A NEW ROBUST FOUR PARAMETER SINE FITTING PROCEDURE

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ABSTRACT

In this paper a new procedure to perform four parameters sine fitting is presented in a closed form, ready for standardization. This new procedure grants convergence of the algorithm, even in those cases where the traditional techniques tend to converge to local minimums of the error function.

This new procedure can also be used to compute the initial values for the traditional four parameters sine fitting algorithm if very high accuracy is required. In such case convergence of the traditional algorithm is also assured.

Keywords: ADC Testing, Sine Fitting.

I. INTRODUCTION

Sine fitting algorithms are traditionally used in Analog to Digital Converters (ADCs) testing to obtain the effective number of bits (ENOB) and signal to noise and distortion ratio (SINAD). These algorithms are also used in time base tests of digitizers when the internal sampling clock is not available to the user to measure aperture uncertainty and fix error in sampling. More recently they were proposed to obtain INL, DNL and the transfer function of ADCs, namely when they present a hysteric behavior [1].

Sine fitting is a very efficient and fast way to measure ENOB and SINAD. The main problems with this testing procedure are the difficulty on assuring coherent sampling in some experimental conditions and divergence of the four parameters algorithm when the initial values of amplitude, offset, phase and especially frequency are incorrectly estimated.

In order to assure coherent sampling, input (f) and sampling frequencies (f_s) must be choose in such a way that a record of M samples with uniformly distributed phases are acquired. This will be accomplished if the frequencies are related by:

$$f = \frac{J}{M} f_s \tag{1}$$

and M and J are relatively prime, this meaning that they have no common factors and that there are exactly J cycles in the record. If M is a power of two, an odd value for J assures coherent sampling. If the signal frequency meets the above condition, the maximum phase difference between successive sampled phases will be $2\pi/M$. In this condition, an ideal n bits ADC, in the absence of random noise, requires a record size greater or equal to $2\pi 2^n$ to ensure that every code bin is stimulated.

There are many algorithms for performing the least squares curve fits of a sine wave. IEEE Standards 1057 [2] and 1241 [3] present two methods which estimate three (A, B and C) or four (A, B, C and f) parameters of a sine wave, defined as $A\cos(2\pi f t_n)+B\sin(2\pi f t_n)+C$, that fit a set of M samples, $y_1,\ldots,y_M$, acquired at a frequency $f_s=1/T_s$. The residuals, r_n, of the fit are given by

$$r_n = y_n - A\cos(2\pi\, f t_n) - B\sin(2\pi\, f t_n) - C\,. \tag{2}$$

These sine fitting algorithms seek the minimum of r_n, but must be used with some precautions and the use of records containing at least five cycles is recommended.

If the ratio between signal and sampling frequency is unknown (or inaccurately known), the four parameter sine fitting algorithm should be used. It seeks solutions of a nonlinear system of equations, which must be solved in an iterative way. From initial estimated values for the frequency and the other three parameters, A, B and C, the algorithm produces a new set of values A_i, B_i, C_i and a correction Δf_i to the frequency to be used in the next iteration. The main problem of this algorithm is that its results are highly dependent on the number of samples and especially on the initial estimated values, including naturally the frequency. The error function presents local minimums in predictable but almost surprising places [4-6]. Occasionally the algorithm generates corrections to the frequency that lead to convergence to local minimums and consequently to erroneous solutions.

The parameter estimation is exposed to errors due to different causes like small number of samples, small number of samples per period, noise and distortion.

In order to understand what could be done to increase its performance an intensive study of the algorithm behavior in different conditions was carried out [6,7]. Fig.1 shows the results obtained by using the three parameter algorithm for the relative error of the amplitude of the ac component, defined as

$$\varepsilon_A = \varepsilon_{rms} / \sqrt{(A^2 + B^2)/2}\,, \text{ where } \varepsilon_{rms} = \sqrt{\sum_{n=1}^{M} r_n^2 / M}\,, \tag{3}$$

as a function of the estimated frequency and of the number of samples, for a pure sine wave digitized by an ideal ADC with an infinite number of bits. It can be seen that when the estimated frequency diverts from the correct value the error increases, reaching the maximum of 100% at approximately a number of samples $N=f_s/\Delta f_0$, where f_s is the sampling frequency and Δf_0 is the absolute error on the estimation of the frequency f. From this point, local minimums of ε_A are found. Their location depends on the number of samples, M. If the sampling interval contains a large number of periods of the signal under test, the gap between the global minimum and the adjacent maximums is approximately equal to $f \pm k f_s/M$ [4-6], where k is a positive integer.

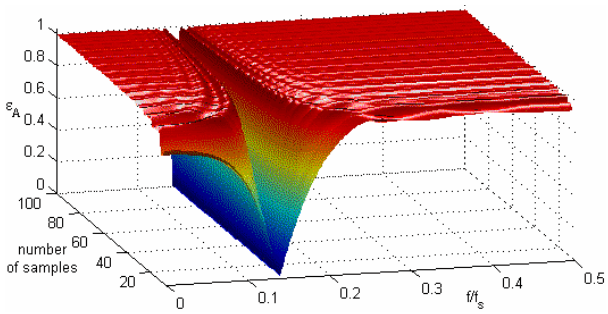

Fig. 1. Three parameter algorithm error ε_A as a function of the number of samples and the estimated frequency, for f_0=0.14247f_s.

II. INCREASING CONVERGENCE OF THE TRADITIONAL FOUR PARAMETER SINE FITTING ALGORITHM

When the frequency used as input for a given iteration of the four parameter algorithm is close to $f \pm f_s/M$ there is a high probability that the algorithm will converge to a local minimum of the error function [6,7]. This will depend on the number of samples, on the values considered for A, B, C and also on the initial phase of the acquired sinewave.

In order to avoid these undesirable situation a few measures described in the next paragraphs can be taken.

First of all <u>frequency variation restrictions should be imposed</u>. Considering that the initial estimation of the frequency is the result of the application of an FFT, the maximum error on frequency estimation is f_s/M. Since the maximums of the error function of the 3 parameter algorithm closest to f are approximately at $f \pm f_s/M$, by limiting frequency corrections to a small fraction of f_s/M we will assure that the frequency is not "over-corrected". Small frequency changes avoid any "over-correction" but will increase the number of iterations. Normally 10% to 25% of f_s/M should be enough, depending on the SINAD.

Figure 1, shows that when the number of samples is very small, the maximum of ε_A closest to f is located at a frequency very far from f. This suggests <u>the use of a small number of samples in the initial iterations</u> in order to avoid convergence to local minimums. Since the order of magnitude of the absolute minimum value of ε_A (dependent on SINAD in each case) can be evaluated from the amplitude spectrum through the FFT, a strategy to increase the number of samples from one iteration to the next can be established by comparing it with the value obtained for ε_A in each iteration. The entire number of samples, M, should be considered on the final iterations since they correspond to the highest sensitivity in the application of the algorithm as can be seen in figure 1. This will assures that the frequency is not "over-corrected" but, on the contrary, it is modified towards the absolute minimum.

<u>The initial frequency to use in the 4 parameter algorithm can be estimated by linear regression</u>, as illustrated in fig. 2. Assuming that the set of samples contains at least one period, the closest maximums in relation to the correct frequency occur approximately for $f \pm f_s/M$ (points L and R in fig. 3) and beyond these maximums the error tends to a large constant value, $\varepsilon_A \cong 1$ when the number of samples is bigger than 1 period; but between these points, the curve may be approximated by two straight lines intercepting in a point which abscissa is the correct frequency and the coordinate is approximately the total distortion of the sine wave; the bigger the number of samples the better will be the approximation.

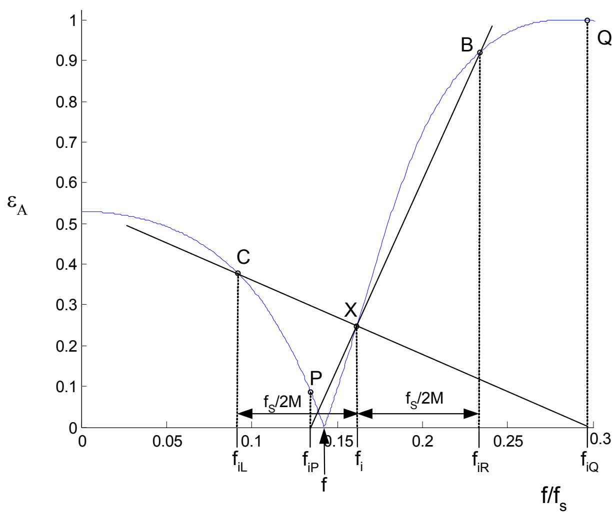

Fig. 2 – Three parameter algorithm error ε_A as a function of the estimated frequency, for 7 samples in the conditions of fig. 1. Estimating the correct frequency value by linear regression as explained in the text.

III. THE NEW PROCEDURE

Based on the measures proposed in the former section the following procedure (see flowchart in fig. 4) to perform four parameter sine fitting or to estimate accurately the initial frequency, amplitude, offset and phase to be used in the traditional algorithm in order to improve convergence, can be established:

(1) Compute the DFT for the record containing all the acquired samples for a first estimation of the stimulus signal frequency (f_0).

(2) In order to minimize spectral leakage, consider the bigger subset of the M samples containing an integer number of periods (k/f_0) plus or minus one sample. With this subset compute DFT again to obtain a new estimation of the input frequency (f_1) and the rms value for noise and distortion (ε_{Amin}). Remember that when f does not satisfy the coherent sampling condition, equation (1), each harmonic of f occupies a band of frequencies instead of a single frequency. This second step is optional, however we recommend it to be performed for a more accurate computation of ε_{Amin}. The estimated frequency, f_1, will be located in the interval bounded by the frequencies correspondent to points L and R in fig. 3.

(3) Define the test condition to stop the round of iterations: minimum frequency correction (δf) and difference between ε_A and the value estimated by the DFT ($\delta\varepsilon_A$).

(4) In order to avoid convergence to local minimums of the error function consider a subset N_1 of the M acquired samples containing approximately one period of the input sine wave.

(5) Let i=1 and α=1000.

(6) Use f_i as input frequency to the three parameter algorithm applied to N_i samples. This will result in a certain amplitude error, ε_i, represented by point X in fig.2

(7) Compute $\alpha_i = \dfrac{\varepsilon_i}{\varepsilon_{i-1}}$. If α_i<1 (it can not happen for i=1) make N_i=N_{i-1} and return to point (6).

(8) Repeat the three parameter algorithm for frequencies half way the maximum frequency error of the DFT, that is $f_{iL}=f_i-f_s/2M$ and $f_{iR}=f_i+f_s/2M$, corresponding respectively to points B and C and leading to ε_{iR} and ε_{iL}. Note that in fig. 2 the value of $f_s/2M$ is exaggerated for illustration purposes.

(9) Perform a linear regression with the two pair of points XB and XC. Let us consider the first pair, located on the same side of the curve relatively to the correct frequency. By the interception with the horizontal axis (ε_A=0) a new frequency $f_{iP} = f_i - \dfrac{\varepsilon_i\left(f_i - f_{iR}\right)}{\varepsilon_i - \varepsilon_{iR}}$ is obtained. Due to the shape of the concavity of the error function the correct frequency will be located between f_i and f_{iP}. The error of the three parameter algorithm at point P will be ε_{iP} eventually greater than ε_i but surely smaller than ε_{iR}. If instead of pair XB, we have used the pair XC, a frequency f_{iQ} would have been obtained; this new frequency should be eliminated since $|f_1-f_{iQ}|>f_s$/M or/and the three parameter amplitude error associated, ε_{iQ}, is probably greater than ε_{iL} but surely greater than ε_i and ε_{iP}.

(10) The better approximation of the input frequency f will be the frequency f_{iB} corresponding to the minimum of (ε_{iL}, ε_{iR}, ε_{iP} and ε_{iQ}). The corresponding error will be designated by ε_{iB}.

(11) Let i=i+1. Estimate a new the frequency by evaluating the weighted average: $f_i = \dfrac{f_{i-1}\varepsilon_{(i-1)B} + f_{(i-1)B}\varepsilon_{i-1}}{\varepsilon_{i-1} + \varepsilon_{(i-1)B}}$.

(12) If N_i is already equal to M than verify the two simultaneous conditions to stop the round of iteration: ($|f_I-f_{i-1}|<\delta f$) and ($\varepsilon_i-\varepsilon_{Amin}<\delta\varepsilon_A$); otherwise compute a new value for the number of samples to use in the three parameters algorithm through N_i= N_{i-1}(2-α_i) and return to point (6).

(13) Use f_i to estimate A, B, and C by applying one last time to the three parameter algorithm.

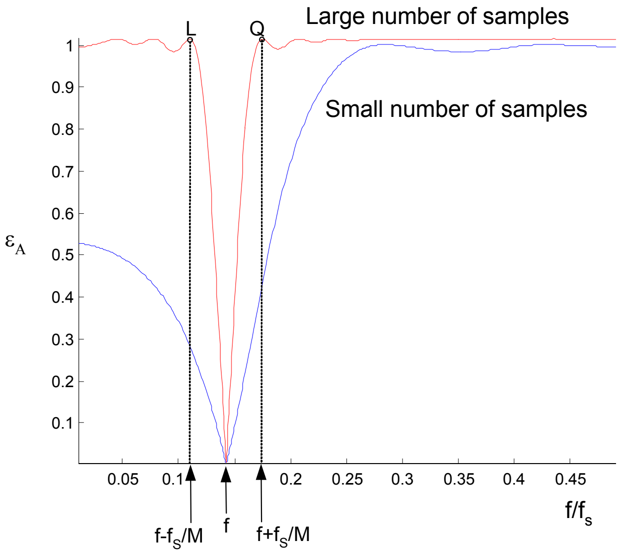

Fig. 3 – Three parameter algorithm error ε_A as a function of the estimated frequency, for a small (7 samples) and a larger number of samples (M=30)

Our experience shows that applying this set of values as an input to the traditional four parameter algorithm very

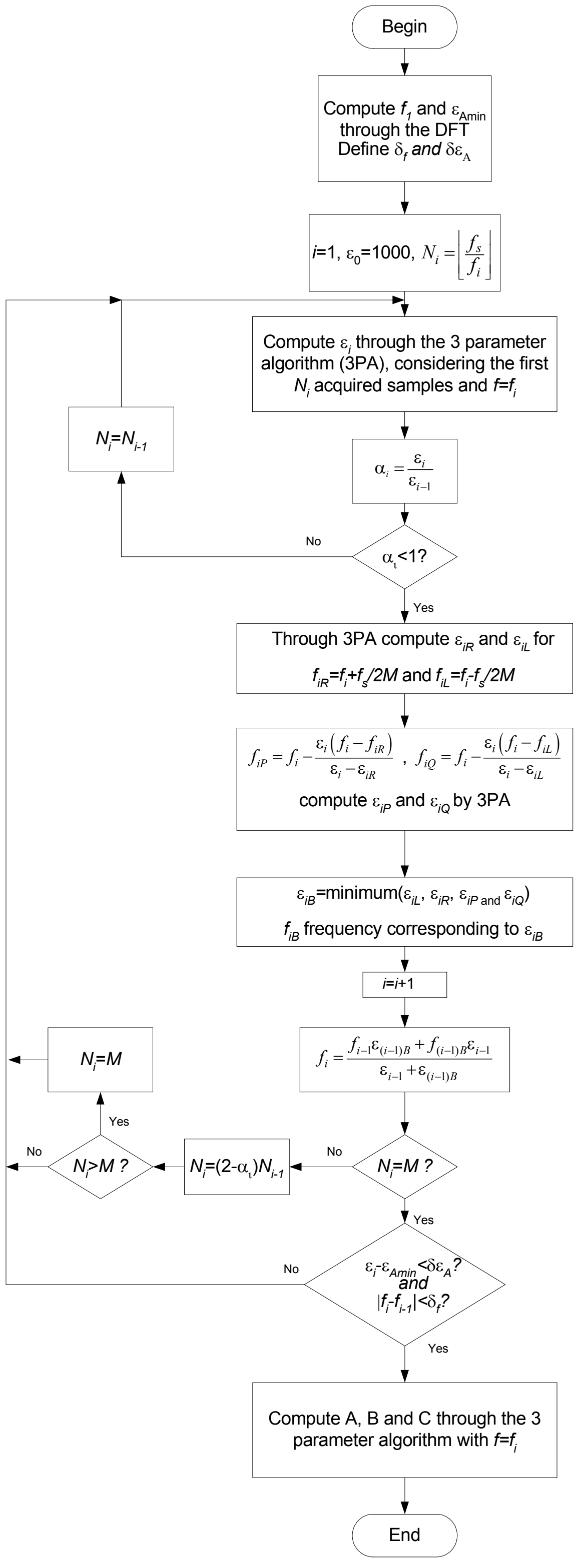

Fig. 4 – Flowchart of the proposed procedure.

small corrections will be done and within 2 or 3 iterations all the parameters will stabilize in their final values, the frequency correction being less than $10^{-8}f_s$.

A large number of tests were performed through experimentally acquired and numerically generated sample records. In all cases a fast convergence was achieved, even for situations with rms additive noise values as bigger as 20% of the sinewave rms value. In those tests, values of δf as small as $10^{-10}f_s$ were considered.

IV. CONCLUSIONS

A new four-parameter sine fitting technique that grants convergence to the absolute minimum of the error function was proposed. The procedure is based on the use of the traditional three parameter sine fitting algorithm performed with an increasing number of acquired samples, in order to avoid convergence to local minimums of the error function, together with a linear regression technique that assures a fast convergence.

The present technique performed extremely well in adverse conditions and was also successfully applied in impedance measurement system.

V. ACKNOWLEDGEMENTS

This work was sponsored by the Portuguese national research project entitled “New measurement methods in Analog to Digital Converters testing”, reference PCTI/ESE/32698/1999, whose support the authors gratefully acknowledges.

VI. REFERENCES

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