UNCERTAINTY IN THE ADC TRANSITION VOLTAGES DETERMINED WITH THE HISTOGRAM METHOD

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Abstract – *This paper, presents an innovative study of the uncertainty in the ADC transition voltages, determined by the Histogram Method for ADC testing. The new approach used, allows for a global and uniform overview of this subject, improving the existing knowledge about the workings of the Histogram Method.*

Keywords – ADC Test, Histogram Method, Transition voltages.

1. INTRODUCTION

The value of a code transition voltage T_k of an analog to digital converter (ADC) is, by definition, the value of a constant input signal that causes half the samples acquired to have an output code equal to or higher than code k [1]. Although the transition voltages are, by definition, deterministic, the values of those transitions, determined by the Histogram method, are random. This is due to several factors that influence the result of the test and that can be grouped in three classes:

1) *Input equivalent wideband noise*. This factor models both amplitude noise sources inside the ADC and unwanted disturbances corrupting the stimulus signal [2].
2) *Phase noise*. This factor represents the phase noise of the stimulus signal generator, the phase noise of the sampling signal generator and the ADC aperture uncertainty.
3) *Initial stimulus signal phase*. In the case of asynchronous sampling represents the phase of the stimulus signal at the instant of acquisition of the first sample. This will vary with uniform distribution from record to record of acquired samples [3].

The value obtained for a particular transition voltage will generally not be equal to the actual transition voltage of the ADC. It is possible however to define an interval where the correct value of the transition voltage will be contained with a certain degree of confidence. To define that interval the probability density function of the determined transition voltage must be known. The interval can be made arbitrary small by increasing the number of acquired samples.

In 1984 Doernberg et al. studied this problem in the case of the determination of the code bin widths with the Histogram method [4]. That work took into account only the effect of input-equivalent noise with a variance much greater than the ideal code bin width. In those conditions the sampling voltage was considered uniformly distributed inside each code bin.

Later on, in 1994, Blair produced great advancements in this field [5]. In his work, both the effects of input-equivalent noise and random phase difference between the stimulus signal and the sampling clock were considered. Uncertainty expressions for the code bin width as well as for the transition voltages were presented and included in the de facto standard [1].

The effect of phase noise, not taken into account by Blair, was studied in 1999 by Chiorbolli et al. [6]. An addition to the expressions from Blair was proposed to take into account this effect.

The analysis of the uncertainty of the code bin widths and transition voltages done in the works referred, took into account each of the three effects separately and the results obtained were added to each other. In this paper a unified approach is presented that studies the influence of these effects together providing a more precise view of the contribution of each effect to the uncertainty of the measured transition voltages. The validity of approximations used in the previous works are analyzed and a new expression for the variance of the measured transition voltages is proposed.

2. TEST BENCH

In the traditional histogram method, a full-scale periodic stimulus signal is applied to the ADC and a certain number of samples are acquired at a constant rate asynchronously with the stimulus signal. The value of the ADC transition voltages is obtained by comparing the number of codes obtained in each code bin with the number expected in the case of an ideal ADC. To determine this last number it is necessary to know the probability distribution of the sample voltages.

Consider each sample j (j=0,1,…,M-1) ideally acquired at instant t_j. Without loss of generality the time origin can be set to the ideal sampling instant of the first sample ($t_0 = 0$). The phase of the samples (γ_j), relative to the stimulus signal of frequency f, is thus

$$\gamma_j = 2\pi f \cdot t_j + \theta + \varphi \quad (1)$$

where θ represents the phase noise and φ the phase of the stimulus signal at the ideal instant of acquisition of the first sample.

The value of the sinusoidal stimulus signal in the sampling instant of sample j can be written as

$$x_j = d - A \cdot \cos\left(2\pi f \cdot t_j + \theta + \varphi\right) \quad (2)$$

where d and A are the stimulus signal offset and amplitude. The values of the sampled voltages (v_j) are equal to the value of the stimulus signal in the sampling instant (x_j) plus the input-equivalent wideband noise (n_v).

$$v_j = n_v + d - A \cdot \cos\left(2\pi f \cdot t_j + \theta + \varphi\right) \quad (3)$$

To simplify the computations, some normalizations are made. Let u_j be the normalized sample voltage.

$$u_j = \frac{v_j - d}{A} = \frac{n_v}{A} - \cos\left(\gamma_j + \theta\right) = n - \cos\left(\gamma_j + \theta\right) \quad (4)$$

where n is the normalized input-equivalent wideband noise and γ_j is the sample phase in the absence of phase noise.

$$\gamma_j = 2\pi f \cdot t_j + \varphi \quad (5)$$

3. GENERAL SAMPLING

In this section the determination of the probability distribution of the number of counts of the cumulative histogram is presented for a general type of sampling (random, synchronous or asynchronous). In the next paragraph the particular case of asynchronous sampling will be considered.

3.1 Probability Density Function of the Sample Voltages

The phase noise can be considered normally distributed with a null mean and a standard deviation σ_θ. Since the normalized value of the stimulus signal in the sampling instant (y_j) is a function of the phase noise

$$y_j = s(\theta) = -\cos\left(\gamma_j + \theta\right) \quad (6)$$

the conditional probability density function (p.d.f.) of y_j can be determined from the p.d.f. of the phase noise [7].

$$f_{y_j}\left(y \mid \gamma_j\right) = \sum_{m=-\infty}^{\infty} \frac{f_\theta\left(\theta_m\right)}{\left|s'\left(\theta_m\right)\right|} \quad (7)$$

where θ_r are the roots of equation (6). Because in each period m of the stimulus signal there are two roots, equation (7) can be rewritten as

$$f_{y_j}\left(y \mid \gamma_j\right) = \sum_{m=-\infty}^{\infty} \frac{f_\theta\left(\theta 1_m\right)}{\left|s'\left(\theta 1_m\right)\right|} + \frac{f_\theta\left(\theta 2_m\right)}{\left|s'\left(\theta 2_m\right)\right|} \quad (8)$$

with the roots given by

$$\theta 1_m = \operatorname{acos}\left(-y\right) - \gamma_j + 2\pi m \quad \theta 2_m = -\operatorname{acos}\left(-y\right) - \gamma_j + 2\pi m \quad (9)$$

Substituting (9) in (8) and considering the stimulus signal derivative $s'\left(\theta\right) = \sqrt{1-y^2}$ we obtain:

$$f_{y_j}\left(y \mid \gamma_j\right) = \frac{\sum_{m=-\infty}^{\infty}\left[e^{-\frac{\operatorname{acos}(-y)-\gamma_j+2\pi m}{2\sigma_\theta^2}} + e^{-\frac{-\operatorname{acos}(-y)-\gamma_j+2\pi m}{2\sigma_\theta^2}}\right]}{\sigma_\theta\sqrt{2\pi}\sqrt{1-y^2}} \quad (10)$$

The p.d.f. of the value of the stimulus signal at the sampling instant is represented in Fig. 1.

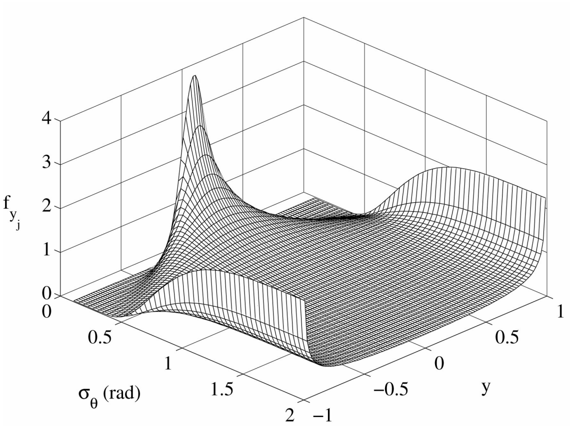

Fig. 1 Representation of the p.d.f. of the value of the stimulus signal in the sampling instant for a sample phase $\gamma_j=\pi/2$ rad as a function of the standard deviation of the phase noise (σ_θ).Note that the case of null phase noise standard deviation ($\sigma_\theta=0$) is not represented because it would imply an infinite value for the p.d.f. for $y=0$.

For values of γ_j different from the considered in Fig. 1 the maximum of f_{y_j} for small values of σ_θ, occurs for different values of y, given by (6) when $\theta=0$. However, as the standard deviation of the phase noise increases, f_{y_j} becomes independent of γ_j.

Considering also that the input-equivalent noise is normally distributed, with a null mean and a standard deviation σ_n, the p.d.f. of the sample voltages (u_j) can be determined by convolving the p.d.f. of the noise (f_n) with the p.d.f. of the value of the stimulus signal in the sampling instant (f_{y_j}) since these variables are independent.

$$f_{u_j}\left(u \mid \gamma_j\right) = f_n * f_{y_j}\left(u\right) \quad (11)$$

In Fig. 2, f_u is represented as a function of the phase noise standard deviation (σ_θ).

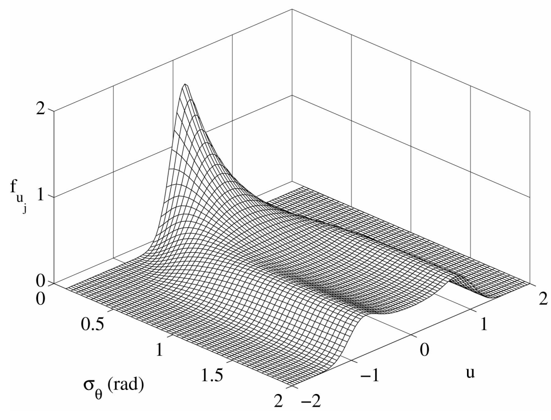

Fig. 2 Representation of the p.d.f. of the sample voltage for a sample phase $\gamma_j=\pi/2$ rad and $\sigma_n=0.2$. This value of σ_n means that the standard deviation of the input-equivalent noise is 5 times lower then the stimulus signal amplitude. Note that the case of null phase noise standard deviation ($\sigma_\theta=0$) is not represented because it would imply an infinite value for the p.d.f. for $u=0$.

3.2 Sample Code

The probability that a sample j belongs to a class k of the cumulative histogram (p_k) is equal to the probability that the normalized sample voltage is lower than or equal to the normalized transition voltage $U[k+1]$ which, by definition, is the value of the probability distribution function of the sample voltage evaluated at $U[k+1]$ (Fig. 3):

$$p_k\left(\gamma_j\right)=F_{u_j}\left(U\left[k+1\right]\mid\gamma_j\right)=\int_{-\infty}^{U[k+1]} f_{u_j}\left(u\mid\gamma_j\right)\cdot du \quad (12)$$

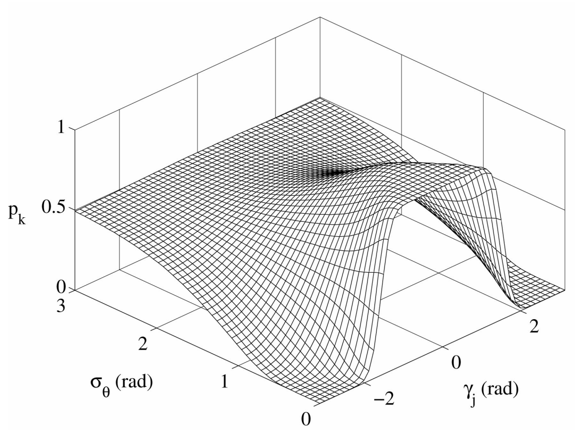

Fig. 3 Representation of the probability that a sample belongs to a class of the cumulative histogram as a function of the sample phase (γ_j) and of the input-equivalent noise standard deviation (σ_θ). Example with $U[k+1]=0$ (transition voltage equal to the DC value of the stimulus signal) and $\sigma_n=0.2$ (standard deviation of the input-equivalent noise 5 times lower then the stimulus signal amplitude).

Consider now a variable w_k that takes the value 1 if a sample belongs to class k of the cumulative histogram and 0 if not.

$$f_{w_k}\left(w\mid\gamma_j\right)=\begin{cases}p_k\left(\gamma_j\right) & , \quad w=1\\ 1-p_k\left(\gamma_j\right) & , \quad w=0\end{cases} \quad (13)$$

This variable has a binomial distribution with mean p_k and variance $p_k\cdot\left(1-p_k\right)$.

The number of counts in class k of the cumulative histogram (c_k) is the sum of variable w_k for all the samples. The p.d.f. of c_k is the convolution of the p.d.f. of the M variables w_k, because they are independent.

$$f_{c_k}\left(c\mid\varphi\right)=f_{w_k}\left(w\mid\gamma_0\right)*\ldots*f_{w_k}\left(w\mid\gamma_{M-1}\right)\left(c\right) \quad (14)$$

Note that, now, the p.d.f. of the number of counts is conditional to the initial phase φ and not to the sampling instant γ_j. The conditional mean of c_k is the sum of the conditional means of the variables w_k for each sample. The same is true for the conditional variance.

The total p.d.f. of the number of counts can be obtained by integrating the product of the conditional p.d.f. with the p.d.f. of the initial stimulus signal phase (f_φ).

$$f_{c_k}\left(c\right)=\int_{-\infty}^{\infty} f_{c_k}\left(c\mid\varphi\right)\cdot f_\varphi\left(\varphi\right)\cdot d\varphi \quad (15)$$

From the total p.d.f. $f_{c_k}\left(c\right)$, it is possible to determine the total mean

$$\mu_{c_k}=\int_{-\infty}^{\infty}\mu_{c_k|\varphi}\cdot f_\varphi\left(\varphi\right)\cdot d\varphi \quad (16)$$

and variance of the number of counts.

$$\sigma_{c_k}^2=\int_{-\infty}^{\infty}\sigma_{c_k|\varphi}^2 f_\varphi\left(\varphi\right)d\varphi+\int_{-\infty}^{\infty}\mu_{c_k|\varphi}^2 f_\varphi\left(\varphi\right)d\varphi-\left[\int_{-\infty}^{\infty}\mu_{c_k|\varphi}f_\varphi\left(\varphi\right)d\varphi\right]^2 \quad (17)$$

4. ASYNCHRONOUS SAMPLING

Considering now that the sampling is performed asynchronously with the stimulus signal, that is, the initial phase of the stimulus signal is not controlled. Let us consider that this initial phase can be taken as uniformly distributed between 0 and 2π. Furthermore, if the frequencies of the sampling clock and the stimulus signal are carefully chosen and are not harmonically related, then the ideal sample phases (no phase noise) are uniformly spaced:

$$\gamma_j=j\frac{2\pi}{M}+\varphi \quad (18)$$

With some manipulation an expression for the mean

$$\mu_{c_k}=\frac{M}{2\pi}\int_{-\pi}^{\pi}p_k\left(\gamma\right)d\gamma \quad (19)$$

and for the variance

$$\sigma_{c_k}^2=\mu_{\sigma_{c_k|\varphi}^2}+\sigma_{\mu_{c_k|\varphi}}^2$$

$$\mu_{\sigma_{c_k|\varphi}^2}=\frac{M}{2\pi}\int_{-\pi}^{\pi}p_k\left(\gamma\right)\left[1-p_k\left(\gamma\right)\right]d\gamma \quad (20)$$

$$\sigma_{\mu_{c_k|\varphi}}^2=\frac{M}{2\pi}\int_0^{\frac{2\cdot\pi}{M}}\left(\sum_{j=0}^{M-1}p_k\left(j\frac{2\pi}{M}+\varphi\right)\right)^2 d\varphi-\left(\frac{M}{2\pi}\int_{-\pi}^{\pi}p_k\left(\gamma\right)d\gamma\right)^2$$

can be easily derived from (15), (16) and (17). These expressions are written in terms of the number of samples acquired (M) and the probability that a sample belongs to a class of the cumulative histogram (p_k). The value of this probability depends, in turn, on the normalized input-equivalent noise standard deviation (σ_n), on the phase noise standard deviation (σ_θ), the normalized transition voltage ($U[k]$) and sample phase (γ_j).

The expression for the variance was divided into two terms: the mean of the conditional variance ($\mu_{\sigma_{c_k|\varphi}^2}$) and the variance of the conditional mean ($\sigma_{\mu_{c_k|\varphi}}^2$).

4.1 Mean of the conditional variance

The term $\mu_{\sigma_{c_k|\varphi}^2}$ of the variance of the number of count of the cumulative histogram is maximum for a transition voltage equal to the stimulus signal offset ($U[k]=0$). Ideally this would be the middle of the ADC input range. In this case the dependence of $\mu_{\sigma^2_{c_k|\varphi}}$ on the standard deviations of phase noise and input-equivalent noise is represented in Fig. 4.

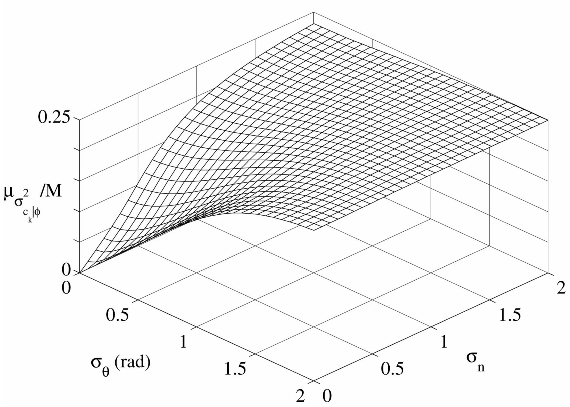

Fig. 4 Representation of the term $\mu_{\sigma^2_{c_k|\varphi}}$ divided by the number of acquired samples (*M*) for a transition voltage equal to the stimulus signal offset (*U*[*k*+1]=0) as a function of the standard deviation of the phase noise (σ_θ) and of the standard deviation of the input-equivalent noise divided by the stimulus signal amplitude (σ_n=1 is equivalent to σ=*A*).

This term, as can be seen from equation (20) is directly proportional to the number of samples (*M*). For small values of the standard deviations this terms increases proportionally to the standard deviation. In particular, in the absence of one of the noise sources,the rates of increase are equal.

$$\begin{aligned} \mu_{\sigma^2_{c_k|\varphi}}\Big|_{\sigma_\theta=0} &\xrightarrow{\sigma_n\to 0} \frac{M}{\pi\sqrt{\pi}}\sigma_n \\ \mu_{\sigma^2_{c_k|\varphi}}\Big|_{\sigma_n=0} &\xrightarrow{\sigma_\theta\to 0} \frac{M}{\pi\sqrt{\pi}}\sigma_\theta \end{aligned} \quad (21)$$

In [5] the same rate was obtained in the case of the presence of input-equivalent noise alone (1.13M/2π). For large values of standard deviation the term $\mu_{\sigma^2_{c_k|\varphi}}$ approaches M/4. This behavior was overlooked in [5] for the case of the cumulative histogram (and the transition voltages). There the dependency on the standard deviation was considered always linear. The same occurred in [6] in the case of phase noise exclusively.

An asymptotic approximate expression for the term $\mu_{\sigma^2_{c_k|\varphi}}$ is proposed.

$$\mu_{\sigma^2_{c_k|\varphi}} \approx M\cdot\min\left(\frac{1}{4},\frac{\sqrt{\sigma_n^2+\sigma_\theta^2}}{\pi\sqrt{\pi}}\right) \quad (22)$$

where the term $\sqrt{\sigma_n^2+\sigma_\theta^2}$ was obtained empirically to account for the joint effect of both noises.

4.2 Variance of the conditional mean

The dependence of the term $\sigma^2_{\mu_{c_k|\varphi}}$ on the number of samples and the transition voltage is less monotone then the term $\mu_{\sigma^2_{c_k|\varphi}}$. Fig. 5 represents this term in the case of absence of phase noise. In the case of absence of input-equivalent noise the dependence on the standard deviation of the phase noise, σ_θ, is similar to the one depicted in Fig. 5.

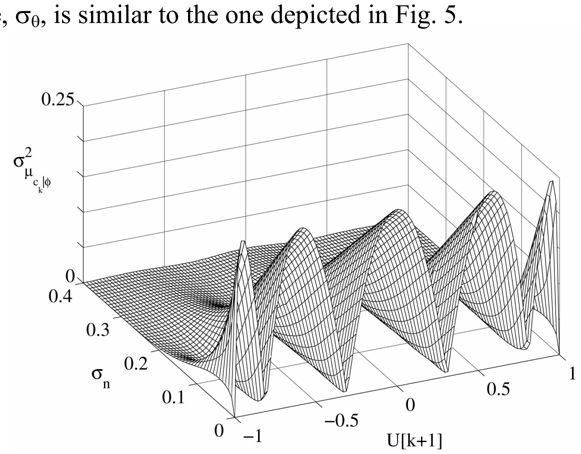

Fig. 5 Representation of the term $\sigma^2_{\mu_{c_k|\varphi}}$ in the absence of phase noise (σ_θ=0) and 5 acquired samples as a function of the normalized transition voltage (*U*[*k*+1]).

For small values of the standard deviation of the noise the term $\sigma^2_{\mu_{c_k|\varphi}}$ depends strongly on the transition voltage. The number of arcs seen in Fig. 5 is equal to the number of samples.

The maximum value of this term occurs in the absence of phase noise and it is equal to ¼. If there is an error in the stimulus signal frequency or in the sampling frequency the value of this term can be higher as was seen in [3].

As the standard deviation of noise increases the term $\sigma^2_{\mu_{c_k|\varphi}}$ decreases until it reaches 0 for high values of the standard deviation (Fig. 6).

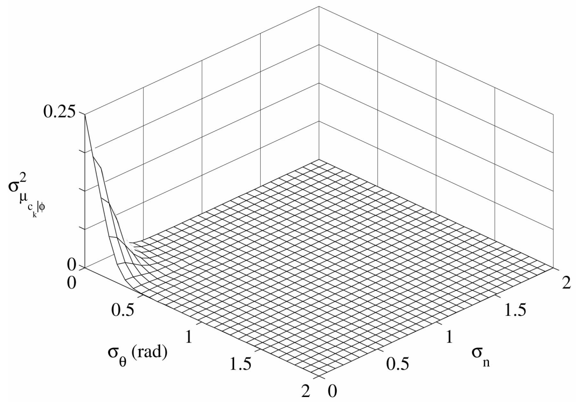

Fig. 6 Representation of the term $\sigma^2_{\mu_{c_k|\varphi}}$ for a transition voltage equal to the stimulus signal offset (*U*[*k*+1]=0) and 5 acquired samples as a function of the standard deviation of the phase noise (σ_θ) and of the standard deviation of the input-equivalent noise divided by the stimulus signal amplitude (σ_n=1 is equivalent to σ=*A*).

In [5] the effect of the random initial phase of the stimulus signal was considered separately from the effect of input-equivalent noise, leading to an overestimation of the variance.

4.3 Variance of the number of counts

The variance of the number of counts is determined by adding the terms $\mu_{\sigma^2_{c_k|\varphi}}$ and $\sigma^2_{\mu_{c_k|\varphi}}$.

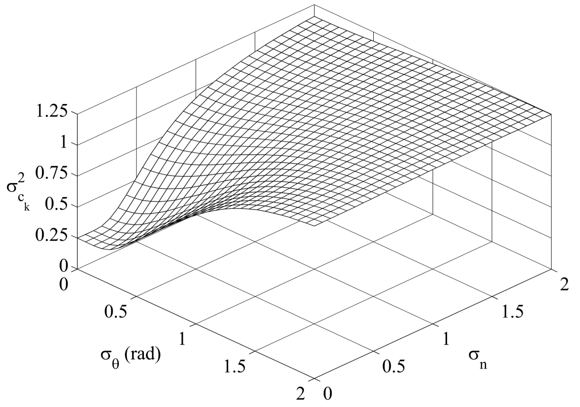

Fig. 7 Representation of the term $\sigma^2_{c_k}$ for a transition voltage equal to the stimulus signal offset ($U[k+1]=0$) and 5 acquired samples as a function of the standard deviation of the phase noise (σ_θ) and of the standard deviation of the input-equivalent noise divided by the stimulus signal amplitude (σ_n=1 is equivalent to σ=A).

For high values of standard deviation, the variance of the number of counts approaches M/4 as was the case for term $\mu_{\sigma^2_{c_k|\varphi}}$. For low values of standard deviation the variance in the number of counts becomes constant and equal to ¼. If frequency errors are greater than the limit determined in [8], the variance will be higher than ¼ for low values of standard deviation.

The following asymptotically approximate expression can be used to estimate the variance of the number of counts.

$$\sigma^2_{c_k} \approx \max\left(\frac{1}{4}, M \cdot \min\left(\frac{1}{4}, \frac{\sqrt{\sigma_n^2+\sigma_\theta^2}}{\pi\sqrt{\pi}}\right)\right) \tag{23}$$

5. VARIANCE OF THE TRANSITION VOLTAGES

The values of the transition voltages can be obtained directly from the number of counts of the cumulative histogram [5].

$$T[k+1] = d - A \cdot \cos\left(\pi \frac{CH[k]}{M}\right) \tag{24}$$

The variance of the transition voltages can thus be approximated by

$$\sigma_T^2 \simeq \left(\frac{A\pi}{M}\right)^2 \sigma^2_{c_k} \tag{25}$$

Substituting (23) in (25):

$$\sigma_T^2 \simeq \left(\frac{A\pi}{M}\right)^2 \max\left(\frac{1}{4}, M \min\left(\frac{1}{4}, \frac{\sqrt{\left(\sigma_{n_v}/A\right)^2+\sigma_\theta^2}}{\pi\sqrt{\pi}}\right)\right) \tag{26}$$

This expression is an asymptotically approximation. There is no exact expression, however, more accurate but also more complex expressions can be used. The maximum relative error in using equation (26) is lower than 17%. Note that this is the error of the determination of the error interval of the transition voltages, determined with the histogram method, and not of the transition voltages themselves.

The expression presented in [6] is now reproduced, with the notation used here, for comparison with the proposed expression.

$$\sigma_T^2 \simeq \left(\frac{A\pi}{M}\right)^2 \left[\frac{M}{\pi\sqrt{\pi}}\frac{\sigma_{n_v}}{A} + \frac{1}{\sqrt{\pi}}\frac{M}{\pi}\sigma_\theta + \frac{1}{4}\right] \tag{27}$$

This expression leads to greater errors than the one proposed, namely the variance increases to infinite when the standard deviation tend to infinite instead of approaching M/4. This behavior is accounted for with the minimum function in equation (26).

The effect of the random initial phase of the stimulus signal is better accounted for with the maximum function in equation (26) instead of being added to the effects of the other factors (term ¼ in equation (27)) as was done in [6].

6. CONCLUSION

In this paper a unified approach was taken in the determination of the uncertainty of the transition voltages determined by the histogram method. An approximate expression was presented that better takes into account the effect of the phase noise, input-equivalent noise and random initial phase of the stimulus signal.

ACKNOWLEDGMENT

This work was sponsored by the Portuguese research project PCTI/ESE/32698/1999, entitled "New Measurement Methods in Analog to Digital Converters Testing", whose support the authors gratefully acknowledge.

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