed method with two DVs [2, 6]; the results are shown in Fig. 3. The shown value for one day represents the arithmetic mean value of 20 repeated measurements in a total duration of ≈ 2 h. In each measurement, the ratio of two resistances is measured, and the typical standard deviation of that ration was $\approx 0,02$ ppm, while the standard deviation of the mean value for one day was at the same level. The dominant source of uncertainty in these measurements was the uncertainty of the reference standard itself, which was at that time estimated to be about 0,87 ppm. However, it was found that the long-term drift of P100M has a relative value of 24,42 ppm/year (or $\approx 0,067$ ppm/day). This means that the change of its resistance drops with aging, which is a good characteristic. Therefore, for nowadays it could only be lower than the last obtained

value. Even it is somewhat high drift, the P100M standard is very useful for the direct connection to the capacitance standard, as explained in section II.

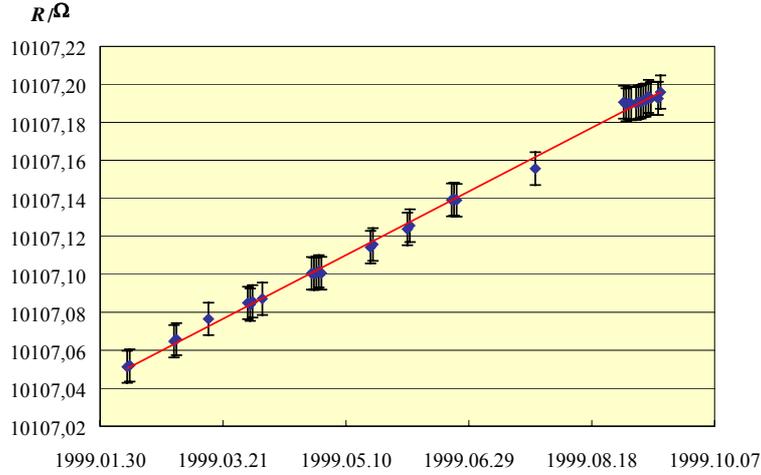


Fig. 3. Change of the resistance of P100M at a level of 10 kΩ (parallel connection) between February 9 and September 15, 1999

II. Measurement of voltage ratio at ≈ 16 Hz

The method for the calibration of standard P100M ($R_X = 100$ MΩ) by means of standard capacitor GR100 ($C_N = 100$ pF) at frequency $f = (2\pi C_N R_X)^{-1} = 15,915$ Hz has been developed in PEL [5, 8]. In this method DVs with $8\frac{1}{2}$ digit resolution (HP 3458A¹ [7]) are used in their dc operation modes to measure both dc and ac voltages. The measuring bridge for this purpose has two dividers (Fig. 4): one of them consists of two resistance standards R_{b1} and R_{b2} (Fluke 742A¹ [7]) which forms a divider 10 MΩ/10 kΩ, while in the second one, as an upper element, the P100M is used in measurement with dc current, and GR100 with ac current. The lower element in this branch is R_{a2} (Fluke 742A¹, 100 kΩ [7]), which yields to the ratio 1000:1 in both dividers.

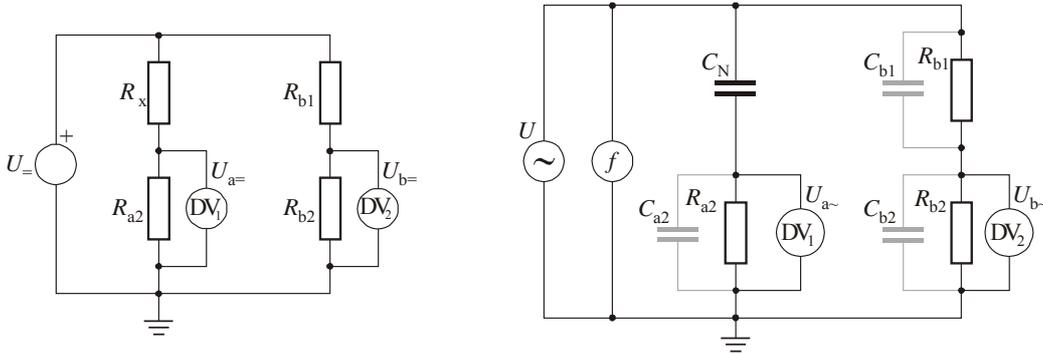


Fig. 4. A measurement method for $C_N \rightarrow R_X$ comparison includes measurements with dc current (left) and ac current (right) of ≈ 16 Hz – in both measurements DVs operate in dcv mode; C_{b1} , C_{b2} and C_{a2} are the parasitic capacitances

From the voltage ratio $U_{a=}/U_{b=}$, measured with dc current, as well as the ratio $U_{a~}/U_{b~}$ with ac current, follows:

$$R_X = \frac{U_{b=}}{U_{a=}} \frac{U_{a~}}{U_{b~}} \frac{K_2 K_3}{\omega C_N} - R_{a2}. \quad (1)$$

The factor K_2 takes into account the influence of the parasitic capacitance C_{a2} :

$$K_2 = \sqrt{1 + [\omega R_{a2} (C_{a2} + C_N)]^2} = \sqrt{1 + \varphi_{km}^2}. \quad (2)$$

The C_{a2} is equal to the sum of the capacitance of: input connector of DV₁, connecting cable to DV₁, resistor standard R_{a2} , and "LO" connector of C_N to its case. To determine factor K_2 with an uncertainty of 0,1 ppm, it is necessary to measure C_{a2} (≈ 500 pF) with an uncertainty of 1,7 pF, which is hard demand for direct measurements of parasitic capacitances. However, the parameter φ_{km} can be calculated from the phase difference between the voltages $U_{a\sim}$ and U , and the needed accuracy will be achieved if the phase difference is measured with an uncertainty of 17 μ rad, which is not easy to realize. In opposite, the factor K_3 in (1) depends on the frequency compensation of the divider with parasitic capacitances C_{b1} and C_{b2} , and is calculated as follows:

$$K_3 = 1 + 0,499 [(\varphi_{b1} - \varphi_{b2})(\varphi_{b1} + \varphi_{b2})], \quad (3)$$

where $\varphi_{b1} = -\omega R_{b1} C_{b1}$ and $\varphi_{b2} = -\omega R_{b2} C_{b2}$. For compensated divider $\varphi_{b1} = \varphi_{b2}$, $\Delta\varphi_b = \varphi_{b2} - \varphi_{b1} = 0$ and factor $K_3 = 1$. This can be verified from the phase measurement between the voltages $U_{b\sim}$ and U , and to determine K_3 with 0,1 ppm it is necessary to measure $|\Delta\varphi_b|$ better than 450 μ rad, which is not a strict requirement. Both factors K_2 and K_3 can be determined with the needed accuracy using the very precise method of phase measurement, as it was found and presented in [6, 9].

II.A. AC voltage measurement

Precise measurement of low-frequency ac voltage is possible using the method in which only one sample per period is taken when DVs integrate on dcV range. For the measured voltage

$$y(t) = Y_m \sin(\omega t + \varphi_1) \quad (4)$$

with period T , the measured value u_i of the i -th sample of the so-called "derived sine" will be:

$$u_i = \frac{Y_m \sin(\pi T_m / T)}{\pi T_m / T} \sin\left[\frac{\pi}{T}(2t_0 i + T_m) + \varphi_1\right], \quad (5)$$

where T_m is the integration time, while the beginning of integration is shifted for the interval $t_0 = T/p$ (p is an integer) for every subsequent period. In a real situation the measured voltage is contaminated by the influence of dc component and higher harmonics as well. If we assume that the dc component is negligible (or can be eliminated), the share of the n -th harmonic in the measured value u_i is equal to:

$$u_{n,i} = \frac{Y_{m,n} \sin(n\pi T_m / T)}{n\pi T_m / T} \sin\left[\frac{n\pi}{T}(2t_0 i + T_m) + \varphi_n\right]. \quad (6)$$

The ratio between T_m and T has been chosen to $1/3$, and in this case the harmonics dividable by 3 do not affect the rms value of the "derived sine". The influence of other harmonics is entirely insignificant if they are lower than 0,01 % of the basic harmonic. Thus, the effective value of the "derived sine" can be determined as the effective value of the main harmonic, described by (5).

A small difference between the sampling rate and the main frequency yields to a non-integer number of samples per period of the "derived sine". Thus, the rms value of the samples will be:

$$U^2 = \frac{1}{k_{eff}} \sum_{i=0}^{k_{eff}-1} u_i^2, \quad k_{eff} = k + \Delta k. \quad (7)$$

Here k indicates the integer number of the "derived sine" partition ($k = m \times p$, where m represents a total number of periods), while Δk is the correction for numeric synchronization. Since the sum of squares needs to be calculated for $u_{k-1+\Delta k}$, an approximation with a 5th order polynomial is used around the samples of the "derived sine" close to zero value at the end of interval for which the rms value is calculated. The method is described elsewhere in details [5, 10].

II.B. Voltage ratio measurements and obtained results

Based on the described procedure of rms value determination, computer controlled measurements were done with nominal values $T_m = 21$ ms, $t_0 = 0,9$ ms and $p = 70$, which gave the interval between two samples equal to $t_0 + T = 63,9$ ms. Thus, frequency $f = 1/63$ ms = 15,873 Hz is used, which is very close to the value $(2\pi C_N R_X)^{-1}$. In the real situation these values should be slightly modified according to the corrective terms for each DV and the final frequency resolution of ac source. For one measurement altogether $n = 2381$ samples (or 34 periods of the "derived sine") were taken during the time of ≈ 3 min,

and that was repeated ten times in the so-called *standard measurement procedure*. Furthermore, the DVs were set for synchronized triggering, where a "master" voltmeter generates a trigger impulse at the beginning of integration, and sends it to a "slave" voltmeter [11].

In that way we are able to measure both voltages, as well as the ratio U_a/U_b . However, to determine R_X by (1) it is also needed to measure the ratio U_a/U_b , and on the other hand, to avoid the voltmeters' errors it is necessary to exchange the position of the voltmeters in the schemes in Fig. 4, as well as the polarity of dc source. All mentioned have a consequence that the duration of the whole measurement procedure is a couple of hours, and from that point of view it is of great interest to have good stability of the measured ratios. Following that approach we tested the influence of the frequency stability of the used ac source (Fluke 5200A¹ [7]) by repeating the *standard measurement procedure* during a longer period of time. Connecting an outer source with a stable frequency to the "Phase Lock" connector can stabilize the frequency of the output voltage of ac source; for that purpose HP 3245A¹ [7] is used. The results of the measurements of the ratio U_a/U_b , obtained in that way during ten hours (each value follows from repeated ten measurements of 3 min) are shown in Fig. 5 and 6, with and without frequency stabilization of the ac voltage, respectively. The used voltage U had an rms value of 80 V, while DVs were operated on 100 mV ranges.

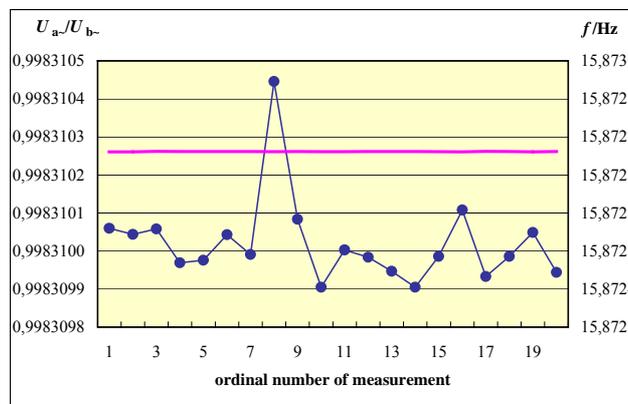


Fig. 5. Measurements of the ratio U_a/U_b with frequency stabilization; right axis is applied frequency

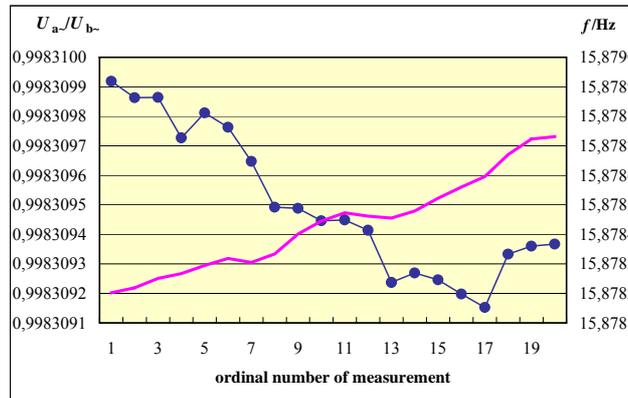


Fig. 6. Measurements of the ratio U_a/U_b without frequency stabilization; right axis is applied frequency

III. Conclusion

It is obvious that, with the frequency stabilization of ac source from the outer source, the ratio U_a/U_b could be as stable as 0,1 ppm for a couple of hours (Fig. 5), while without this stabilization (Fig. 6) this ratio exhibits drift, which depends of the frequency drift of the used source. These information are of interest for the $C_N \rightarrow R_X$ comparison. The resistance of the used 100 M Ω standard (P100M) change in time for $\approx 0,067$ ppm/day, which is significant drift, but nevertheless this standard is very convenient for the comparison with the 100 pF capacitance standard. The used DVs enable very accurate determination of the low-frequency voltage ratio. The further investigations are in progress.

References

- [1] D. Ilić and M. Krešić: "Maintenance of the reference resistance standards of the Primary Electromagnetic Laboratory in Croatia", *Proc. of the 13th IMEKO TC4 Symposium* (paper D6), Athens, 2004.
- [2] R. Malarić and I. Leniček, "Resistance measurement capabilities of the Primary Electromagnetic Laboratory", *Proc. of the 12th IMEKO TC-4 Symposium, Part 1*, pp. 96–98, Zagreb, 2002.
- [3] V. Bego, J. Butorac and K. Poljančić, "Voltage balance for replacing the Kilogram", *IEEE Trans. Instrum. Meas.*, vol. 44, no. 2, pp. 579–582, April 1995.
- [4] J. Butorac and I. Kunšt: "Achievement of 0,1 ppm accuracy with the resistance standard of 100 M Ω " (in Croatian), *Proc. of the JUKEM '90*, Part 1, pp. 127–133, Sarajevo, 1990.
- [5] V. Bego, J. Butorac and D. Hrsan, "Calibration of the standard resistors by means of the standard capacitors and precise digital voltmeters", *CPEM '96 Digest*, pp. 392–393, Braunschweig, 1996.
- [6] D. Ilić, "Use of digital measuring instruments in maintenance of standards" (in Croatian), Ph.D. dissertation, pp. 219, FER, Zagreb, 1999.
- [7] *** Instruction manuals for: General Radio 1404-B Capacitance Standard, Fluke 742A Series Resistance Standards, HP 3458A Multimeter, HP 3245A Universal Source, and Fluke 5200A AC Calibrator.
- [8] V. Bego, K. Poljančić and D. Hrsan, "Precise measurement of power and RMS values of AC voltages and currents with digital multimeters", *Proc. of the 7th IMEKO TC-4 Symposium*, pp. 639–644, Prague, 1995.
- [9] D. Ilić and J. Butorac, "Use of precise digital voltmeters for phase measurements", *IEEE Trans. Instrum. Meas.*, vol. 50, no. 2, pp. 449–452, April 2001.
- [10] D. Ilić and J. Butorac, "Measurement of AC voltages with digital voltmeters", *Proc. of the XVI IMEKO World Congress*, vol. X, pp. 185–190, Vienna, 2000.
- [11] U. Pogliano, "High precision measurement of electrical power by means of synchronisation of integrative analog to digital converters", *Proc. of the 8th IMEKO TC-4 Symposium*, pp. 33-36, Budapest, 1996.

¹ Brand names are used for the purpose of identification. Such use does not imply endorsement by authors nor assume that the equipment is the best available.