The Goertzel filter-bank usage in the non-stationary impedance measurement

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Abstract-The paper presents the method of measurement of non-stationary impedance parameters based on measurement signals sampling and DSP. The Goertzel filter-bank approach has been proposed and analysed. The proposed method has been compared to traditional STDFT approach. The results of simulations and experiments have been presented.

I. Introduction

In many cases of impedance measurements, it is assumed that the object under measurement is stationary or quasi-stationary. It means that the impedance characteristic does not change during measurement series and impedance spectrum of the object can be achieved point-by-point at specified frequencies. When the tested object is expected to be dynamically changing [1] the traditional impedance measurement technique cannot be used. To acquire the time-varying impedance spectra, it is necessary to employ the joint time-frequency analysis (JTFA) methods [2]. Unfortunately, methods used widely in signal energy spectrum estimation, are unusable for impedance measurements, due to the phase information loss. There are some proposals [3] of the usage of Short Time Discrete Fourier Transformation (STDFT) to extract information of the impedance spectra. To circumvent disadvantages of STDFT, a new method of obtaining time frequency spectra by means of filter banks has been proposed.

II. STDFT approach to non-stationary impedance measurement

The application of DFT in single frequency static impedance measurements has been discussed in [4]. Multifrequency impedance measurements have been presented in [5]. As STDFT inherits the properties of DFT, it forms the basis for non-stationary impedance measurements in time-frequency domain. STDFT approach is based on applying multifrequency perturbation signal, coherent sampling of voltage and current signals, and then, calculating their transforms. Complex division of voltage and current 2D time-frequency spectra (point-by-point) produces impedance time-frequency spectra for every time sample [n] and every frequency. STDFT is defined by (1).

$$X[n,k] = \sum_{m=0}^{L-1} x[n+m] \cdot w[m] \cdot e^{-j\frac{2\pi}{L}km} \tag{1}$$

It can be calculated by so-called "sliding window" method, based on calculating L-point DFT for block of samples cut from record of data *x[n]* by *L*-length window w[m]. STDFT calculation produces a regular grid of results: every *L*-length DFT produces time-localised complex values for *L/2* positive frequency components of signal *x[n]*.

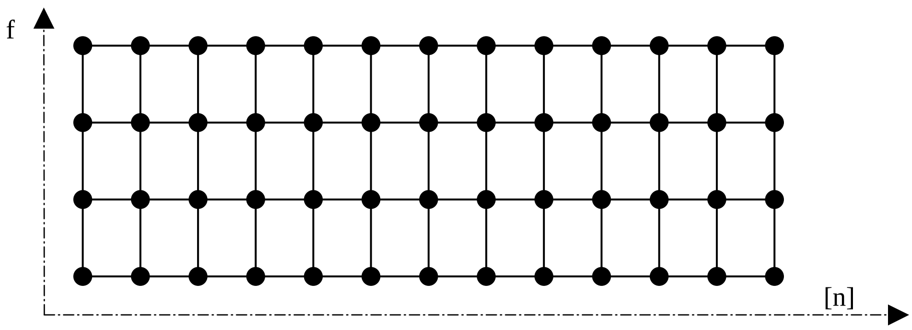

Figure 1. STDFT result grid

The length of *L* and shape of window *w[m]* influences on

STDFT resolution in time and frequency domain, due to uncertainty principle – one can not achieve perfect resolution in both domains.
In theoretical approach, a DFT is calculated for every *L*-length block, with value of *n* increased by 1. However, with record length of several thousands of samples, and with large *L*-length (non-stationary impedance measurement) that solution would be rather redundant. It requires calculating DFT several thousand times, while the values of *X[n,k]* and *X[n+1,k]* can not differ significantly – averaging effect of L-length DFT. A popular solution is to use non-overlapping or 50% overlapping blocks. Using non-overlapping segments is equal to block-by-block DFT processing a record of data. If we take into consideration only positive frequencies, the size of results grid can be approximated as in (2):

$$\frac{N_{STDFT}}{L_{STDFT}} \times \frac{L_{STDFT}}{2} \tag{2}$$

Although very popular, STDFT has several disadvantages, especially in case of perturbation signal with components from several decades. The sampling frequency must fulfil the Nyquist criterion for the highest frequency component present in perturbation signal – that leads to very high number of samples acquired. STDFT window length *L* is same for all frequency components, and must be greater than period of lowest frequency component of perturbation signal. If the signal contains components from several decades, this unnecessary limits time resolution for higher frequencies. A single *L*-length block contains few periods of low frequency sinusoids, but thousands of periods of high frequency sinusoids. Usually low frequency components of multifrequency perturbation signal are calculated with too low selectivity in frequency domain. On the other hand high frequency components are unnecessary selective in frequency domain (every block result contains thousands of high frequency periods), but they are poor localised in time domain. Moreover, to avoid leakage effects of DFT, *L* has to be the least common multiple of periods of all frequencies. In worst case scenario, *L* would be a multiplication of periods of all *K* perturbation components, normalised by sampling frequency:

$$L = \prod_{k=1}^{K} \hat{T}_k = \prod_{k=1}^{K} \frac{1}{\hat{F}_k} \tag{3}$$

As this condition is very hard to fulfil, leakage effect limits spectral selectivity of transformation, bringing necessity of using advanced spectral estimation techniques. Another disadvantage of using STDFT in non-stationary impedance measurements is its frequency spectra redundance. To calculate time-varying impedance at certain frequencies (present in perturbation signal), one is interested in frequency bins only for these frequencies, but DFT calculates *L* spectral bins.

III. The filter-bank approach

To circumvent disadvantages of STDFT, a new method of obtaining time varying frequency spectra has been proposed, by means of filter banks. Both DFT and STDFT transformations can be realized by *L*-channel filter bank representation. Filter banks implement in digital signal processing domain the idea of classic analog analyser with synchronous amplitude detection. Filter bank consists of several parallel structures, each of them with modulator and low-pass filter. Modulator shifts the spectrum in frequency domain according to modulation principle:

$$\begin{gathered} x(t) \Leftrightarrow X(j\omega) \\ x(t) \cdot e^{j\omega_s t} \Leftrightarrow X(j(\omega - \omega_s)) \end{gathered} \tag{4}$$

After shift, the bin corresponding to complex value of desired frequency is placed at zero frequency, and its value can be extracted from modulator output by simple averaging or low-pass filtering.

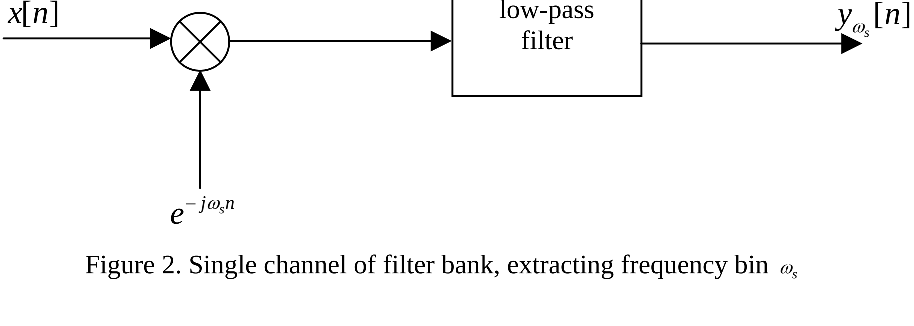

Figure 2. Single channel of filter bank, extracting frequency bin ω_s

If we denote *K*-component perturbation signal as:

$$x[n] = X_0 + \sum_{k=1}^{K} (X_k e^{j\omega_k n} + X_k^* e^{-j\omega_k n}) \tag{5}$$

The signal after modulation by ω_s in *s*-th frequency filter bank is:

$$x_{\omega_s}[n] = x[n] \cdot e^{j\omega_s n} = X_0 \cdot e^{-j\omega_s n} + \boxed{\sum_{k=1}^{N} (X_k e^{j(\omega_k - \omega_s)n}} + X_k^* e^{-j(\omega_k + \omega_s)n}) \tag{6}$$

As it can be seen, if *k*=*s*, the DC value at output of filter bank channel, extracted by averaging is the amplitude of component with frequency f_S. If one creates *L*-channel filter bank, with all filters in all channel with same length *L*, such filter bank will calculate DFT of signal *x[n]*. If we assume, that averaging filter is a simple summator and ignore the amplification of signal, the output of single channel *s* can be written as:

$$X_s = \sum_{n=0}^{L-1} x[n] e^{\frac{-j2\pi s n}{L}} \tag{7}$$

and is equal to definition of the *s*-th bin of *L*-point DFT. The equality of DFT and filter bank approaches has been proved in [6]. The filter bank realisation of STDFT for non-stationary signals has been presented in [7].

Although we can calculate STDFT with filter bank, there is no need to do so, as the STDFT approach to non-stationary multisine impedance measurements is disadvantageous. The filter bank realisation is much more flexible. If the object under measurement is linear, then according to the superposition principle, an response to multifrequency perturbation should be similar to sum of responses to every component – there should be no cross-talks to other frequency bins of STDFT. If we assume the linearity of object, we can only evaluate several spectral bins, instead of L. Moreover, the length of filter *L* need not be the same in every channel, thus allowing to reduce leakage effects without the demand for *L* being the least common multiplier of periods of all multisine components. A possibility to vary length of filter in every channel independently allows optimising time-frequency selectivity for every channel. High frequency time selectivity is no longer limited by lowest frequency period.

Filter banks can be oriented for block-by-block calculation or sample-by-sample continuous computation. The first kind, which is to be presented in this paper, produces an irregular grid of results:

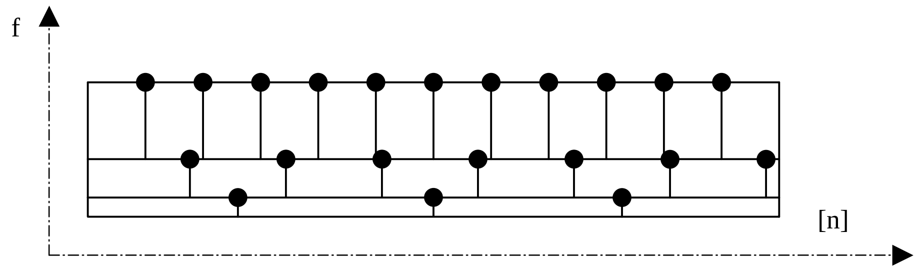

Figure 3. Block-oriented Goertzel filter bank grid of results

As a test engine for filter bank method, a well-known in literature Goertzel algorithm has been selected. Goertzel algorithm calculates the value of single DFT bin and can be treated as a modulator and 2nd order IIR filter or as a filter with harmonic contents in an impulse response. The Goertzel algorithm is a complete realisation of single filter bank channel and is parameterised by two values: block length and number of frequency bin extracted. Before further research, some parameters of algorithm have been tested. It has been noticed that filter selectivity depends on number of periods of extracted frequency present in one block of data processed by filter. It was decided, that length of every channel will be tied with discrete period (period normalised with sampling interval) of frequency component extracted at that channel by coefficient *periodcount*:

$$L = [L_1\ L_2\ L_3 \ldots L_k\ ] = pc \cdot [\hat{T}_1\ \hat{T}_2\ \hat{T}_3 \ldots \hat{T}_k\ ] = \frac{pc}{T_s} \cdot [T_1\ T_2\ T_3 \ldots T_k\ ] \tag{8}$$

In further experiments, the *pc* value varied from 5 to 25 for non-stationary simulations. It is worth noticing, that in stationary case, authors [5] averaged 64-256 DFT results in order to gain stable and credible results.

IV. Simulation and test results

STDFT and Goertzel Filter Bank methods of obtaining time-varying impedance spectrum were compared by simulation in Matlab environment. The same set of perturbation and object response signals has been processed by both STDFT and Goertzel Filter Bank (GFB) methods. Simulated voltage perturbation signal consisted of 4 components with different amplitudes and frequencies 200Hz, 2000Hz, 20000Hz and 100000Hz, sampled at 1MHz. The characteristic of an object was assumed as presented in fig. 4, with two amplitude steps and one 180º phase step in time domain. It does not represent any electrical circuit's behaviour – it was chosen to compare both algorithms at rapid changes of object's properties during measurement.

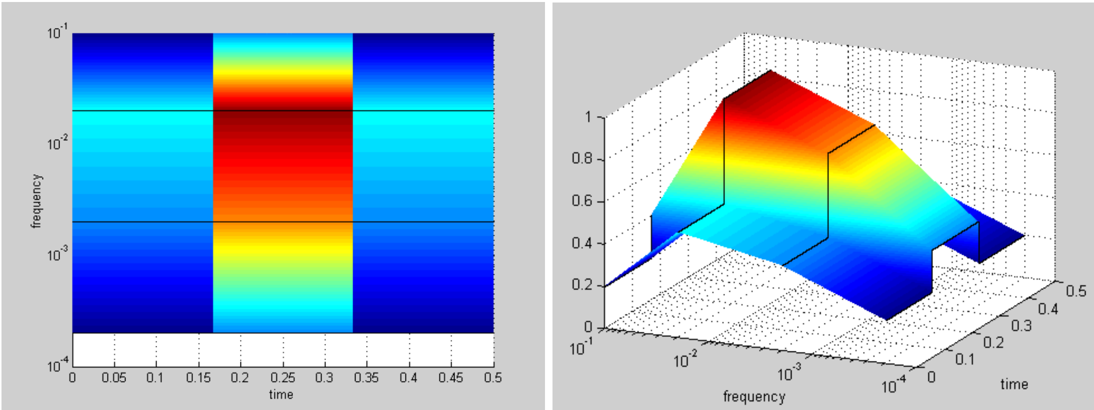

Figure 4. Assumed characteristic of object

As the changes of amplitude in time for every frequency component k of perturbation and response signal was assumed, these signals could be easy simulated:

$$u[n] = \sum_{k=1}^{K} U_k[n] \sin\left(2\pi \hat{f}_k n\right) \qquad i[n] = \sum_{k=1}^{K} I_k[n] \sin\left(2\pi \hat{f}_k n\right) \tag{9}$$

Perturbation and response signals spectra were calculated and divided point-by-point in order to calculate impedance spectra as in real DSP impedance measurements. These operations were conducted for non-overlapping STDFT with two different values of L, for 50% overlapping STDFT and with Goertzel Filter Bank approach. The value of L for STDFT has been chosen equal to Goertzel filter length for the lowest frequency component of perturbation signal, in order to provide similar low frequency selectivity.

$$L_{STDFT} = L_1 = pc \cdot \hat{T}_1 \tag{10}$$

As STDFT and GFB methods produce different grid of results (regular and irregular), they are incomparable directly with each other and with assumed (ideal) characteristics. To allow reliable comparison, the results were linearly interpolated to the same (regular) time-frequency grid. Interpolated results were the basis of further post processing. To visualise results of STDFT and GFB methods estimated spectra were plotted. In order to compare accuracy, the results were compared by means of especially created numerical error criteria. An averaged module of relative spectra estimation error was defined as:

$$\overline{\delta} = \frac{1}{KJ} \cdot \sum_{k=1}^{K} \sum_{j=1}^{J} \left| \frac{Z_{calculated[k,j]} - Z_{assumed[k,j]}}{Z_{assumed[k,j]}} \right| \cdot 100\% \tag{11}$$

where K is the number of components in perturbation signal, and J is the number of grid knots on time axis (number of block-calculated results, equal to data record length N divided by block length L).
Numerical comparison of methods is presented in table:

	GFB *pc=10*	STDFT $L=L_1$	STDFT $L=L_1$ overlapping 50%	STDFT $L=0.1L_1$
$\vert\delta\vert_{avg}$ [%]	**2.7134%**	**11.4587%**	**8.5077%**	**0.9366%**

GFB method had an average relative error three times smaller than STDFT (L_1) with same low

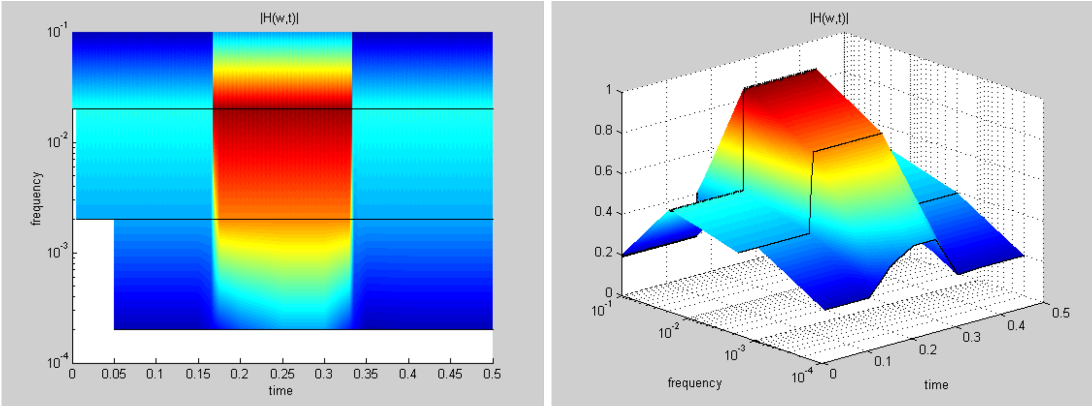

Figure 5. Spectra reconstructed by GFB

frequency resolution. Spectra reconstructed by GFB and STDFT are presented on Fig. 5 and Fig. 6. It can be seen that high frequency components are better localised in time domain with GFB than with STDFT, due to variable filter (block) length. Lowest frequency component is smothered with both methods – its rapid change has been averaged by long filter (block) length L or L_1. STDFT results show the disadvantage of STDFT non-overlapping transformation: same block length for all components. As a result, every signal component is poorly localised in time domain. Although poorer localisation in time domain should benefit with better frequency domain selectivity, that does not matter in case of described experiment, as GFB high frequency filter channels are already selective enough.

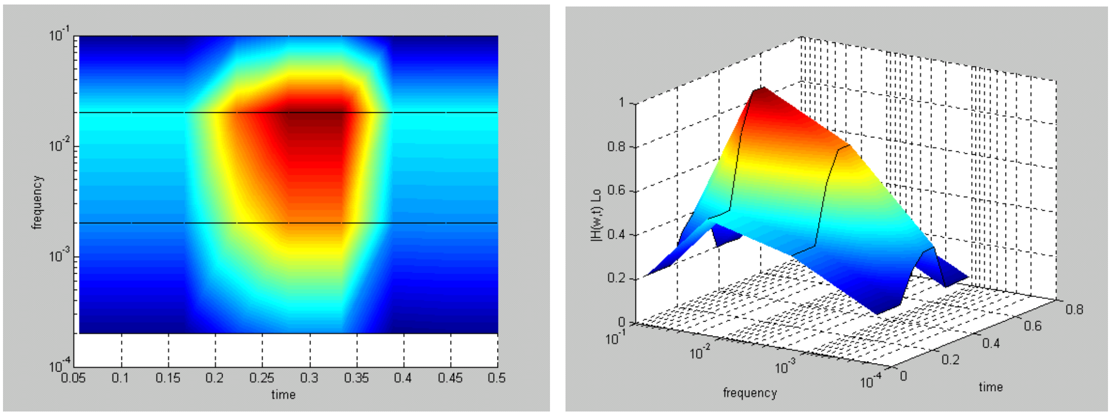

Figure 6. Spectra reconstructed by STDFT ($L=L_1$)

Applying SDFT method with 50% overlapping, gave a reduction of estimation error from ~11.45% to ~8.50%, at a cost of doubling numerical complexity. However, overlapping produced an artificial "hollow" in estimated spectra, at point where response phase has changed by 180º, presented at Fig. 7. That could lead to erroneous analysis of results.

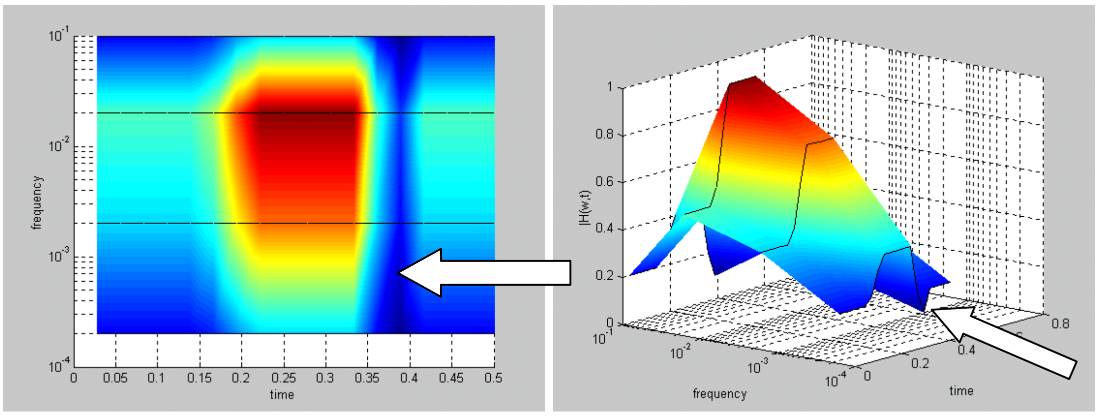

Figure 7. Spectra reconstructed by 50% overlapped STDFT ($L=L_1$)

Spectrum reconstruction with error comparable to GFB could be obtained by STDFT with *L* length reduced ten times, to $0.1L_1$ (Fig. 8). It is worth noticing, that this would lead to ten times reduced selectivity of STDFT transform for lowest frequency component of perturbation, as compared with previous experiments. That is a disadvantage, as in wide spectrum multifrequency measurements of dynamic objects, the selectivity of low frequency component is often critical to distinguish offset fluctuations from lowest component changes.

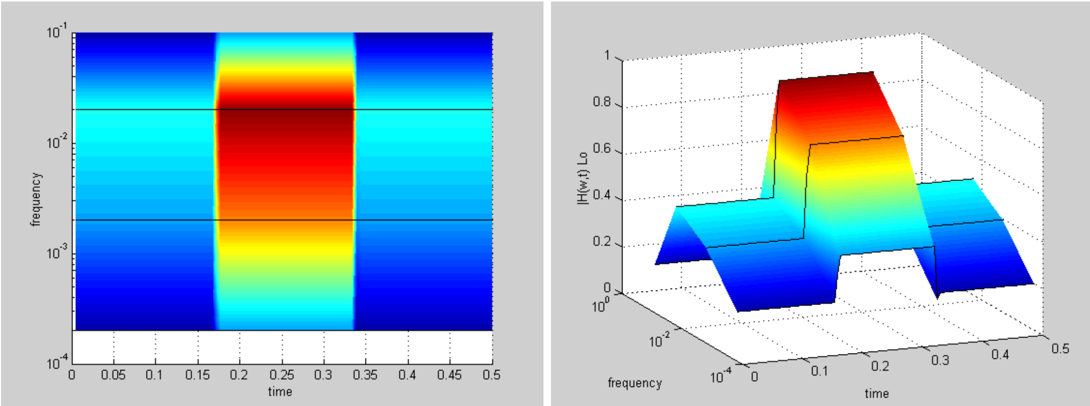

Figure 8. Spectra reconstructed by STDFT ($L=0.1L_1$)

V. Conclusions

During numerical simulation of calculating time-varying impedance spectra, the Goertzel Filter Bank approach has been proved to be superior to STDFT method, especially in case of few frequency components from different decades. Plots of spectra reconstructed by new method and results of error criterion show that Goertzel Filter Bank allows to gain better time localisation for higher frequency components, while maintaining the same frequency resolution for lowest frequency component. This variable time-frequency resolution property is similar to wavelet transformation. However, it is obtained while still operating in time and frequency domains with simple, physical meaning as opposed to wavelet transformation time scale. The new proposed Goertzel Filter Bank approach to non-stationary impedance measurement is oriented for experimentation with dynamic objects where both good frequency resolution for low frequency components and good time localisation for high frequency components is required.

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