The link between the Near-End and the Far-End Crosstalk Coupling Constants

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Abstract- A model is proposed to describe the local connection between the capacitive and the inductive unbalance functions of two neighbouring twisted pairs. A realistic simulation of the unbalance functions indicates that the constant ratio of the inductive unbalance with respect to the capacitive unbalance is a workable model. Using this model, it is demonstrated that the calculation of the reflective coupling constant for the calculation of reflective crosstalk is possible when one has disposal of the widely used coupling constants to predict Near-End and Far-End crosstalk in case the line ends are perfectly terminated. The reflective coupling constant is necessary to model the power spectral densities of the crosstalk signals when the line ends are not matched.

I. Introduction

Crosstalk is the dominating channel impairment for all realizations of Digital Subscriber Lines (xDSL). Two types of crosstalk functions are used in the telecommunication world, Near-End Crosstalk (NEXT) and Far-End Crosstalk (FEXT). Workable equations to describe the frequency behaviour of both crosstalk types have been derived by Campell [1] and Cravis and Crater [2].

The crosstalk models used in the telecommunication industry and recommended by the standards, such as ETSI, ITU and ANSI, assume that the lines are matched at the generator side and at the load side [3]. However, the lines of the access network of an operator do not fulfil this requirement. A subscriber line (loop) exists out a number of cascaded line sections. In most cases the line segment connected with the Central Office has the smallest wire diameter and when one moves towards the customer location thicker wires are used. Moreover, a variety of cable types exists which makes that the specifications, such as the characteristic impedance, vary even if the cables have the same wire diameters. These impedance irregularities cause reflections and the matched crosstalk model will not be valid anymore. Another phenomenon that causes reflections on transmission lines is the presence of one or more bridged taps in the loop. The junction of the in-line transmission line with the bridged tap causes strong reflections, which propagate back to the ends of the loop. Taken into account the above mentioned remarks, crosstalk caused by reflections will occur on an access network of a telecom operator.

In the past, models for predicting the crosstalk in case a bridged taps is present in the network were proposed. E.g. these models were used to study the performance of power back-off methods for ADSL and VDSL [4]. It has been experienced that the crosstalk predictions were not valid anymore when the tap is located too close to the Central Office. Recently, crosstalk models are used to study the impact of crosstalk on Multiple Input Multiple Output (MIMO) transmission over cable bundles. In case bridged taps exists, the existing crosstalk models differ too much from reality, because the used models contain many heuristic approximations.

Nowadays, it is investigated how the loop quality can be obtained from measurements performed at the Central Office only. A number of methodologies for Single-Ended Line Identification have been researched and tested out [5]. E.g. if one tries to estimate the data capacity, the noise level should be predicted at the customer side. Hereto, the noise is measured at the Central Office side and using crosstalk modelling and disturber identification, the noise power spectral density at the customer side will be estimated [6][7]. A better crosstalk model can improve the prediction of the far-end crosstalk noise level.

It is the intention to formulate better crosstalk models starting from the phenomena that are part of the foundations of crosstalk: the capacitive and inductive point-coupling between 2 lines. It will be demonstrated that the capacitive and inductive unbalance functions at a point location are not independent from each other. Therefore, a model will be presented to relate both unbalance functions with each other. Starting from this and using the expressions of [8] to calculate crosstalk when the line ends are not matched, will lead to a workable model to predict crosstalk between 2 lines. The crosstalk model (see [8]) will serve as a basic building block to compute crosstalk for complex loop topologies.

II. Crosstalk and assumptions

Consider two pairs of metallic wires where each line end of the pair is terminated with an impedance. A generator E with internal impedance Z_{dn} drives the disturber line at the near-end (see Figure 1).

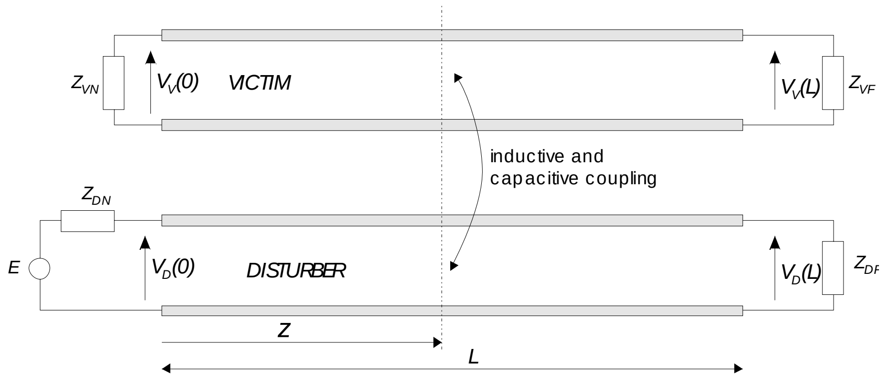

Figure 1. Two coupled transmission lines.

In practice, one is only interested in the crosstalk between pairs in the same cable bundle. So, all wire pairs have the same physical dimensions, except from the twist rate and small locally varying wire positions. In order to keep the analysis workable, the following assumptions are used:

- The victim and disturber lines are uniform and of the same type meaning that the propagation function $\gamma(\omega)$ and the characteristic impedance $Z_c(\omega)$ are equal for both lines.
- The propagation function and the characteristic impedance are independent of z .
- The electromagnetic coupling is weak.

Under these assumptions the power spectral densities (PSDs) of the crosstalk voltages $V_v(z=0,\omega)$ and $V_v(z=L,\omega)$ were elaborated in [8]. It was found that they depend on three crosstalk constants k_n , k_f and k_{nf} . Considering the rather voluminous expressions to describe the crosstalk PSDs, they will not be repeated in this contribution. The authors refer to [8].

III. Coupling between the unbalance functions

In order to detect the relation between the inductive unbalance function $M_u(z)$ and the capacitive unbalance function $C_u(z)$ at a distance z from the generator a simulation was carried out. Consider hereto two twisted pairs at height h above a perfect ground plane as shown in figure 2. Each pair consists of 2 insulated conductors that rotate around a common rotation axis if the distance z varies. Pair 1 has a wire twist period of T_1 and Pair 2 has a wire twist period of T_2 .

The following geometric parameters were used in the simulation example: $a = 0.5mm$, $t = 0.5mm$, $d = 3mm$, $h = 1m$, $T_1 = 5cm$ and $T_2 = 8cm$. The line length amounted to $L = 15cm$ and the number of simulation steps $N = 1001$. So, the angle increment for each simulation step is given by $d\phi = (L/T_1)2\pi/(N-1)$ and $d\varphi = (L/T_2)2\pi/(N-1)$.

The physical parameters are: the permittivity $\varepsilon_{insulator} = 2.26\, 10^{-9}/(36\pi)$ [F/m] for polyethylene, the permeability $\mu_{insulator} = \mu_{conductor} = 4\pi 10^{-7}$ [H/m], the conductivity $\sigma_{conductor} = 58.7\, 10^{6}$ [S/m] for copper and $\sigma_{insulator} = 10^{-12}$ [S/m].

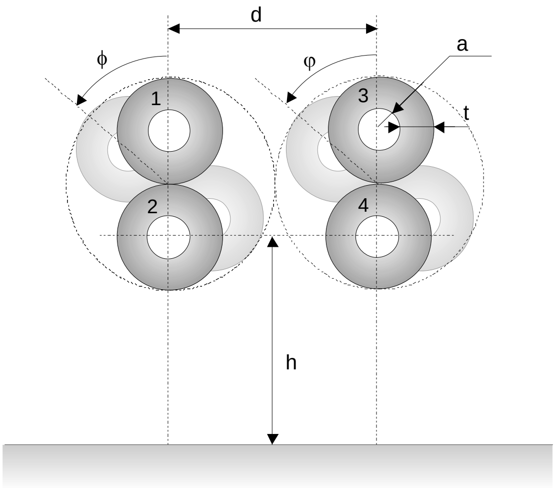

Figure 2. The cross-sectional geometry of 2 twisted pairs above a perfectly conducting plane.

The unbalance functions in case of a weak coupling between the wires can be calculated using the following equations (see [8] for their definition and [1] for their derivation):

$$C_u(z) = (C_{13} + C_{24}) - (C_{23} + C_{14}) \quad (1)$$

and the per-unit-length capacitance between wire i and wire j is denoted by C_{ij} (see figure 2 for the wire numbers). The inductive unbalance is given by:

$$M_u(z) = (M_{13} + M_{24}) - (M_{23} + M_{14}) \quad (2)$$

and the per-unit-length inductance between wire i and wire j is denoted by M_{ij} (see also figure 2).

No analytical expressions exist to calculate the wire capacitances when the wires are very close to each other. Therefore, the moment method was used to obtain the per-unit-length capacitance matrix $\boldsymbol{C}$ [9][10]. It was also assumed that the medium surrounding the conductors is a homogeneous dielectric. The per-unit-length inductance matrix $\boldsymbol{L}$ can be obtained by inverting the capacitance matrix in the following way: $\boldsymbol{L} = \varepsilon_{insulator}\mu_{insulator}\, \boldsymbol{C}^{-1}$.

The calculated values of the unbalance functions (1) and (2) are shown in figure 3a. It can be observed that there is a strong correlation between $M_u(z)$ and $C_u(z)$. Hence, the following coupling model between both unbalance functions is proposed:

$$M_u(z) = k \cdot C_u(z) \quad (3)$$

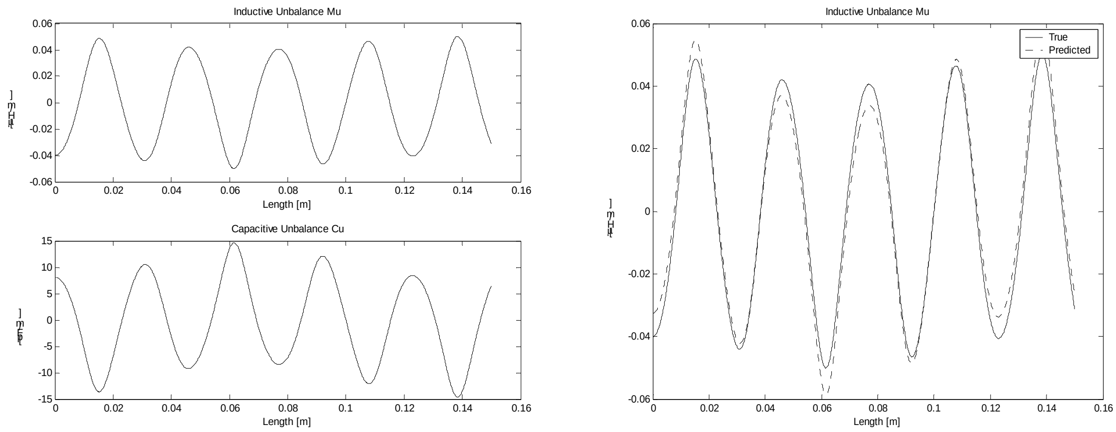

Figure 3. (a) The variation of the inductive and capacitive unbalance over the distance z. (b) The true and predicted inductive unbalance as a function of the distance z.

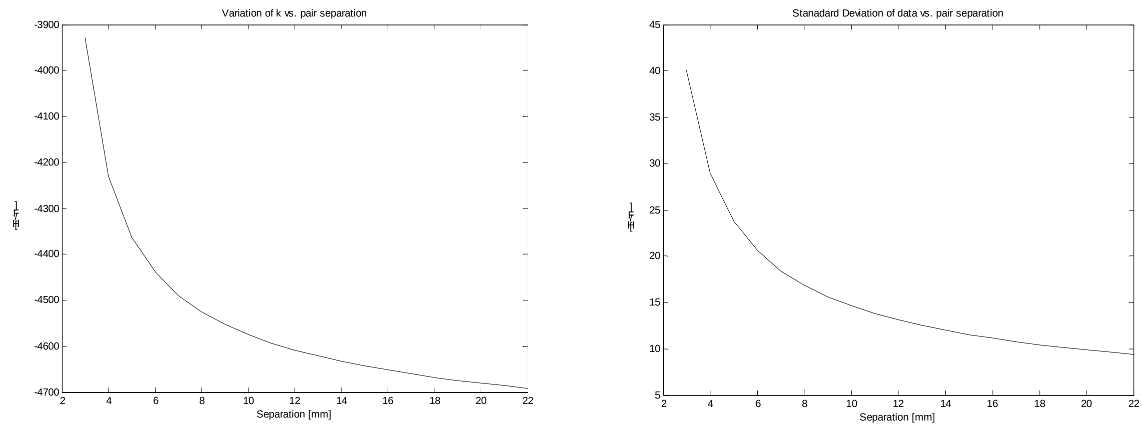

Figure 4. (a) The variation of the parameter *k* as a function of the pair separation *d*. (b) The standard deviation of the estimated parameter *k* as a function of the pair separation *d*.

The value of k was found using a least squares estimator with $\Delta z = L/(N-1)$: $k = \sum_{n=0}^{N-1} M_u(n\Delta z) C_u(n\Delta z) \Big/ \sum_{n=0}^{N-1} C_u^2(n\Delta z)$. For this specific simulation example $k = -3.9\,10^3$ [H/F]. The effectiveness of this simple model is shown in figure 3b. Two inductive unbalance curves are shown. Firstly, the inductive unbalance $M_u(z)$ calculated via the moment method (true) and the predicted one using (3) where $C_u(z)$ was calculated using the moment method. The predicted values approximate the true inductive unbalance values sufficiently. Even, the approximation error decreases further if the pair separation d increases. This effect was simulated and is shown in figure 4b. Another outcome of the simulation shows that the value of k decreases if the pair separation increases. This means when the pairs are more close to each other, the capacitive unbalance becomes more dominant. This effect is depicted in figure 4a.

Using the proposed model for the inductive unbalance $M_u(z) = k \cdot C_u(z)$ the unbalance functions for NEXT and FEXT can now be further elaborated.

The covariance of the unbalance function for NEXT

The unbalance function for NEXT is given by [8]:

$$C_n(z,\omega) = \frac{C_u(z)}{4} Z_c(\omega) + \frac{M_u(z)}{2Z_c(\omega)} \quad (4)$$

Using the coupling between the inductive and capacitive unbalance (substitute (3) in (4)), the covariance of the same unbalance function can be transformed into its well-known form:

$$E[C_n(z)C_n^*(y)] = \left\{ \frac{|Z_c|^2}{16} + \frac{k^2}{4|Z_c|^2} + \frac{k}{8}\left(\frac{Z_c}{Z_c^*} + \frac{Z_c^*}{Z_c}\right) \right\} E[C_u(z)C_u(y)]$$

Using the fact that $\frac{Z_c}{Z_c^*} + \frac{Z_c^*}{Z_c} \approx 2$ because Z_c is almost constant (e.g. 100Ω) and almost real valued (e.g. for frequencies above 25.875kHz which is the start frequency for an ADSL modem) then

$$E[C_n(z)C_n^*(y)] = \frac{1}{4}\left(\frac{|Z_c|}{2} + \frac{k}{|Z_c|} \right)^2 E[C_u(z)C_u(y)] = \frac{1}{4}\left(\frac{|Z_c|}{2} + \frac{k}{|Z_c|} \right)^2 \tilde{k}\delta(z-y)$$

The assumption that the covariance of the capacitive unbalance function $E[C_u(z)C_u(y)] = \tilde{k}\delta(z-y)$ was used because the standardised models use the same assumption. Moreover, it keeps the final equations simple. Another argumentation is that the wavelength of the used signals is much larger than the twist period of any wire pair. Therefore, if the distance granularity is not too fine then the assumption is probably acceptable. So, finally, one obtains

$$E[C_n(z)C_n^*(y)] = k_n \delta(z-y) \quad (5)$$

with

$$k_n = \frac{1}{4}\left(\frac{|Z_c|}{2} + \frac{k}{|Z_c|} \right)^2 \tilde{k} \quad (6)$$

The covariance of the unbalance function for FEXT

The unbalance function for FEXT is given by [8]:

$$C_f(z,\omega) = \frac{C_u(z)}{4} Z_c(\omega) - \frac{M_u(z)}{2Z_c(\omega)} \quad (7)$$

and the covariance of the same unbalance function can be transformed into its well-known form:

$$E[C_f(z)C_f^*(y)] = \left\{ \frac{|Z_c|^2}{16} + \frac{k^2}{4|Z_c|^2} - \frac{k}{8}\left(\frac{Z_c}{Z_c^*} + \frac{Z_c^*}{Z_c}\right) \right\} E[C_u(z)C_u(y)]$$

$$E[C_f(z)C_f^*(y)] = \frac{1}{4}\left(\frac{|Z_c|}{2} - \frac{k}{|Z_c|} \right)^2 E[C_u(z)C_u(y)] = \frac{1}{4}\left(\frac{|Z_c|}{2} - \frac{k}{|Z_c|} \right)^2 \tilde{k}\delta(z-y)$$

Finally, one obtains

$$E[C_f(z)C_f^*(y)] = k_f \delta(z-y) \quad (8)$$

with

$$k_f = \frac{1}{4}\left(\frac{|Z_c|}{2} - \frac{k}{|Z_c|} \right)^2 \tilde{k} \quad (9)$$

The covariance of the unbalance function for NEXT and FEXT

The covariance of the unbalance function for NEXT and FEXT is can be calculated in a similar fashion.

$$E[C_n(z)C_f^*(y)] = \left\{ \frac{|Z_c|^2}{16} - \frac{k^2}{4|Z_c|^2} + \frac{k}{8}\left(\frac{Z_c^*}{Z_c} - \frac{Z_c}{Z_c^*}\right) \right\} E[C_u(z)C_u(y)]$$

Using the fact that $\frac{Z_c^*}{Z_c} - \frac{Z_c}{Z_c^*} \approx 0$ then

$$E[C_n(z)C_f^*(y)] = \frac{1}{4}\left(\frac{|Z_c|^2}{4} - \frac{k^2}{|Z_c|^2} \right) E[C_u(z)C_u(y)] = \frac{1}{4}\left(\frac{|Z_c|}{2} - \frac{k}{|Z_c|} \right)\left(\frac{|Z_c|}{2} + \frac{k}{|Z_c|} \right) \tilde{k}\,\delta(z-y)$$

Finally, one obtains

$$E[C_n(z)C_f^*(y)] = k_{nf}\,\delta(z-y) \tag{10}$$

with

$$k_{nf} = \frac{1}{4}\left(\frac{|Z_c|}{2} - \frac{k}{|Z_c|} \right)\left(\frac{|Z_c|}{2} + \frac{k}{|Z_c|} \right) \tilde{k} \tag{11}$$

The coupling between the crosstalk constants

Using (6), (9) and (11) brings forward the following relation between the coupling constants:

$$k_{nf} = \sqrt{k_n k_f} \tag{12}$$

So, starting from the simple model $M_u(z) = k \cdot C_u(z)$ that relates the capacitive and inductive unbalance it was found that the crosstalk coupling constant k_{nf} can be calculated from the knowledge of the well-known coupling constants k_n and k_f. If the line ends are perfectly matched (setup shown in figure 1), then it is not necessary to use k_{nf} to model the crosstalk. However, using (6) and (9) it is clear that k_n and k_f are not independent from each other. In fact:

$$\frac{k_n}{k_f} = \left(\frac{|Z_c|^2 + 2k}{|Z_c|^2 - 2k} \right)^2 \tag{13}$$

In case crosstalk has to be modelled or simulated when the line ends are badly terminated, it was demonstrated in [8] that an advanced model should be used to predict the PSD of the crosstalk signals. This model makes use of k_{nf} but in [8] no argument for its value could be given. However, using [8] and (12) it is clear that crosstalk can be modelled even if the line ends are not perfectly terminated. Further, any loop topology is allowed now: a single line segment not perfectly terminated, cascaded line segments, loops with bridged taps, etc.

IV. Conclusion

A first order model was proposed to calculate the inductive unbalance from the capacitive unbalance between two wire pairs. It was shown that using this simple model the approximation error is acceptable. Starting from this model, the connection between the widely used crosstalk coupling constants k_n and k_f can be revealed and moreover, the crosstalk constant k_{nf} introduced by the authors in [8] is directly given in terms of the previous ones. This result allows an operator to calculate k_{nf} from k_n and k_f, which are known from early measurement campaigns carried out on their access network.

V. References

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