USING AN INTERPOLATION METHOD FOR NOISE SHAPING IN A/D CONVERTERS

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Abstract: *Digital post processing of output data is often used to enhance the ADC Effective Number Of Bits (ENOB). In particular, it can be used to partially recover ENOB restrictions caused by nonlinearities. The paper deals with advantages and disadvantages coming from the application of a proposed nonlinearity correction method based on the Bayes theorem. It allows the reduction of large scale errors in output signal by means of the use of dithering with low peak-peak voltage instead of a high amplitude one. The paper gives a brief description of the method. Then, the results of an experimental investigation carried out on actual ADC output data are presented and discussed.*

Keywords: Interpolation, Noise Shaping, ADC Correction.

1. Introduction

In ideal ADCs, if the value of the analogue input is equal to the transition threshold value T(k), one of the two codes, k and k+1, will be produced as output with a 50% probability. Actual ADCs are affected by random error processes and transfer characteristic nonlinearities that change such property. A probabilistic model of the ADC can be thought as shown in Fig. 1. The first stage is represented by an ideal ADC which provides the input amplitude quantisation. In Fig. 1, the ideally quantised i-th sample x[i] is reported as output of the first stage with its probability density function (PDF) p(x[i]). In the following stage, the ideal output codes are changed to the real ones k[i]. This code alteration could be described as a data transfer across a noisy channel. It includes effects of both random behaviour around the ADC transition level T(k) and its shift caused by Integral NonLinearities INL(k).

After the noisy channel, the data flux could be recovered by using a suitable post-processing method. In particular, by providing an opportune oversampling factor [1] and adding a dithering signal, an inverse Bayesan filter followed by a low-pass filter could return damaged data in their right positions [2].

It is well known that dithering can be used in general to compensate only such nonlinearities whose amplitude are overlapped by the dithering signal amplitude [3]. So, large scale errors require large dithering amplitude. The application of the method based on the Bayesan filter to the oversampled data flux has shown that low amplitude peak-peak dithering can be used to reduce also the effect of large scale INLs. Interpolation process based on Bayes theorem, in fact, smoothes the distortion in the output signal caused by the INL in the ADC transfer characteristic. In the following sections, first the interpolation method is briefly recalled, then the results obtained by an experimental investigation carried out on an actual ADC are discussed.

2. Bayesan filtering on ADC outputs

As above mentioned, in a probabilistic approach the transition threshold T(k) can be defined as the input voltage value that cause an output code oscillation between the k and k+1 values with a 50% probability. In order to obtain an estimate of such thresholds, a suitable amount of samples should be taken for each input voltage. In actual ADCs the actual T(k)s are different from the ideal ones. The resulting quantisation characteristic could be nonlinear, causing significant distortion on the output data. As a consequence of INL presence, the output code corresponding to the ideal thresholds oscillates

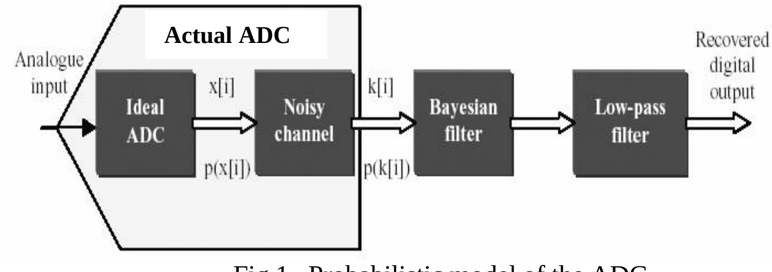

Fig.1. Probabilistic model of the ADC.

between k and k+1 with probabilities different from 50%. In the worst case, if the differential nonlinearity for the code bin k is –1, such code cannot be produced as an ADC output. By using a probabilistic approach, INL errors could be transferred into conditional probabilities [4]. In order to describe the transformation of the ideal code bins x into their actual counterparts k (output of noisy channel) the transfer characteristic of the channel has been described in terms of conditioned probabilities p(k[i]/x[i]). These probabilities describe the effect of the INL on the occurrences of the code k[i] caused by input samples with value x[i]. The probability density function p(k[i]/x[i]) is calculated, starting from known values of threshold levels T[k], by means of the algorithm proposed in [2]. The values T[k] are determined following the testing procedure of the ADC reported in [5].

According to the Bayes theorem, the probability of a certain k[i] output code given the input voltage x[i] is:

$$p(x[i]/k[i]) = \frac{p(k[i]/x[i]).p(x[i])}{\sum_{x=0}^{2^N-1} p(x[i]).p(k[i]/x[i])} \qquad (1)$$

where N is the number of bits.

The relation (1) gives the probability that, given an output code k[i], the corresponding ideal code is x[i]. So, it can be used to estimate the ideal codes from the actual ones. In such a way it is possible to recover the distortion caused by the ADC characteristic nonlinearity.

The method can be summarized as follows [2]:

1. Add a dithering signal with a low peak-peak amplitude to the input signal.
2. Oversample the signal by an opportune factor.
3. Model the channel by using the p(k[i]/x[i]).
4. Determine the p(x[i]/k[i]).
5. Determine the most probable input sample x_e[i].
6. Remove the dithering by a low-pass filtering.

The dithering can be removed by a simple low-pass FIR filtering [6] carried out by using a moving window with length L_s.

Finally, the following formula can be used in order to obtain the correct output [2]:

$$\begin{aligned}\bar{x}[i] &= \frac{1}{2L_s+1}\sum_{j=-L_s}^{L_s} x_e[i+j] = \\ &= \frac{1}{2L_s+1}\sum_{j=-L_s}^{L_s}\sum_{x[i]=0}^{2^{N-1}} x[i]\frac{p(k[i+j]/x[i+j]).p(x[i+j])}{p(k[i+j])}\end{aligned} \qquad (2)$$

which can be simplified by assuming that the probability p(x[i]) can be approximated by p(k[i]).

Assuming that p(k[i]) is evaluated by drawing the output histogram, and p(k[i]/x[i]) is evaluated as above quoted, the following approximated formula can be used [2]:

$$\bar{x}[i] \approx \frac{1}{2L_s+1}\sum_{j=-L_s}^{L_s}\sum_{x[i]=0}^{2^{N-1}} x[i].\, p(x[i+j]/k[i+j]) \cdot \qquad (3)$$

A first evaluation of the proposed approach on simulated signals was carried out by adding a high frequency dithering signal to a triangular wave input. Such simulations, with a dither peak-peak value lower than an LSB, gave good results on a simulated ADC, modelled with a known polynomial INL [2]. In particular, when a FIR filter with a small L_s was used, the Bayesan filtering assured some reduction of the quantisation noise floor at higher frequencies, while it caused a slight increase of the noise effect at lower frequencies. On the contrary, it caused a noise reduction over all frequencies by using a greater L_s. Thus, a significant ENOB improvement has been achieved, provided a suitable selection of the L_s. The price to pay is, of course, the oversampling and the post-processing time.

3. Experimental results

The simulations shown that the efficiency of the proposed Bayesan filtering method depends on the precision with which the conditioned probabilities *p(x[i]/k[i])* are calculated. Moreover, it has been observed that a relevant increase in the window width L_s significantly decreases the level of spectral components at higher frequencies while slightly decreases the ones at lower frequencies.

In order to assess the efficiency of the proposed method a proper figure of merit was proposed. It is an effective number of bits defined by the formula:

$$G\_ENOB = \frac{1}{2}\log_2\frac{e_r^2}{e_{out}^2} \qquad (4)$$

where e_r is the actual quantisation noise power on the ADC output without using the proposed method. The e_{out} is the quantisation noise power after dithering, Bayesan estimation and FIR filtering. The same figure of merit has been used for the method validation on the actual data. A suitable test bench has been set up in order to proof the method correction capabilities on an actual ADC as reported in Fig.2. In particular, a Stanford Research DS360 generator has been used to produce the test signal, which have been chosen to be a sine wave. A 12 bit data acquisition board (DAQ) LAB-PC-1200 from National Instruments has been used for the A/D conversion. The DAQ input-output

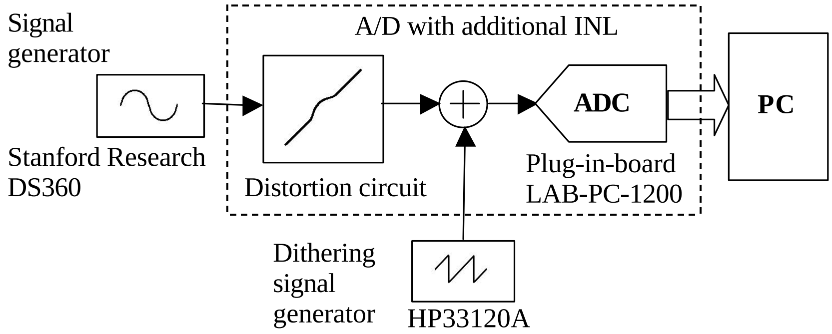

Fig.2. Test setup for the method validation on actual data.

characteristic has been considered ideal while the distorted characteristic has been emulated by means of a self built pre-processing circuitry. Assuming that the emulated INL is much greater than the native DAQ one, the system will correctly emulate the behaviour of an actual ADC. The dithering signal has been generated by using an HP33120A generator.

The pre-processing circuitry has been built with the capability of adjusting the INL position over the full-scale range of the DAQ A/D. Moreover, it is possible to set its amplitude and width.

In Fig. 3, an example of INL generated in such a way is given.

For the purposes of the present work, a sinusoidal test signal has been generated with a 10 V_{pp} amplitude and a 21.777 Hz frequency. The dithering signal has been generated as a triangular wave with a 40 kHz frequency and 5 mV_{pp} amplitude. The DAQ input range is ± 5 V while the sampling frequency has been set to 100 kHz.

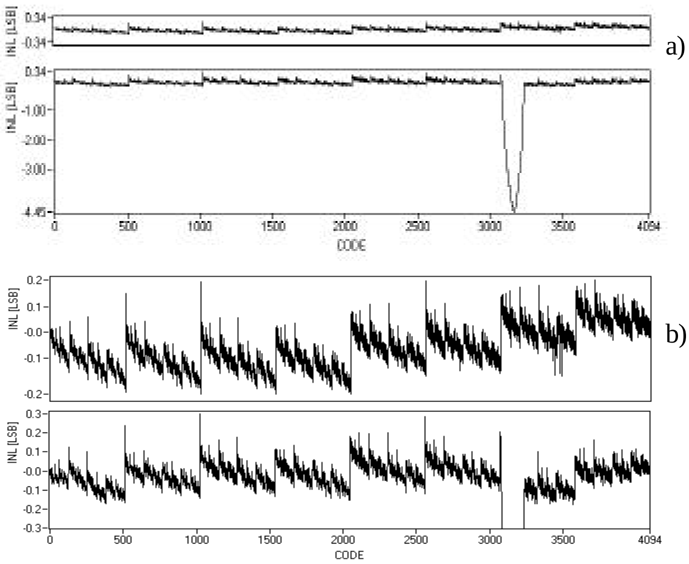

Fig. 3. a) An example of INL characteristics of the emulated A/D without and with the added nonlinearity. b) Zoom on both characteristics shown in a).

Fig. 4 shows a zoom on the input sine wave corrupted by an imposed INL with 15 LSB amplitude. It can also be seen the presence of the dithering.

On the same figure, the output signal, obtained by applying the proposed method with an L_s = 50 is shown. As it can be seen, the proposed Bayes interpolation can restore large scale nonlinearities without using a large amplitude dithering signal.

In order to assert the influence of the window length L_s also on the actual data, an investigation has been carried out by varying L_s. In Fig.5 the resulting corrected outputs are reported in the time domain for L_s = 2, 10 and 50 for a visual comparison. The same outputs are reported in the frequency domain in Fig. 6. The signal spectrum has been obtained by acquiring a 32768 samples record and using an FFT with a Hanning window.

As it can be seen from Fig. 6, the proposed method application on the actual data can reduce the noise floor by 15 to 20 dB at higher frequencies with an L_s = 10 low pass filtering, and more with an L_s = 50, while, its effect is almost negligible with a short L_s. It should be noticed that an increase of L_s leads to an attenuation of the noise also to lower frequencies, as it can be retrieved from the comparison of Fig. 6c and d.

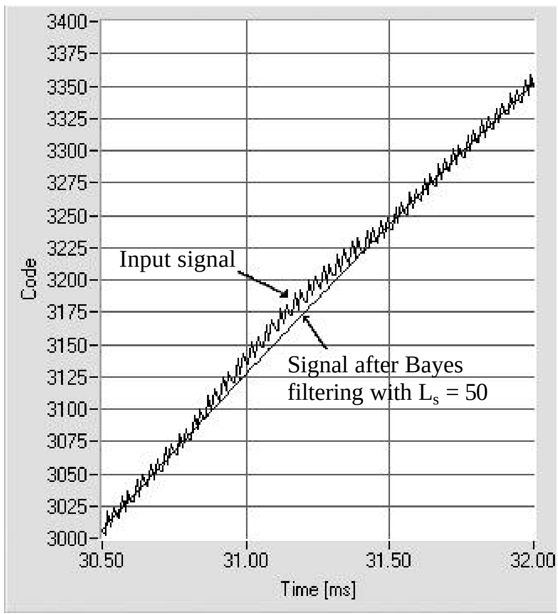

Fig. 4. The input signal after the application of an INL of 15 LSB and the dithering, and the output one, obtained by the application of the proposed method.

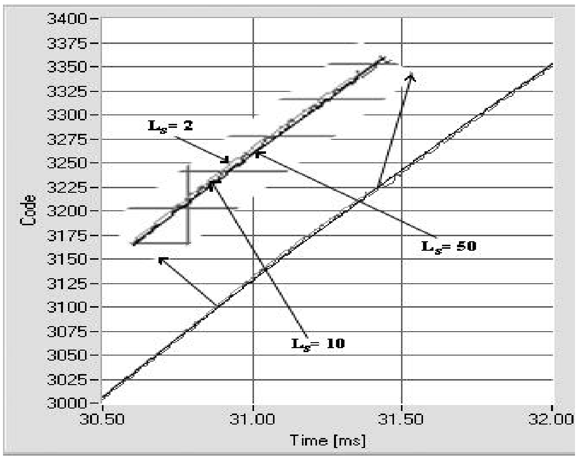

Fig. 5. Time domain comparison of the corrections obtained by using a window length L_s= 2, 10 and 50.

4. Conclusions

A suitable Bayesan filtering based method has been proposed with the aim of suppressing the spectral components of the output noise due to the INL presence on the ADCs. In that direction the proposed method allows to use a dithering signal with small peak - peak value to suppress large scale nonlinearity errors. Moreover, it offers another advantage: the distortion is significantly reduced by using a smoothing FIR filter with a length starting from L_s=10. This enables fast calculations to obtain an output spectrum with a good correction of the INL effects.

In accordance with the simulation results, the experimental ones show that a consistent increase of the window length causes the decrease of spectral components not only at high frequencies but also at lower ones. In this case the proposed method increases the G_ENOB.

At this time the main constrains which limit a time efficient implementation of the proposed method regard the calculation of the p(x) probabilities for a sine wave signal. A faster algorithm and the symmetry of the error features could accelerate the digital processing time.

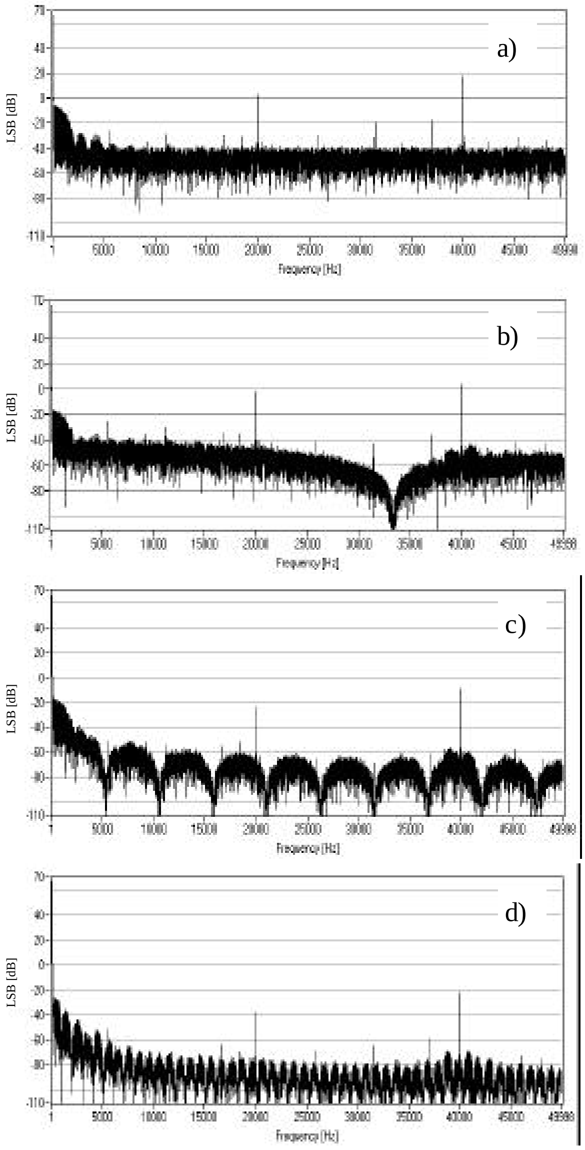

Fig. 6. Amplitude spectrum of (a) the input signal; and of the output signal after the method application with (b) L_s=2; (c) L_s=10; (d) L_s=50.

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