Calibration of 100 MΩ Hamon resistor using current-sensing Wheatstone bridge

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Abstract **-** Primary electromagnetic laboratory in Zagreb maintains standards of four major electromagnetic units: volt, farad, ohm and second. Reference standards are periodically calibrated in PTB (Germany) and several methods has been realized for internal laboratory traceability check. *RC*-comparison method uses Hamon divider constructed of 100 equally balanced 1-megaohm resistors that connected in parallel or serially exhibit Hamon ratio of 100 MΩ : 10 kΩ with relative uncertainty of 10^{-8} [1]. Hamon resistance of 100 MΩ is directly compared with impedance of 100 pF capacitance reference at frequency of 15,873 Hz. Measurement of low-frequency voltage ratio is based on two digital voltmeters HP 3458A with improved algorithm for data processing [2]. Accuracy of reference standards comparison 10 kΩ →100 pF depends on consistency of Hamon divider that can be disturbed by aging of used metal-film components. In this paper a recently realized method for accurate measurement of 100 MΩ Hamon resistor by means of comparison with calibrated and stable 10 MΩ resistance standard at higher voltage levels has been presented.

I. Introduction

Method of *RC*-comparison begins with determination of 1:1 ratio between Hamon divider connected in parallel and 10 kΩ reference standard that has been previously calibrated at PTB. This comparison is based on voltage ratio measurement using two digital voltmeters (DVs) positioned across each of 10 kΩ resistor and one voltage source of 20 V, hence each resistor has voltage drop of 10 V. Making two voltage measurements by interchanging voltemeter's positions, instability of voltage source and systematic errors of the DVs have no influence on resistance ratio uncertainty. The resistors of Hamon divider are balanced within 10^{-4}, therefore the error of the $1:100^2$ Hamon ratio should not exceed 10^{-8}. However, aging of metal-film resistors during 12 years of Hamon resistor exploatation could significantly disturb Hamon ratio. This possibility launched development of modified Wheatstone bridge which can be used for comparison of Hamon divider, now in 100 MΩ mode, with another reference standard, such as 10 MΩ.

II. DVM-based system for high-ohm resistance comparison

Currently used method for high-ohm resistance comparison by means of two digital voltmeters and electrometer interface (E) is shown in Figure 1. Influence of input resistance instability of digital voltmeters has been lowered with much higher input resistance of electrometer interface [3,4]. Therefore voltage measurement on two resistors' positions gives exact ratio R_1/R_2 without influence of DV2's input resistance paralleling effect. Entire measuring system is carefully shielded and uncertainty of ratio is practically caused only by nonlinearity of DV2 on 10 V measuring range, what can be find out separately. The best results are obtained for 1:1 resistance ratio when uncertainty of the method is only influenced by short-term instability of DV2's readings. The estimated uncertainty for the ratio 100 MΩ : 10 MΩ is less than $0{,}8\cdot10^{-6}$ (coverage factor $k = 1$). Limitation of this method lies in maximal permitted input voltage of the electrometer interface of 10 V, what is the reason why comparison that incorporates interchanging positions of the resistors couldn’t be done at 100 V. Since *RC*-comparison is performing on measuring voltage of 100 V (calibrating voltage for capacitance standard), there is possibility of influence of metal-film voltage coefficient on Hamon divider accuracy. Therefore a new method for resistance comparison at higher voltage levels has been investigated.

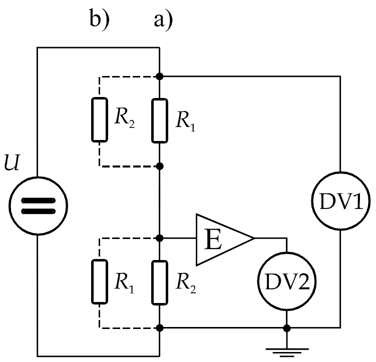

Figure 1: Measuring system for high-ohm resistance comparison uses two digital voltmeters and electrometer interface.

III. Modified Wheatstone bridge-based system for high-ohm resistance comparison

Modified Wheatstone bridge shown in Figure 2. consists of two voltage sources and current-sensing detector. The use of effective low-impedance current-sensing detector instead of high-impedance electrometer null-detector insures that the only one sensitive point of bridge (A) is continuously held at potential near ground (virtual zero), even when the bridge is unbalanced, thus eliminating parasitic currents paths from point "A" towards ground.

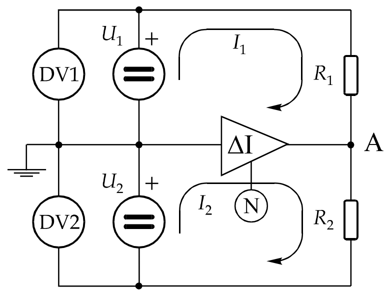

Figure 2. Modified Wheatstone bridge realized with current-sensing detector
as an indicator of bridge balance.

A. Realization of current-sensing detector

The current-sensing detector used in this method is transimpedance amplifier that converts current difference $\Delta I = I_1 - I_2$ into voltage measured by sensitive null-detector (N), such as analog null-detector FLUKE 745AB or programmable digital voltmeter HP 3458A. The principle of operation is shown in Figure 3. The non-inverting input of operational amplifier (OPamp) is connected to the ground, keeping as well the inverting input at potential of "virtual zero". The difference of currents I_1 and I_2 that flow through compared resistors R_1 and R_2 respectively is converted to output voltage by factor R_F V/A. The feedback resistor R_F defines sensitivity of the detector, depending on values of compared resistors, applied voltages and Johnson noise of R_F itself.

The operational amplifier used in this design has been carefully chosen between commercially available OPamps having ultra-low input offset current and low input offset voltage with low temperature coefficient. This parameters has been measured on few OPamps of AD 549K type (Analog Devices) and the chosen one exhibited approximately 30 fA of input offset current and less than 10 µV of input offset voltage. The feedback resistor used for 100 MΩ : 10 MΩ comparison is epoxy encapsulated cermet type 1 GΩ (Meggit HB). Since the measured voltage $\Delta I \cdot R_F$ is considered to be an error of resistors ratio, the true value of R_F, as well as its temperature stability, are not of great significance. The capacitor C_F is added as a bypass capacitor for reduction of 50 Hz noise. With value of 1 nF, time constant of the detector is 1 second, therefore 50 Hz noise is reduced more than two orderds of quantity. This simple detector design also has a few components added for automation of measuring process. One is relay Re1 that bypasses feedback resistor during voltage sources activation.

After the voltage sources were set to approximate value needed for bridge balance and enabled by computer, the relay Re1 is opened and the voltage on detector output is being measured. On that way a current shock through R_F caused by nonsynchronous voltage applying is avoided, decreasing also the setting time of the detector. Second are relays Re2 and Re3 that are used for shorting resistors R_1 and R_2 in step of "zero of the bridge" determination. After the balance of the bridge was adjusted, relays Re4 and Re5 are interchanging positions of the voltmeters DV1 and DV2 (digital voltmeters HP 3458A) during voltage ratio measurement in order to eliminate their systematic errors.

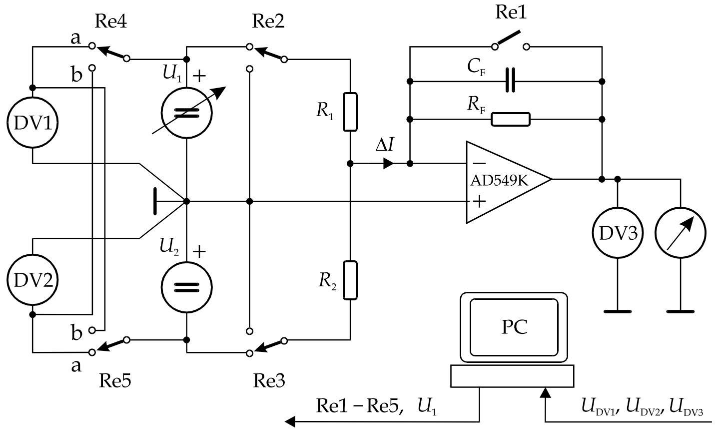

Figure 3. Functional schematic of modified Wheatstone bridge for high-ohm resistance comparison.

B. Principle of measuring procedure

After the balance of the bridge was achieved, the ratio of compared resistors is determined by means of voltage ratio using simple following equation:

$$\frac{R_1}{R_2} = \frac{U_{1a}}{U_{2a}} \cdot \frac{U_{2b}}{U_{1b}}, \tag{1}$$

where indexes "1" and "2" designate the voltmeters DV1 and DV2, and indexes "a" and "b" their positions. Procedure of balancing modified Wheatstone bridge has two steps. For the period of first step resistors R_1 and R_2 are connected to the ground and output voltage U_0 of the detector is measured with DV3. This voltage is a consequence of input offset voltage of detector's amplifier and its input offset current flowing through the feedback resistor:

$$U_0 = U_{of} \cdot \left(1 + \frac{R_F}{R_1 \| R_2}\right) - R_F \cdot I_{of} . \tag{2}$$

Assuming that offset parameters of OPamp are constant, U_0 therefore represents "zero" that has to be established by bridge balancing. In second measuring step, when voltage sources are enabled, PC establishes feedback loop of bridge balancing with programmable voltage calibrator FLUKE 5700A controlled over IEEE-488 bus. The bridge is considered "in balance" when readings of DV3 are not showing deviation from U_0 more than previously set margins of error. Since 100 MΩ Hamon resistor has to be measured at voltage of 100 V, that will produce current flow in upper branch of bridge of 1 μA. The balance of bridge will be achieved with 10 V across reference standard of FLUKE 742A - 10 MΩ in lower branch, and for that purpose FLUKE 731B voltage standard is used. If relative current difference of 10^{-6} (1 pA) is sensed, the detector will provide output voltage of 10^9 V/A· 1pA = 1 mV.

IV. Results

The measurement setup for the first step of balancing procedure includes 10 subsequent readings with DV3 (HP 3458A) to gain needed "zero" of the bridge. It was found to be $U_0 = -0{,}51$ mV with standard deviation of $s_{U0} = \pm 70$ μV. This "zero" measurement was repeated several times during two weeks and no significant changes were observed. Prior to the start of the balancing procedure the margins of bridge balance were set on $U_m = \pm 0{,}1$ mV around measured U_0. In second measuring step, after initial voltages had been enabled, the mean value of 10 readings of DV3 was taken as a measure of bridge unbalance. That "error" voltage has been used for correction of calibrator voltage via PC and balancing procedure was repeated. The balance was found for such voltage of calibrator that puts mean value and associated standard deviation of 10 subsequent readings of detector “error voltage” within calculated margins ($U_0 \pm U_m$). That balancing algorithm is a result of number of bridge testing on sensitivity, influence of parasitic impedances and balancing speed, and is found to be satisfactory with regard to reliable zero finding. Table 1. shows results of 100 MΩ : 10 MΩ comparisons made in the same laboratory conditions within twelve days.

Table 1: Results of comparison of 100 MΩ Hamon divider with 10 MΩ reference resistor.

Day	Number of measurements	Measured ratio	Standard deviation / 10^{-6}	Value of 100 MΩ Hamon resistor / MΩ
2004-02-16	10	10,1074430	0,19	101,078928
2004-02-19	10	10,1074434	0,15	101,078932
2004-02-22	10	10,1074449	0,11	101,078947
2004-02-25	10	10,1074459	0,13	101,078957
2004-02-28	10	10,1074477	0,13	101,078975
Reference resistance: R_1 = 10,000445 MΩ				

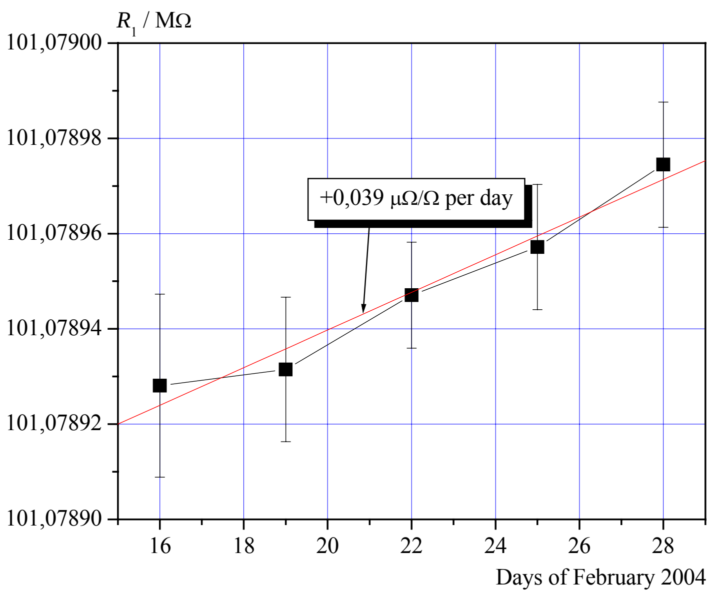

Figure 4: Graphic analysis of 100 MΩ : 10 MΩ comparisons during 12 days of measurement. Error bars include only uncertainty of estimated resistance ratio.

Assuming that 10 MΩ reference resistor is stable over measuring time, the calculated values for Hamon resistor given in Figure 4. are showing positive time drift, what is an expected behaviour thanks to series of *RC*-comparisons done in 1994 [1] and 1999. It must be emphasized that the exact value of Hamon resistor also depends on accuracy of 10 MΩ standard. Its value can be established with uncertainty of $\pm 0{,}2 \cdot 10^{-6}$ [4] using chain resistance comparisons down to 10 kΩ primary resistance standard.

V. Uncertainty calculation for ratio measurement

The uncertainty budget for the realized modified Wheatstone bridge includes several components described in Table 2. The combined relative standard uncertainty (k = 1) for 100 MΩ : 10 MΩ ratio was calculated $u = 0{,}19 \cdot 10^{-6}$. The first line in Table 2. is contribution due to mean standard deviation of five measuring results stated in Table 1. Other contributions are consequences of limitations of used components and measuring instruments.

Table 2: Uncertainty budget for 100 MΩ : 10 MΩ comparison on 100 V using realized current-sensing detector and modified Wheatstone bridge (*k* = 1).

Quantity	Relative standard uncertainty / 10^{-6}	Probability distribution	Uncertainty contribution / 10^{-6}
Deviation of measuring results (type A)	0,14	normal	0,14
Nonlinearity of voltmeters on 100 V range	0,1	rectangular	0,06
Margins of bridge balance	0,1	rectangular	0,06
Stability of "zero" value including max. temperature coefficient of OPamp offset voltage	0,08	normal	0,08
Noise of feedback resistor	< 0,01	normal	0,01
Resolution of voltage calibrator	0,1	rectangular	0,06

VI. Conclusion

The described modified Wheatstone bridge with current-sensing detector is capable of measuring resistance ratios at voltage levels up to 1000 V. The sensitivity of detector can be simply altered by changing the value of feedback resistor. First measurements described in this paper confirm excellent repeatability of 100 MΩ : 10 MΩ ratio at 100 V with extended uncertainty (*k* = 2) better than $5 \cdot 10^{-7}$. This is very good result having in mind simplicity and inexpensiveness of current-sensing detector realization.

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