TIME-DOMAIN EXPERIMENTAL CHARACTERIZATION OF NON-LINEAR DYNAMIC EFFECTS IN S/H-ADC DEVICES

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Abstract – *The input/output relationship of a Sample/Hold and Analogue-to-Digital Conversion device (S/H-ADC) can be described as the response of a non-linear system with memory. A general-purpose "black-box" behavioural approach, based on a modified Volterra representation, has been proposed by authors for the modelling of a wide class of non-linear dynamic systems and specifically applied to the characterization of S/H-ADCs. In this paper, the instrumentation set-up and the experimental procedure for the extraction of S/H-ADC model parameters are presented and a novel standard for the characterization of non-linear dynamic effects in this family of measurement systems is proposed.*

Keywords - S/H-ADC, system modelling, non-linear dynamic effects.

1. THE S/H-ADC FINITE-MEMORY MODEL

A S/H-ADC is usually described in terms of its static input/output characteristic and conventional models are derived from simple DC measurements. This kind of approach, although capable to accurately predict the device response at moderately low frequencies, does not take into account any empirical information about system dynamics, which on the contrary can become important effects when the input frequency raises up, leading to the introduction of a relevant additional contribution on the output. In order to identify a general approach to the problem of modelling also the dynamic effects of a S/H-ADC, authors proposed in the last few years the functional description shown in Fig.1 [1-3]. The actual device is described as an ideal instrument sampling and converting to digital a signal $y(t)$ which is the result of the input/output relationship of a cascade of two block with memory, the first one being purely-linear and the second one characterized by short-lasting non-linear dynamic effects. The first system in the cascade takes into account those dynamics which are associated with the input signal conditioning section of the device (i.e. amplifiers, filters, etc.), and can be suitably characterized by linear operators with quite "long" memory. The second block, instead, is introduced in order to describe the system non-linearities, both static and dynamic. In particular, being the non-linear dynamic effects in S/H-ADCs associated with the active devices (diodes, transistors), which are usually described by highly non-linear behaviour and "fast" dynamics, the time duration of memory of the second element in the cascade can be considered "short" if compared to the typical minimum period of the input signal $s_I(t)$. Under such an assumption, the non-linear dynamic system can be accurately characterized by means of the modified Volterra series expansion proposed in [4], truncated to the first-order term. More precisely, the second block in the cascade of Fig.1 can be further described as the sum of two non-linear subsystems (Fig.2), which are respectively memoryless and purely-dynamic with "short" memory. The response $y^{(S)}(t)$ of the first block can be written as a power series of the signal $s(t)$:

$$y^{(S)}(t) = z_0[s(t)] = y_0 + \sum_{r=1}^{\infty} \frac{a_r}{r!} s^r(t) \tag{1}$$

while the output $y^{(D)}(t)$ of the purely-dynamic system can be approximated as a discrete convolution over the memory time $[-T_A, T_B] \equiv [-P_A\Delta\tau, P_B\Delta\tau]$ with respect to the *dynamic deviation* function $e(t,\tau)$:

$$\begin{aligned} y^{(D)}(t) &\cong \sum_{\substack{p=-P_A \\ p\neq 0}}^{+P_B} w[s(t), p\Delta\tau]\cdot e(t, p\Delta\tau)\Delta\tau = \\ &= \sum_{\substack{p=-P_A \\ p\neq 0}}^{+P_B} e(t, p\Delta\tau) \sum_{n=0}^{N} \beta_{pn} s^n(t) \quad , \quad e(t,\tau) = s(t-\tau) - s(t) \end{aligned} \tag{2}$$

In Eq. (2) $w[s(t),\tau]$ is the first-order kernel of the modified series, which is non-linearly controlled by the signal $s(t)$ and can be theoretically derived from conventional Volterra kernels [3][4]. By expanding each term $w[s(t), p\Delta\tau]\Delta\tau$ into a polynomial series truncated to the Nth-order, coefficients β_{pn} become model parameters which unambiguously characterise the dynamic non-linearities of response $y^{(D)}(t)$ (i.e. the non-linear dynamic effects of the device). S/H-ADC behavioural description of Fig.2 clearly points out the intrinsic separation, taken into account by such an approach, between static and respectively linear and non-linear dynamic effects, which allows the identification of a reliable and unambiguous experimental procedure for the characterization of each block of the model. In addition, the exploitation of the modified series approach for the modelling of the non-linear dynamic system allows to describe it by considering only one convolution integral (i.e. the first-order kernel contribution in the series expansion), instead of the three or four terms needed at least in the classical Volterra method. In fact, the separation between static and purely-dynamic non-linearities, typical of the modified series formulation, leads to higher-order integral contributions which are functional with respect to the dynamic deviation $e(t,\tau)$. Since this function assumes small values in the short memory time even in the presence of large fluctuations of the signal $s(t)$, the series expansion terms of order higher than the first are superior-order infinitesimals and can be neglected.

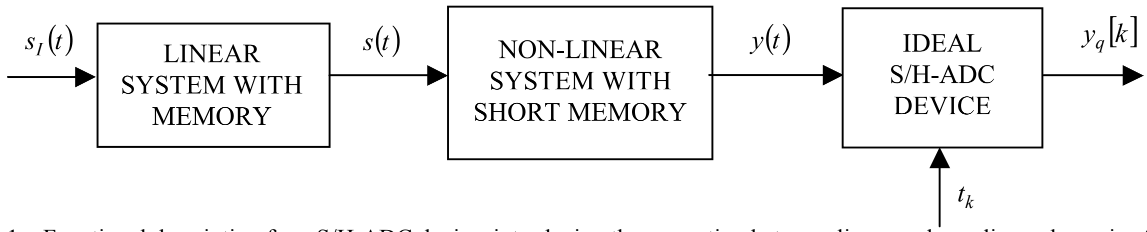

Fig.1 – Functional description for a S/H-ADC device, introducing the separation between linear and non-linear dynamic effects.

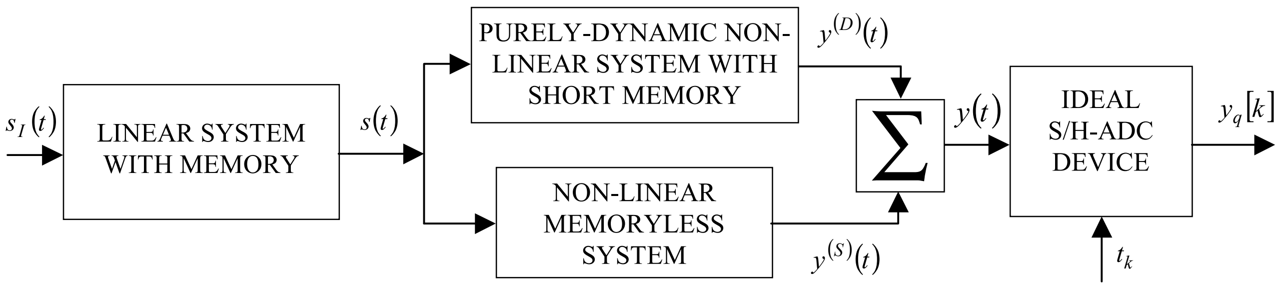

Fig.2 – Final functional model for the S/H-ADC device.

2. EXPERIMENTAL IDENTIFICATION OF THE S/H-ADC MODEL

The S/H-ADC functional description of Fig.2, based on the model equations (1) and (2), has been applied to the modelling of the non-linear dynamic response of a general-purpose 250kS/s successive approximation digital acquisition board with 12-bit resolution. The board presented a PCI interface which has allowed its connection to the bus of a PC architecture: thus, it has been possible to control the device under test (DUT) through the National Instrument LabVIEW software tool. An auxiliary S/H-ADC board, capable of a 5MS/s real-time acquisition frequency and in general characterized by better performances in terms of accuracy and linearity than the DUT, has been connected to the same PC system with reference purposes. Fig.3 shows the instrumentation set-up which has been used to the aim of model parameter characterization.

Since the non-linear system with response $y^{(D)}(t)$ in Fig.2 is purely-dynamic and it is always possible to impose, without loss of generality, $H(0)=1$ to the transfer function of the linear block, the non-linear memoryless system is characterized by a response $y^{(S)}(t)=z_0[s(t)]$ which coincides with the S/H-ADC static characteristic: simple DC measurements in the device operating region [-5V,5V] have allowed the extraction of the coefficients a_r which appear in the polynomial series expansion of function z_0 (Eq. (1)). These coefficients give also a "global" information about conventional Volterra kernels, since the following expression can be theoretically demonstrated [4] for $r=1,2,\ldots$:

$$a_r = \int \cdots \int_{-\infty}^{+\infty} h_r(\tau_1,\tau_2,\ldots,\tau_r)\prod_{i=1}^{r} d\tau_i \qquad (3)$$

As far as the characterization of the purely-linear system is concerned, it can be shown [2] that at zero bias, small-signal operating conditions the non-linear dynamic block gives no contribution to $y(t)$. A 80MHz waveform generator has been used as an input source for both DUT and REF acquisition boards. This has been possible by splitting the test signal through a suitably matched (50Ω) transmission network. A very low-amplitude sinusoidal test signal has been synthesised by the generator at different frequencies, not only in the linear bandwidth of the S/H-ADC (up to 500 kHz), but also outside this region, where the linear dynamics start to introduce a relevant attenuation. By calculating the first derivative g_{DC0} of function z_0 with respect to $s(t)\equiv 0$, the transfer function $H(f)$ of the linear subsystem can be obtained, at each operating frequency f_i, from the expression:

$$H(f_i) = \frac{Y(f_i)}{g_{DC0} \cdot S_I(f_i)} \quad (4)$$

where $Y(f_i)$ is the discrete transform of *y(t)* deduced by the samples acquired by the DUT, while $S_I(f_i)$ is the complex number representing the input signal, which can be accurately characterized starting from the samples at the output of the REF S/H-ADC operating at 5MS/s (maximum frequency). No synchronism between the two boards is needed during this procedure step, since each experimental parameter is related to a sinusoidal signal defined on a independent time axis.

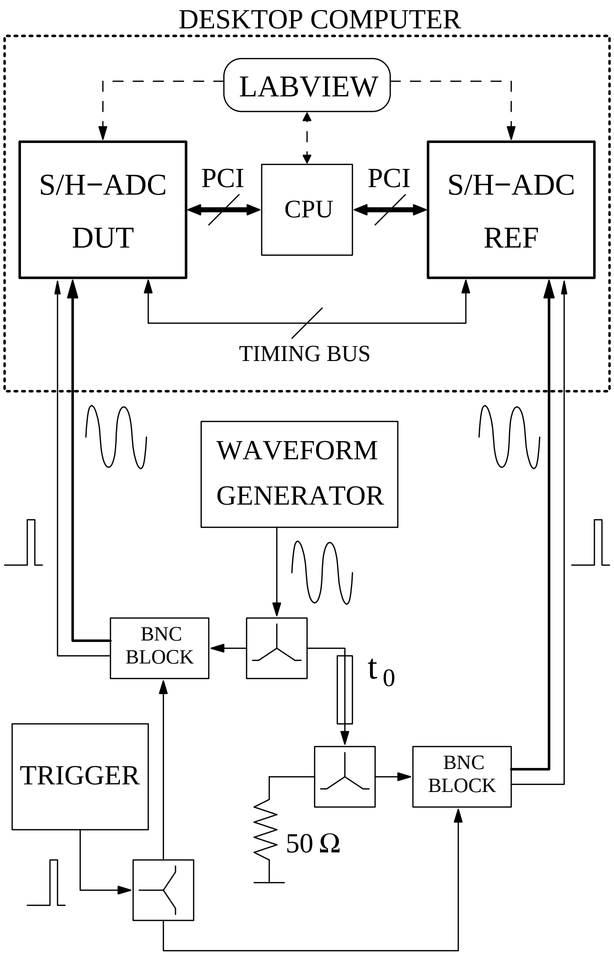

Fig. 3 – Instrumentation set-up used in the aim of S/H-ADC model characterization.

The two procedures, which have easily carried out the experimental characterization of DC characteristic $z_0[\cdot]$ and transfer function *H(f)*, have been followed by a final step for the identification of the non-linear purely-dynamic subsystem of Fig.2. The same waveform generator has been exploited to synthesis a set of *R* large-amplitude (0.5V) sinusoidal test signals $s_{Ir}(t)$, for different bias values equally-spaced in the interval $[-4\text{V},4\text{V}]$ (step 1V) and at different frequencies f_k (from 1kHz up to 1MHz). If each input signal is sampled by both DUT and REF S/H-ADCs in *M* time instants t_m, an over-determined system of *RxM* linear equations can be obtained from Eq. (2) $(P_A = P_B = 3\ ;\ \Delta\tau = 20\text{ns})$:

$$\sum_{\substack{p=-P_A \\ p\neq 0}}^{P_B} \left[s_r(t_m - p\Delta\tau) - s_r(t_m)\right] \sum_{n=0}^{N} \beta_{pn} s_r^n(t_m) = \quad (5)$$
$$= y_r(t_m) - z_0\left[s_r(t_m)\right] \quad (r = 1,\ldots,R)(m = 1,\ldots,M)$$

Samples $y_r(t_m)$ are directly available at the output of the DUT, while delayed signal samples $s_r(t_m - p\Delta\tau)$ can be computed starting from the input Fourier transform $\widetilde{S}_{Ir}(f)$ (derived through data acquired by the REF device) and by applying to it the linear transformation:

$$H_p(f) = H(f)\exp(-j2\pi f p\Delta\tau) \quad (6)$$

Since values $z_0[s_r(t_m)]$ can be deduced through the previously identified DC characteristic, the system (5) in the $(P_A + P_B)(N+1) << RM$ unknowns β_{pn} can be solved by means of reliable least-square algorithms, which don't suffer from convergence problems and lead to an unambiguous solution. In this final step of the experimental procedure a particular attention has been paid to the synchronism between the two S/H-ADC devices. In fact, the empirical data, which appear in system (5), acquired by the two boards must be strictly referred to the same coherent time axis. To this aim, a timing bus has been used to connect DUT and REF devices and the same TTL trigger signal has been applied to them through the BNC blocks, which also provided the input test signal final connection to the boards (a 20% duty-cycle trigger performed the best synchronism between the devices). Moreover, since it is not possible to practically carry out signal paths to REF and DUT S/H-ADCs which present exactly the same length, the time delay t_o suffered from the wave propagation, through an auxiliary coaxial cable (see Fig.3), to the reference device, has been characterized (t_o=2.5ns) and empirical data have been suitably depurated from such a non-ideality.

3. EXPERIMENTAL RESULTS

The experimental procedure described in the previous section allowed a complete identification of S/H-ADC model parameters. Both static and respectively linear and non-linear dynamic effects in the device behaviour have been characterized, by means of an input/output empirical data-based approach using conventional instrumentation and without the need for synthesising complex test signals or introducing higher-order statistics, typical instead of classical Volterra methods. In particular, the S/H-ADC non-linear dynamics can be characterized simply by a matrix $\underline{B}$ of scalar parameters β_{pn} whose magnitude variations with respect to order n can carry out an estimation of device dynamic non-linearity level and allow a comparison, from this point of view, between different devices. The parameter dependence from index p provides instead information about the actual non-linear memory time duration, and can suggest the best choice for values of P_A, P_B and $\Delta\tau$. Two different experimental identifications of the non-linear dynamic response have been carried out, by considering N=3 and N=5, respectively. Fig.4 shows the magnitude of $\underline{B}$ elements with respect to p and n. Tab.1 reports the actual value for $\underline{B}$. It is interesting to notice that, with the 5th-order polynomial expansion, the first-order modified kernel is characterized by an anti-symmetric dependence with respect to $s(t)$ (i.e. odd-order coefficients prevail in the expansion over the even ones), which reveals a behaviour of non-linear dynamics similar to that of static characteristic $z_0[s(t)]$ (whose polynomial series coefficients are reported in Tab.2), typical of bipolar S/H-ADC devices.

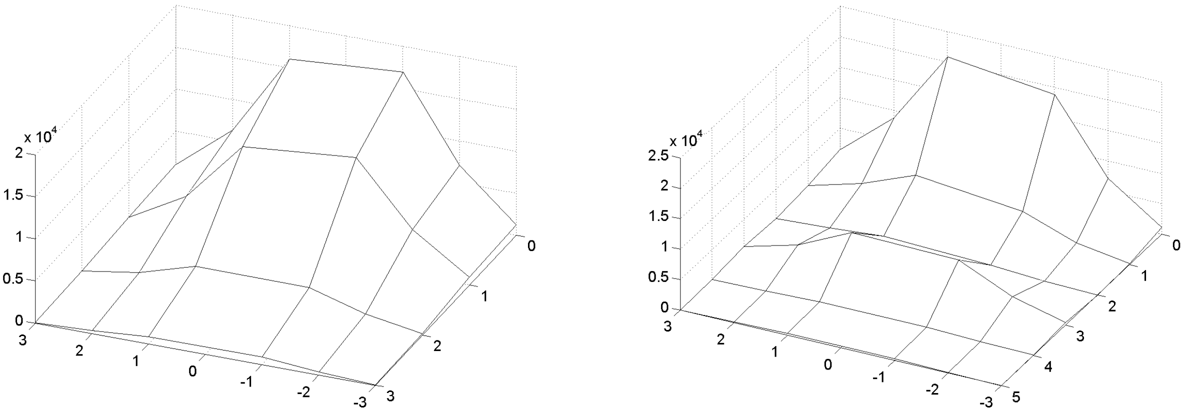

Fig.4 – Magnitude of coefficients β_{pn} in the polynomial expansion of the first-order modified kernel, which describe the S/H-ADC non-linear dynamics. Model experimental characterization has been performed taking into account both order N=3 (*left*) and order N=5 (*right*).

p \ n	0	1	2	3
–3	-1.25	0.950	0.211	0.0606
–2	7.09	-5.37	-1.27	0.355
–1	-16.9	12.7	3.19	-0.871
1	-16.0	11.5	3.25	-0.859
2	6.34	-4.41	-1.32	0.345
3	-1.06	0.702	0.224	0.0581

p \ n	0	1	2	3	4	5
–3	1.12	-0.198	0.0138	0.428	0.0348	-0.0210
–2	-7.10	1.51	0.184	-2.60	-0.211	0.128
–1	18.8	-4.60	-0.733	6.60	0.536	-0.326
1	20.8	-6.39	-1.376	6.93	0.571	-0.344
2	-8.78	2.96	0.707	-2.87	-0.239	0.143
3	1.55	0.570	-0.148	0.496	0.0421	-0.0247

Tab.1 – Actual value (10e3 unit) for elements β_{pn} of matrix $\underline{B}$ for modified kernel polynomial expansion of order N=3 (*left*) and N=5 (*right*).

r	0	1	2	3	4	5
a_r	-8.54E-4	1.00	2.14E-4	-4.80E-4	-1.38E-4	3.29E-4

Tab.2 – Polynomial expansion coefficients of DC characteristic.

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