Efficient evaluation of ELF field

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Abstract- In this work, efficient techniques for the computation of the low frequency magnetic field due to the electrical circuit in buildings or industrial environment are discussed. Existing models used to compute the magnetic field caused by power lines are recalled at first. Then, an algorithm for the fast evaluation of the magnetic field due to electrical wires is proposed. It is based on the partitioning of the various sources into simpler magnetic sources: for each elementary magnetic source, some closed-form formulae are provided and combination rules are suggested in order to provide the calculation of the magnetic field from the segments associated to the elementary sources themselves.

The algorithm has been validated by comparing the results to the ones provided by well assessed computation method. It is shown that the computational speed of the proposed technique is approximately 200 times faster than the direct application of the Laplace equation.

I. Introduction

The number of electrical and electronic devices in everyday use has grown steadily in recent years. Electrical machinery are essential equipment in industry practice, while a number of different appliances ranging from domestic cooking aids to cooling or heating systems are daylife tools that no one would like to dismiss. Civil and industrial buildings are hence crowded with power and utility lines bringing both signals and energy, assuming different current and voltage levels during time [1].

Despite the demand for these everyday facilities, people are now also concerned about the effects of electromagnetic (EM) fields on health. Researchers have done or are currently having a hard work in order to understand the biological effects of EM fields. One among the most difficult questions is which is the level of an EM field intensity that can be considered safe when a subject is exposed to that field for a very long time. Even though the debate is still far from providing a clear answer, government agencies are now imposing limits on the maximum level of electromagnetic fields affecting people life. Such exposure limits differ among countries and depends on the actual destination of an area, typically distinguishing from home and working areas [2] [3] [4].

In any case, the common idea underlying the different laws approved or under discussion is that the intensity of the total EM field due to all the equipment at a given spatial point should be lower than a given limit. The numerical value of such limit depends on the frequency of electromagnetic emission; commonly fields are divided into two categories: industrial frequency, also referred to as Extremely Low Frequency (ELF), ranging from DC to a few tens of kHz, and radio frequencies for higher values. ELF field limits are usually referred to the magnetic field, while the electric field is considered due to radio-frequency sources, although everybody may derive relationships between electrical and magnetic field of a radiated wave.

In the above context, the problem simply stated is to provide a sufficiently accurate evaluation of the surrounding EM field. In this paper ELF fields are considered only. One should note that ELF fields are due to rather low-intensity sources in residential areas, but exposition time is longer, and radiations have a stronger emotional impact. For these reasons, law limits are usually more severe, so that they can be easily exceeded by radiation sources having a limited intensity. On the other side, high intensity sources are usually found in an industrial environment, where a complex layout of circuits and wiring makes the problem of EM field analysis a non-trivial task.

In any case, the designer of a civil or industrial plant has now a new previously unknown problem. In order to satisfy functionality and security issues, the designers have to deal with the problem of determining in advance whether the circuit will be in compliance with exposure limits or not. Noticeably, in some countries, government long-term plans are under discussion with the aim of gradually lower limits down to very low values, so that such problem can be no longer ignored. It is well known that shielding ELF radiations may be a viable solution only in a very limited number of special cases. Planning in advance the physical layout of lines or devices seems currently to be the simpler and cheaper solution.

The algorithms for the determination of EM fields are well known, and many commercial solutions can be purchased at a reasonably price. Anyway, off-the-shelf tools are based on finite element algorithms that directly solve Maxwell's equations. Hence they require an accurate definition of the problem being solved both in terms of geometrical conditions and in terms of electrical materials and sources being involved. Another side effect of such accurate tools is that the time required for the determination of the EM field under a well stated condition is often rather high even for objects having a limited extension in space. Moreover, one of the critical point of the EM determination problem is the current injected into conductors: a value that in many real-life situations is not exactly known, since it depends on the devices switched on, and on their operating conditions.

II. Computational algorithm

The magnetic field in buildings is normally due to electrical circuits, i.e. wires. Other sources such as electrical rotating machines are elsewhere analysed, or the magnetic field produced in the surrounding space can be determined by direct measurements [2] [3] [4] [5], when the calculation is an overwhelming task. In any case, as concern to environmental magnetic field, combined effect of devices and wires can be obtained by a direct summation of the respective magnitude contributions, proving an overestimated value not too far from the actual one.

Moreover, since the field is investigated in the air, the magnetic field, $\overline{H}$, is related to the magnetic induction, $\overline{B}$, by $\overline{B} \cong \mu_0 \cdot \overline{H}$, where μ_0 is the magnetic permeability of the vacuum, and hence the induction $\overline{B}$ will be considered in the following without loss of generality.

It is well-known that if a current I flows through a wire having negligible section and described by a curve S, the induction $\overline{B}$ at the observation point P is:

$$\overline{B}(P) = \int_S \frac{\mu_0 \cdot I}{4 \cdot \pi} \cdot \frac{d\overline{S} \times \overline{u}_r}{r^2} \qquad (1)$$

where $\overline{u}_r$ is the unit vector directed from $d\overline{S}$ to P, the elementary source $d\overline{S}$ is along S in the direction of I and r is the distance between P and $d\overline{S}$.

For the more general case of a set of wires, modelled by the curves { S_k, $k = 1,....N$ }, each carrying a current I_k , the induction $\overline{B}$ is obtained as the summation of the values $\overline{B}_k(P)$ calculated applying (1) to each curve S_k .

Equation (1) can be used for the evaluation of the magnetic induction $\overline{B}$ in a very simple manner. One has to approximate the integral with a summation by splitting the curve S into a suitable number of elementary contributions of length ΔS , each being sufficiently small. Standard numerical techniques are available to this aim [6]. In the following, this calculation technique will be referred to as “direct approximation of Ampère-Laplace law”.

When a real-life electrical circuit is considered, in practice the curve S can be viewed as the combination of straight lines so that (1) can be split into the summation (2):

$$\overline{B}(P) = \frac{\mu_0}{4 \cdot \pi} \cdot \sum_k I_k \cdot \int_S \frac{d\overline{S} \times \overline{u}_r}{r_k^{\,2}(P)} \qquad (2)$$

where the index k indicates the single straight wire, and each integral can be resolved in a closed form. In fact, for the case of a wire having a finite length a, as the seen in Fig. 1a, the magnetic induction $\overline{B}$**,** at the point $P = P(x, R)$, is:

$$B(P) = \frac{\mu_0 \cdot I}{4 \cdot \pi} \cdot \left\{ \frac{x}{\sqrt{R^2 + x^2}} + \frac{a - x}{\sqrt{R^2 + (a-x)^2}} \right\} \qquad (3)$$

where R is the distance along the normal direction between P and the wire and x is the coordinate of P along the wire.

One should note the for an infinite length straight wire, i.e. for a and x much greater than R, (3) leads to the well-known Biot-Savart law (4):

$$B_{BS}(P) = \frac{\mu_0 \cdot I}{2 \cdot \pi \cdot R} \qquad (4)$$

where the subscript *"BS"* indicates the magnetic induction obtained using the Biot-Savart law.

It is interesting to observe that the previous equation is commonly used for the evaluation of the magnetic field in the proximity of electrical power lines [7].

Moreover, the equations (3) and (4) provide similar results for points P sufficiently close to the wire and far from the wire endpoints. In particular, one may easily find the set of points P for which (3) and (4) differ for less than a given factor α, namely:

$$A = \left\{ x = x(r) : \frac{B(P)}{B_{BS}(P)} \leq \alpha < 1,\ R\ given \right\} \qquad (5)$$

The set A, for R in a given interval, represents a space region for which the magnetic field is close to the one generated by an infinite length wire. Fig. 1b represents this fact for the special case α equals to 3 dB; providing for $x > R$, $x < a - R$.

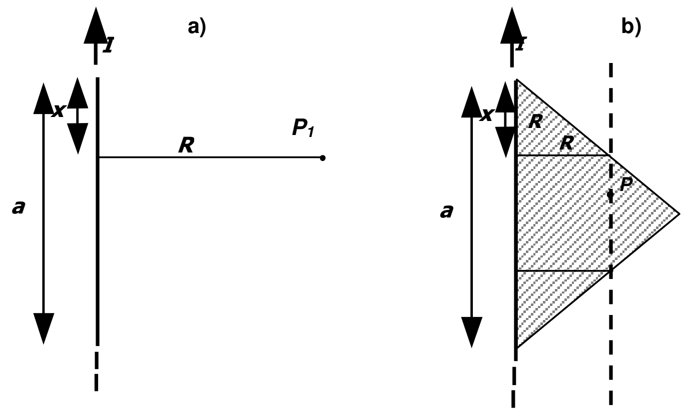


Figure 1: Geometrical model for the evaluation of the magnetic induction B of a finite length straight wire

The rule (5), together with equation (4), shows that one can easily divide the analyzed 3D region into smaller areas where the computation of the field $\overline{B}(P)$ can be efficiently performed. Or, equivalently, given a point P and a set of sources, one may easily evaluate the contribution of each source by checking whether P belongs to one of the possible areas for which the approximation (4), or similar ones, can be assumed under a pre-determined approximation level. This means that by investigating a limited number of geometrical configurations of wires, after simpler algebraic manipulations, a set of straightforward rules for a fast computation of the magnetic induction $\overline{B}$ can be determined. For the sake of brevity, further details on the analysed sources are omitted; anyway couple of wires at a given distance, three-phase conductors and so on have been modelled.

One should note that the above calculation strategy has a computational complexity directly proportional to the number of sources and the number of points P for which the calculation is performed, while it is almost independent of the length of wires. Direct application of Ampère-Laplace equation additionally depends on the number of infinitesimal elements in which each wire is divided and hence has an intrinsically higher computational complexity.

Both direct evaluation of Ampère-Laplace equation and the explained efficient computational schemes (referred to as "fast algorithm") have been implemented. The former has been used as reference due to its intrinsically high accuracy.

III. Algorithm validation

Comparison in a number of cases taken from practical situations have been performed. It has been seen that the "fast algorithm" provides an evaluation of the magnetic induction $\overline{B}$ rules for about a

10% maximum deviation from the Ampère-Laplace law.

As concern to the algorithm efficiency, Table 2 represents a comparison analysis under a very simple circuit layout having up to 10 wires.

A 2D slice of ~900 points and a 3D volume of ~18000 (20 slices of 900 points) points have been considered while 3 m long wires have been used. The grid step along in the volume was of 10 cm.

Table 1 : computation time (ms) for the two algorithms with respect to the number of wires composing the circuit (F= “fast algorithm”, A-L= “Ampère-Laplace equation”).

	1wire	**2 wires**	**3 wires**	**5 wires**	**10 wires**
2D F	16	16	16	31	62
2D A-L	1250	2422	3672	3031	12141
3D F	125	250	343	5563	1094
3D A-L	23141	45953	69828	111657	230219
2D F/A-L	0.0128	0.0066	0.0044	0.0051	0.0051
3D F/A-L	0.0054	0.0054	0.0049	0.0050	0.0048

The Table1 shows that the execution speed is approximately 200 time faster for the “fast-algorithm”, as expected, than the Ampère-Laplace method.

The Table 2 shows the impact of the wire length on the execution speed. In this simulation only one wire is considered having different length values. It is observed that while the computational time does not significantly change for the “fast algorithm”, it linearly increases with the wire length for the other calculation algorithm.

Table 2 : computation time (ms) for the two algorithms with respect to the wire length (F= “fast algorithm”, A-L= “Ampère-Laplace equation”).

	1m	**2m**	**4m**	**5m**	**8m**	**10m**
2D F	16	16	16	15	15	16
2D A-L	265	500	1188	1672	3781	5641
3D F	47	47	46	47	46	94
3D A-L	2468	4906	10766	14657	30609	51610

Both the results showed in Table 1 and 2 have been obtained using a Java Virtual Machine and a Pentium 4 at 2.4 GHz (PC).

IV. Experimental validation

To validate the proposed models, a suitable experiment has been carried out.

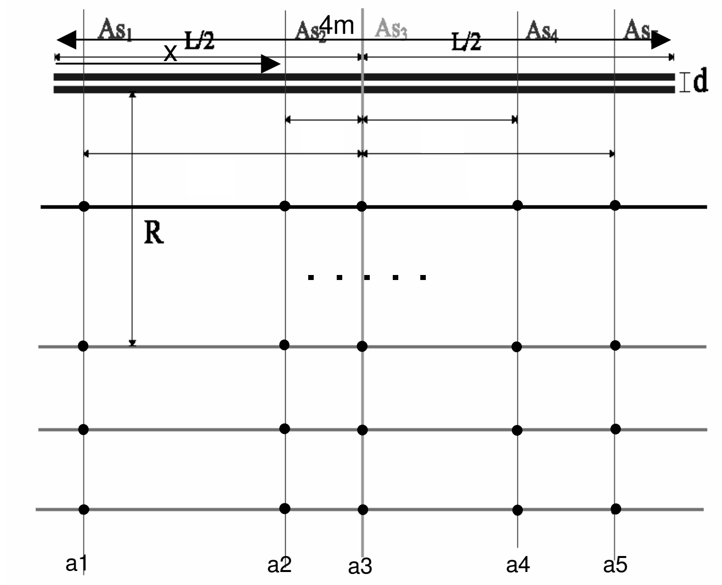
A wood guide having a length of 4 m was taken in which two wires were placed. The guide was provided of two grooves that kept the wires at a known and fixed distance. At the endpoints of the guide the wires were twisted to reduce the external field generated by the wires connected to the current source (a transformer was used to obtain a sufficiently high current of 20 A) and 2 Ω resistive load. In this way the guide emulated a segment of 2 wires of a length of 4 m.

The magnetic induction was measured by a ELF field probe PMM EHP50A taking only the 50 Hz contribution.

A schematic representation of the measurement points is showed in Fig. 2. A grid in the plane containing the wire is considered. Denoting with R the direction in the plane normal to the wire and x the direction parallel to the wire, five values of x and ten values of R have been chosen; 50

measurement points were then gathered into five sets, a_i, (i= 1,..5), where in each set x was fixed. The Fig. 2 shows this arrangement.

The Fig. 3 reports the difference between the measurements field values and this estimated one. It is seen the good agreement between the measured and the estimated values almost for any values of x and R.



$$x = \{0.4, \quad 1.4, \quad 2, \quad 2.4, \quad 3.2\} \quad m$$
$$R = \{0.4 \quad 0.5, \quad 0.6, \quad 0.7, \quad 0.8, \quad 0.9, \quad 1, \quad 1.1, \quad 1.2, \quad 1.3\} \quad m$$

Figure 2: measurement grid

For the case of points close to the wire end and close to the wire itself large differences values are observed: this is an expected result for the fast algorithm, while for the case of the Ampère-Laplace method it is due to lack of modelling of the wire connection to the load.

In any case, the fast algorithm always provided over-estimated values this is an advisable effect when health problems are under consideration.

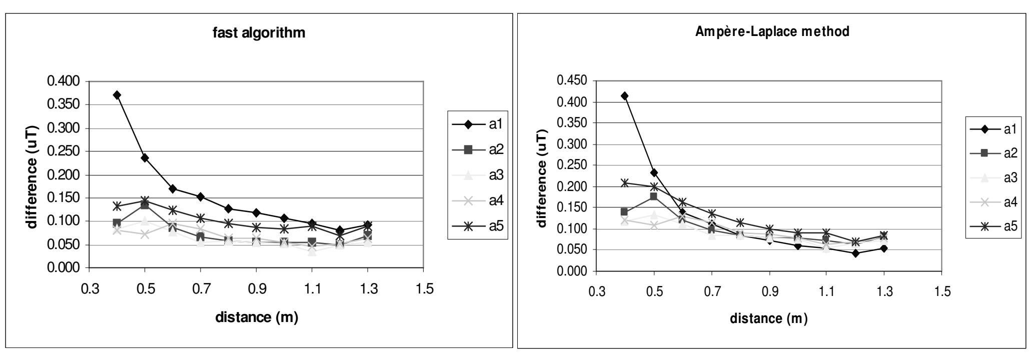


Figure 3: absolute difference (μT) between the measured magnetic field and the estimated values: (a) fast algorithm and (b) the Ampere-Laplace method.

III. Conclusions

Techniques for the evaluation of the magnetic field inside buildings have been discussed. The first technique is based on direct approximation of Ampère-Laplace equation, while the second is based on a volume partitioning strategy combined with the usage of closed-form equations derived for simple geometrical configurations of wires.

Both the methods cannot be applied for the case of complex geometrical structures such as an electrical engine where finite elements methods should be used. This limitation is indeed not a heavy drawback when environmental analysis is under consideration. Instead, the rather high computation speed enable the presented algorithms to be used as an interesting tool for the designers of an electrical circuit concerned about electromagnetic pollution.

The developed tools are now under consideration at institutions involved in the redraw of electrical circuits in public buildings.

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