

DIGITAL MEASUREMENT OF POWER SYSTEM HARMONIC MAGNITUDE AND PHASE ANGLE

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Abstract - *The paper deals with the measurement of harmonic magnitude and phase angle of a periodic voltage $v(t)$ in power systems. The proposed procedure is based on Walsh spectrum measurement followed by digital computation to provide the Fourier spectrum of $v(t)$ in magnitude and phase. Computer simulation results are presented to validate this method.*

Keywords - Harmonic magnitude and phase angle, Power systems, Walsh spectrum, Fourier spectrum.

1. INTRODUCTION

The usual representation of a periodic time-varying signal is the Fourier series. The coefficients of the equation can be obtained according to two main approaches. In the first, the input signal is sampled, converted into digital form and the Fourier integral calculation is used in order to obtain the coefficients. If the period is known, this method requires only one cycle of the signal: however the main drawback is the awkward and the time-consuming multiplications by cosine and sine coefficients of Fourier processing. Variations have been suggested processing the digitized data using Fast Fourier Transform algorithms: they allow accurate measurements but the disadvantage is the large amount of computation involved [1].

In this paper a digital technique of measuring the phase angle of the harmonics as well their magnitudes is presented. The method is firstly based on Walsh spectrum measurement with successive digital computation to provide the Fourier spectrum of the periodic voltage in magnitude and phase [2]. This measurement method is fast and accurate and doesn't require expensive instrumentation; moreover it is particularly well-suited to power-frequency waveform measurements [3]-[6].

The Walsh method, in common with Fourier method, requires that the periodic voltage $v(t)$ be frequency limited, which implies that its Walsh spectrum is made up of an infinite series of c_{al} and s_{al} terms. However, if the highest harmonic order of $v(t)$ is H , then measurement of only S c_{al} components and S s_{al} components (with $S \geq H$) permits

to obtain exactly the Fourier spectrum of $v(t)$ except for the roundoff and truncation errors in digital processing.

The extraction of the cosine and sine coefficients of $v(t)$ from the truncated measure of c_{al} and s_{al} coefficients is achieved by way of a Walsh to Fourier matrix conversion process; a subsequent matrix multiplication provides for compensation for the Walsh spectral truncation.

The compensation matrix takes the form of a diagonal matrix with fixed elements; each converted Fourier spectral component requires to be multiplied by a correction factor which is fixed for a given harmonic.

The Walsh method is attractive for use at power system frequency, for then the synchronization of the Walsh waves with $v(t)$ is not difficult. It is evident that Walsh spectra are superior to the Fourier spectra, in that the multiplication with sine and cosine functions, respectively, is obviated by simple reversal of signs (Walsh waves assume values of plus and minus one). Moreover both odd and even harmonics can be measured.

2. PRINCIPLE OF OPERATION

The Fourier series of a periodic waveform $v(t)$ is written

$$v(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega t + b_n \sin n\omega t] \quad (1)$$

where

$$a_n = \frac{2}{T} \int_0^T v(t) \cos n\omega t dt \quad (2)$$

$$b_n = \frac{2}{T} \int_0^T v(t) \sin n\omega t dt \quad (3)$$

and $T=2\pi/\omega$ is the period of the fundamental frequency at 50 Hz.

Walsh functions, which are shown in Fig. 1 up to $s_{al8}(t)$, can be used as an alternative orthogonal family for the series representation of a periodic waveform according to

$$v(t) = A_0 + \sum_{n=1}^{\infty} [A_n \text{cal}_n(t) + B_n \text{sal}_n(t)] \quad (4)$$

where

$$A_n = \frac{1}{T} \int_0^T v(t) \text{cal}_n(t) dt \quad (5)$$

$$B_n = \frac{1}{T} \int_0^T v(t) \text{sal}_n(t) dt \quad (6)$$

$$A_0 = \frac{1}{T} \int_0^T v(t) \text{wal}_0(t) dt \quad (7)$$

$$\mathbf{A} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_S \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_S \end{bmatrix} \quad (9)$$

represent the cal spectrum and sal spectrum of $v(t)$, respectively.

The element of the Walsh to Fourier conversion matrices \mathbf{F}_a and \mathbf{F}_b , are the Fourier coefficients of the Walsh functions $\text{cal}_m(t)$ and $\text{sal}_m(t)$, respectively.

Thus

$$a_{n,m} = \frac{2}{T} \int_0^T \text{cal}_m(t) \cos n\omega t dt \quad (10)$$

$$b_{n,m} = \frac{2}{T} \int_0^T \text{sal}_m(t) \sin n\omega t dt \quad (11)$$

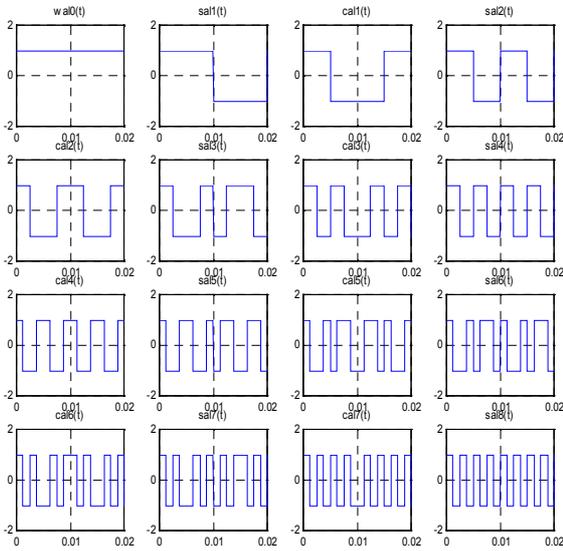


Fig. 1 Walsh functions up to the eighth order.

The procedure for the Walsh series determination and the corresponding Fourier derivation is given below.

Consider a periodic voltage $v(t)$ whose period is T at the fundamental frequency of 50 Hz. Signal $v(t)$ is sampled systematically with sampling frequency f_s .

The Walsh coefficients A_n and B_n are obtained according to (5), (6), (7). To yield the Fourier spectrum of $v(t)$, A_n and B_n are first multiplied by Walsh-Fourier conversion matrices \mathbf{F}_a and \mathbf{F}_b , respectively, to give approximations \mathbf{a}^* , \mathbf{b}^* to the cosine and sine coefficients of $v(t)$:

$$\mathbf{a}^* = \mathbf{F}_a \mathbf{A} \quad \mathbf{b}^* = \mathbf{F}_b \mathbf{B} \quad (8)$$

where

where n is the index of row (corresponding to the harmonic order) and m is the index of column. Moreover the elements of the conversion matrix \mathbf{F}_a assume the same absolute value of the conversion matrix \mathbf{F}_b , but their signs may differ. The conversion matrices are essentially semi-infinite; when applied in (8) however they are truncated to $S \times S$ element square matrices. Also an additional matrix multiplication is required which compensated for the Walsh spectral truncation

$$\mathbf{K}_a \mathbf{a}^* = \mathbf{a} \quad \mathbf{K}_b \mathbf{b}^* = \mathbf{b} \quad (12)$$

where \mathbf{a} and \mathbf{b} are the desired cosine and sine coefficients, respectively. The simplest form of compensation occurs when $S = 2^k - 1$, i.e., the highest harmonic order in $v(t)$ doesn't exceed the third, or the seventh, or the fifteenth, ...

Then compensation takes the form of a diagonal matrix, with fixed elements

$$\left[\frac{\pi n / 2^{k+1}}{\sin(\pi n / 2^{k+1})} \right]^2 \quad (13)$$

where n is the harmonic order.

3. COMPUTER SIMULATION RESULTS

The measurement algorithm has been verified by computer simulation to investigate the validity of this

technique. The program generates an input voltage that is sampled with sampling frequency $f_s = 100$ kHz; its waveform is shown in Fig. 2.

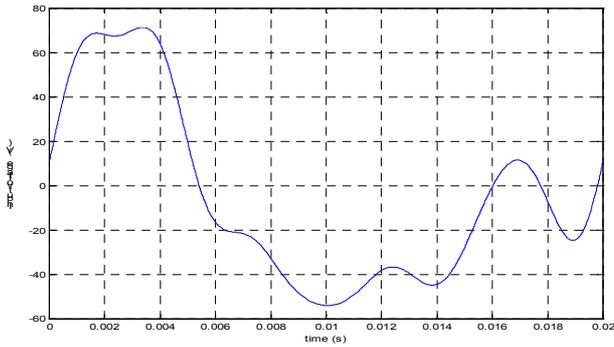


Fig. 2 Waveform of the input voltage

Let the highest harmonic in the input voltage be the seventh (i.e., $k=3$) and that the Walsh truncated spectrum, according to (4), (5), (6), (7) results

$$\begin{aligned}
 & -5.00\text{wal}_0(t) + 30.5046\text{cal}_1(t) - 6.3403\text{cal}_2(t) + 2.5416\text{cal}_3(t) - \\
 & 5.0116\text{cal}_4(t) - 3.8010\text{cal}_5(t) - 2.0578\text{cal}_6(t) + 3.6125\text{cal}_7(t) \\
 & + 19.5930\text{sal}_1(t) + 12.1964\text{sal}_2(t) - 2.0463\text{sal}_3(t) + 3.9259\text{sal}_4(t) \\
 & - 0.7127\text{sal}_5(t) - 2.1051\text{sal}_6(t) - 2.5048\text{sal}_7(t)
 \end{aligned}$$

In applying (8) the Fourier spectrum is obtained in terms of Walsh spectrum

$$\begin{vmatrix}
 1.273 & 0 & 0.527 & 0 & -0.105 & 0 & 0.253 \\
 0 & 1.273 & 0 & 0 & 0 & 0.527 & 0 \\
 -0.424 & 0 & 1.025 & 0 & 0.685 & 0 & 0.284 \\
 0 & 0 & 0 & 1.273 & 0 & 0 & 0 \\
 0.255 & 0 & -0.615 & 0 & 0.920 & 0 & 0.381 \\
 0 & -0.424 & 0 & 0 & 0 & 1.025 & 0 \\
 -0.182 & 0 & -0.075 & 0 & -0.379 & 0 & 0.914
 \end{vmatrix}$$

$$\begin{vmatrix}
 30.5046 \\
 -6.3403 \\
 2.5416 \\
 -5.0116 \\
 -3.8010 \\
 -2.0578 \\
 3.6125
 \end{vmatrix}
 =
 \begin{vmatrix}
 41.4398 \\
 -9.1580 \\
 -11.9204 \\
 -6.3810 \\
 4.0863 \\
 0.5825 \\
 -0.9971
 \end{vmatrix}
 = \mathbf{a}^*$$

$$\begin{vmatrix}
 1.273 & 0 & -0.527 & 0 & -0.105 & 0 & -0.253 \\
 0 & 1.273 & 0 & 0 & 0 & -0.527 & 0 \\
 0.424 & 0 & 1.025 & 0 & -0.685 & 0 & 0.284 \\
 0 & 0 & 0 & 1.273 & 0 & 0 & 0 \\
 0.255 & 0 & 0.615 & 0 & 0.920 & 0 & -0.381 \\
 0 & 0.424 & 0 & 0 & 0 & 1.025 & 0 \\
 0.182 & 0 & -0.075 & 0 & 0.379 & 0 & 0.914
 \end{vmatrix}$$

$$\begin{vmatrix}
 19.5930 \\
 12.1964 \\
 -2.0463 \\
 3.9259 \\
 -0.7127 \\
 -2.1051 \\
 -2.5048
 \end{vmatrix}
 =
 \begin{vmatrix}
 26.7350 \\
 16.6391 \\
 5.9965 \\
 4.9986 \\
 4.0311 \\
 3.0194 \\
 1.1577
 \end{vmatrix}
 = \mathbf{b}^*$$

Next we apply the compensation matrices (12):

Results obtained with the proposed method are then compared with those obtained via the fast Fourier transform (FFT) algorithm.

Fig. 5 shows the frequency content of the input voltage.

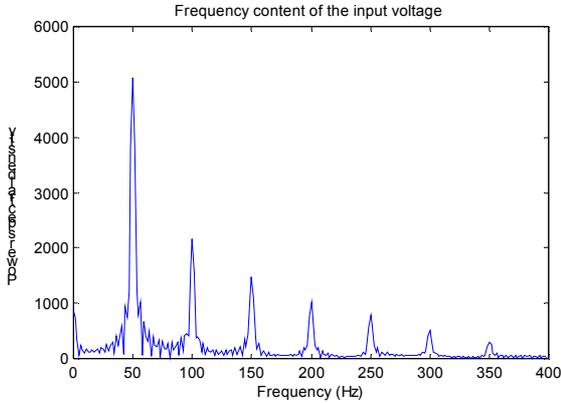


Fig. 5 Frequency content of the input voltage.

Inspection of Fig. 5 confirms that the proposed method based on Walsh spectrum gives good results compared with those obtained via the FFT algorithm.

4. CONCLUSION

This paper has described a digital technique of measuring the magnitude and phase angle of the harmonics of a periodic voltage $v(t)$ in power systems.

The method is based on Walsh spectrum measurement followed by digital computation for obtaining the Fourier spectrum.

This measurement method is fast and accurate and doesn't require expensive instrumentation; moreover it is particularly well-suited to power-frequency waveform measurements, for then the synchronization of the Walsh waves with the input voltage is not difficult. It is evident that Walsh spectra are superior to the Fourier spectra, in that the multiplication with sine and cosine functions, respectively, is obviated by simple reversal of signs (Walsh waves assume values ± 1). Moreover both odd and even harmonics can be measured.

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