THE EQUALITY OF QUANTIZATION WITH DITHER SIGNAL AND OVERSAMPLING
FROM THE POINT OF VIEW OF THE CRITERION OF *SNR*

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Abstract: SNR is a characteristic of analog-to-digital (A/D) conversion. One of its applications is the valuation of the "effect" of the change of the resolution of A/D conversion. SNR may be improved by modifying the process of discretization in terms of time (sampling) and value (quantization).

The present article discusses the equality of these two types of modification from the point of view of the criterion of SNR.

Keywords: quantization, dither, oversampling, signal-to-noise ratio (SNR).

1. INTRODUCTION

The characteristics of an analog-to-digital (A/D) converter clearly define the class of signals that may be converted and determine the accuracy of the applied measuring systems. Attempts at overcoming the technological limitations of A/D converters and at developing a procedure for adapting the characteristics of a-d conversion to the type of signal and of the measurand, have made it possible to design "smart" a-d systems [1].

The adaptation of the characteristics of a-d conversion to the type of signal involves the selection of a proper resolution of the conversion.

A-d conversion is a variety of a-d transformation in which the signal is sampled and quantized, i.e., it undergoes an actual discretization in terms of time and value. The "effect" of the change of the resolution of a-d conversion may be achieved by modifying the process of discretization in terms of time (by applying oversampling and low-pass filtering) or in terms of value (by applying quantization with dither signal). The present paper discusses the equivalence of the two types of modification, with SNR as the criterion of comparison. Only cases of ideal sampling and quantization are considered.

2. MODIFICATION OF THE DISCRETIZATION

If the resolution of the transformation is of N bits, then SNR is expressed by the equation [2]:

$$SNR_0 = 6,02 \cdot N + 1,76 \quad [dB] \quad (1)$$

The term refers to the fact that the only noise source being considered is quantization noise and RMS signal value is

$$U_{RMS} = \frac{1}{\sqrt{2}} \frac{U_{pp}}{2} = \frac{1}{\sqrt{2}} \frac{2^N q}{2}.$$

A modification of the discretization in terms of value consists of applying quantization with dither signal. If the variance of the dither signal equals σ_d^2, and the signal is averaged n times after the A/D conversion, then SNR changes with respect to SNR_0, amounting to:

$$SNR_1 = 6,02 \cdot N + 1,76 + 10 \log \frac{n}{1 + \left(\dfrac{\sigma_d}{\sigma_0}\right)^2}$$

$$[dB] \quad (2)$$

where σ_0^2 represents the variance of the quantization error. The probability density function of a dither signal is symmetrical. In order to ensure that the samples are not correlated, the variance of a dither signal can not be far too small.

SNR_1 is also a characteristic of an A/D conversion with a hypothetical resolution of N_1 bits:

$$SNR_1 = 6,02 \cdot N_1 + 1,76 =$$
$$= 6,02 \cdot N + 1,76 + 6,02 \cdot \Delta N_1 \quad [dB] \quad (3)$$

Accordingly, the "effect" of change of resolution is expressed by the equation:

$$\Delta N_1 = 1,66 \log \frac{n}{1 + \left(\dfrac{\sigma_d}{\sigma_0}\right)^2} \quad [dB] \quad (4)$$

It will be increase resolution "effect" when ΔN_1 is higher than 0. Thence, the multiplicity factor of averaging should be selected in such a way that the condition in Equation 5 is met:

$$n > 1 + \left(\frac{\sigma_d}{\sigma_0}\right)^2 \quad (5)$$

A modification of the discretization in terms of time consists of applying oversampling. If the cut-off frequency of a low-pass signal amounts to f_B, and the frequency of sampling, is equal f_S, then SNR

changes with respect to SNR_0, amounting to [3]:

$$SNR_2 = 6,02 \cdot N + 1,76 + 10\log\frac{f_S}{2f_B} \quad \text{[dB]} \quad (6)$$

$$f_S \gg 2f_B$$

SNR_2 is also a characteristic of an A/D conversion with a hypothetical resolution of N_2 bits:

$$\begin{aligned}SNR_2 &= 6{,}02 \cdot N_2 + 1{,}76 = \\ &= 6{,}02 \cdot N + 1{,}76 + 6{,}02 \cdot \Delta N_2\end{aligned} \quad \text{[dB]} \quad (7)$$

Accordingly, the "effect" of change of resolution is expressed by the equation:

$$\Delta N_2 = 1{,}66\log\frac{f_S}{2f_B} \quad \text{[dB]} \quad (8)$$

3. THE EQUALITY OF QUANTIZATION WITH DITHER SIGNAL AND OVERSAMPLING

The equality of the two methods (dithering and oversampling) from the point of view of the criterion of SNR may be expressed by the equation:

$$\Delta N_1 = \Delta N_2 \quad (9)$$

from which it is inferred that:

$$\frac{n}{1 + \left(\dfrac{\sigma_d}{\sigma_0}\right)^2} = \frac{f_S}{2f_B} \quad (10)$$

Assuming that $f_S = M \cdot 2f_B$ and $\sigma_d / \sigma_0 = k$, it follows that

$$\frac{n}{1 + k^2} = M \quad (11)$$

M – oversampling factor

Table 1 summarizes the conditions in which resolution increases by ΔN under each method.

Table 1

ΔN	n	M	f_S [MHz] (f_B=20 kHz)
2	$16 \cdot (1 + k^2)$	16	0,64
4	$256 \cdot (1 + k^2)$	256	10,24
6	$4116 \cdot (1 + k^2)$	4116	164,64
8	$65960 \cdot (1 + k^2)$	65960	2638,40

As we can see, both methods equally easily produce a small ΔN. The higher the required ΔN, the more advantageous it is to apply the modification of discretization in terms of value. It is possible to realize it on two

manners - through increasing the multiplication factor of averaging n or the variance of dither signal σ_d^2. If the modification of discretization in terms of time is applied and a higher ΔN is required, then the sampling frequency must be raised to a sufficiently high value. Thus, e.g., to produce a $\Delta N = 8$, the sampling frequency must rise from 40 kHz to 2.64 GHz if the signal has the cut-off frequency $f_B = 20$ kHz.

4. CONCLUSIONS

SNR may be improved in the signal band by applying either oversampling and low-pass filtering (SNR_2) or quantization with dither signal and with a multiplicity factor of averaging n suitable for the ratio of the variance of the dither signal to the variance of the quantization error (SNR_1).

"Smart" a/c conversion systems are expected to allow a choice of the applied method of the modification of discretization - either in terms of time or in terms of value.

Difference exists between dithering and oversampling. Using dither we have not only influence on SNR but also on change of the model of operation of quantization (improvement of reconstruction conditions for quantization in Widrow sense, reduction of the non-linear distortion, reduction of the nonlinearity errors of a real A/C converter).

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