Frequency performance of Hamon transfer standards

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***Abstract*-** Shielded and temperature controlled 1 000 Ω / 10 Ω and 10 000 Ω / 100 Ω transfer standards have been realized to be used in the frequency range up to 5 kHz. Their frequency performances have been evaluated by comparison with calculable resistors of quadrifilar and octofilar design. Thanks to the known frequency performances, both standards can serve as reference devices for precision AC measurements of resistance ratios made by means of commercial four terminal-pair bridges.

Introduction

The advent of quantum standards of voltage and resistance has increased the importance of highly accurate resistance-ratio devices, and especially Hamon transfer standards [1] proved to be useful tools for precision measurements of voltage and resistance ratios. These standards consist of n resistors of the same nominal value R permanently connected in series, additional potential and current terminals being connected to their junctions to make four-terminal measurement of the contribution of each resistor to the total series resistance possible (Fig. 1). In case that these contributions are

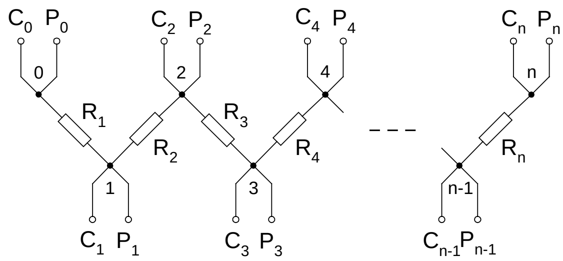

Figure 1. Resistors in series

$$R_i = R\left(1+\delta_i\right), \quad i = 1, 2, ..., n,$$

the total series resistance is

$$R_s = n\,R\left(1+\frac{1}{n}\sum_{i=1}^{n}\delta_i\right)$$

Parallel connection of the same resistors can be obtained by adding four terminal fans, as in Fig. 2.

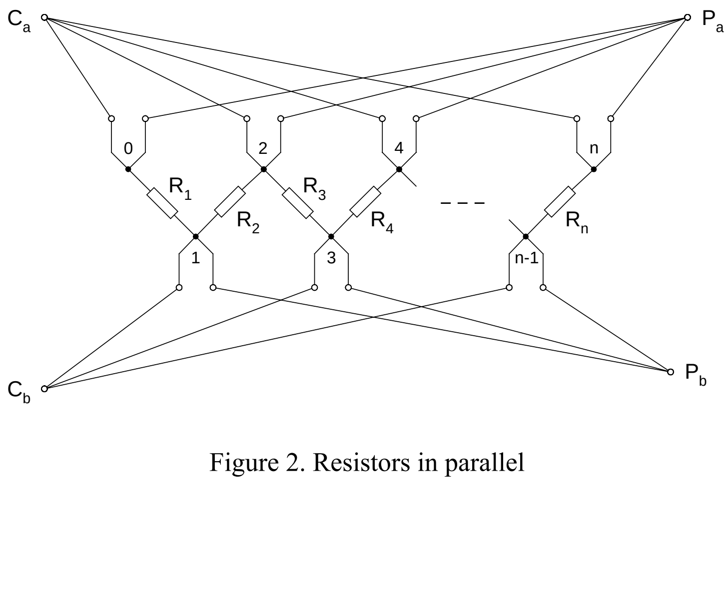

Figure 2. Resistors in parallel

It can be shown [1-3] that fan-conductor resistances do not contribute to the four-terminal resistance of the parallel connection in case that the following is valid for the fans interconnecting the current terminals:

a) Resistances of the end arms of the fan connected to C_a (i.e. of the arms $C_a - 0$ and $C_a - n$) are of the same value ${}^{c}r_a$. Resistances of all remaining arms of this fan are ${}^{c}r_a/2$.
b) Resistances of all arms of the fan connected to C_b have the same resistance ${}^{c}r_b$, which can be different from both ${}^{c}r_a$ and ${}^{c}r_a/2$.

Independence of R_p on the fan-conductor resistances is also ensured if the following is valid for the fans interconnecting the potential terminals:

c) Resistances of the end arms of the fan connected to P_a (i.e. of the arms $P_a - 0$ and $P_a - n$) are of the same value ${}^{p}r_a$. Resistances of all remaining arms of this fan are ${}^{p}r_a/2$.
d) Resistances of all arms of the fan connected to P_b have the same resistance ${}^{p}r_b$, which can be different from both ${}^{p}r_a$ and ${}^{p}r_a/2$.

In case that the conditions a) and b) or c) and d) are fulfilled, resistance of the paralleled resistors can be expressed as

$$R_p = \frac{R}{n}\left(1 + \frac{1}{n}\sum_{i=1}^{n}\delta_i + \text{additional terms containing second and higher powers of } \delta_i\right)$$

where the additional terms become insignificant for small deviations δ_i . In that case,

$$R_s / R_p = n^2$$

to a high degree of accuracy. For example, the ratio R_s / R_p differs from n^2 by less than 1 part in 10^8 when $|\delta_i| < 1.10^{-4}$.

Hamon standards for AC applications

Shielded and temperature controlled 1 000 Ω / 10 Ω and 10 000 Ω / 100 Ω transfer standards have been realized to be used in the frequency range up to 5 kHz. In these standards, ten 100 Ω or ten 1 000 Ω metal foil resistors with tolerances less than ± 0.005 % are permanently connected in series by means of four-terminal copper junctions shaped like equilateral triangular slabs. Two coaxial connectors are connected to each slab, one of them to its centre, the other to one of its vertices. The series connection of the resistors can be converted to a parallel one by means of two shielded shorting bars and two shielded compensation networks (Fig. 3, where r = 1 Ω). By application of the compensation networks fulfilment of the above conditions c) and d) is ensured.

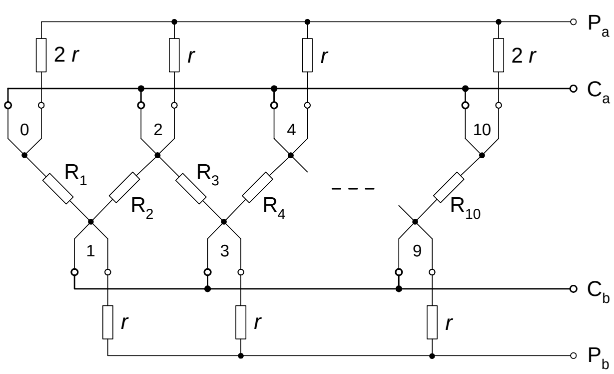

Figure 3. Hamon standard equipped with two shorting bars and two compensation networks

Evaluation of frequency dependences of ratios R_s / R_p

For both transfer standards, frequency dependence of the ratio R_s / R_p has been deduced from results of separate measurements of frequency dependences of resistances R_s and R_p. These have been measured by 1:1 comparisons with frequency dependences of suitable calculable resistors by means of a coaxial four terminal-pair R-R bridge [4]. An 8-decade two-stage inductive voltage divider forms ratio arms of this bridge and special circuitry ensures zero currents in the high potential leads of resistors to be compared. An adjustable combining network makes the main bridge detector insensitive to unwanted potential difference between the low potential terminals of these resistors.

Calculable resistors are resistors constructed in such a way that the frequency dependences of their values can be calculated, with a sufficient accuracy, from a knowledge of their constructional parameters. In calculating the frequency performances of these resistors changes in resistance due to parasitic inductances and capacitances, as well as changes due to eddy currents induced both in the resistive elements and in the surrounding shields have to be evaluated. The effects of parasitic inductances and capacitances are calculated from uniform-transmission-line models.

Calculable resistors of quadrifilar and octofilar design [5-7] have been used as reference standards in the above comparisons. In Table 1 the calculated AC-DC differences of these resistors are given for a frequency of 5 kHz (AC-DC difference is the relative change of the parallel equivalent resistance from its DC value).

Table 1. AC-DC differences of calculable resistors used in evaluating frequency dependences of ratios R_s / R_p

Nominal value [Ω]	AC-DC difference for f = 5 kHz	Note
10	1.8×10^{-6}	quadrifilar design
100	2.9×10^{-8}	quadrifilar design
1 000	6.1×10^{-10}	quadrifilar design
10 000	1.7×10^{-8}	octofilar design

Results and conclusion

In Figs. 4 and 5 the measured changes in ratios R_s / R_p of both standards are plotted as functions of frequency.

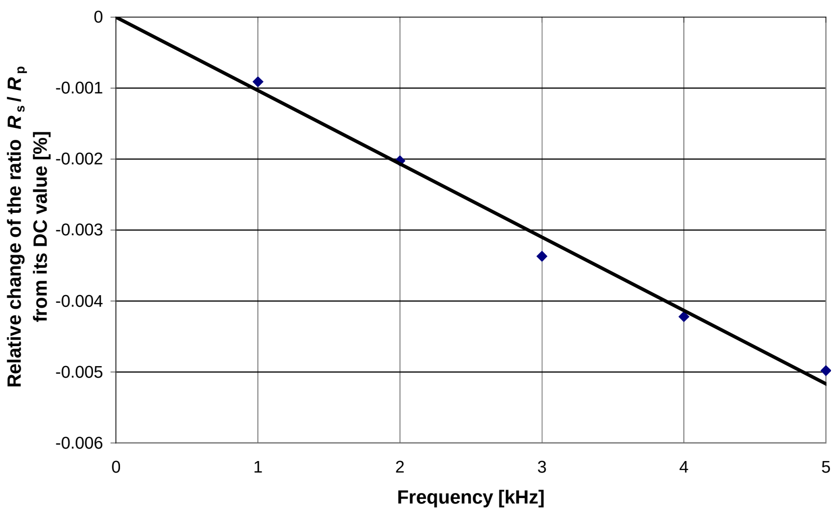

Figure 4. Frequency performance of the 1 000 Ω / 10 Ω standard

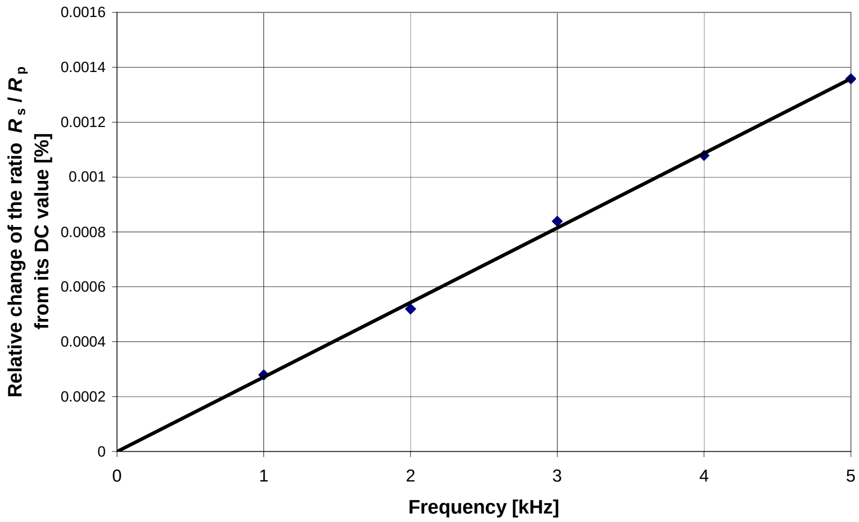

Figure 5. Frequency performance of the 10 000 Ω / 100 Ω standard

In the frequency range up to 5 kHz, both the 1 000 Ω / 10 Ω and the 10 000 Ω / 100 Ω standard show linear frequency dependence of the ratio R_s / R_p , the respective frequency coefficients being -1.0×10^{-3} % / kHz and 2.7×10^{-4} % / kHz. Thanks to the known frequency performance, the above standards can be used as reference devices for precision AC measurements of resistance ratios made by commercial four terminal-pair bridges.

References

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