

DEVELOPING A KIBBLE BALANCE FOR MEASURING ULTRASONIC RADIATION FORCE AT 1 MEGAHERTZ

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Abstract:

Various approaches to bench-top Kibble balances are available worldwide. However, the literature has not disclosed a Kibble balance to measure ultrasonic radiation force (URF). Due to the nature of Kibble balances, assessing URF could be useful and even more valuable. This paper describes the development of a bench-top Kibble balance to measure URF. The system was compared to the Brazilian National Metrology Institute's primary standard for ultrasonic power measurement. Regarding the regular microbalance used for that purpose, the Kibble balance approach led to a smaller uncertainty in the 10 W to 20 W range.

Keywords: Kibble balance; ultrasonic power; radiation force balance; metrological validation

1. INTRODUCTION

1.1. Ultrasonic Power and Radiation Force

Ultrasound is widely used in medical equipment for therapeutic and diagnostic purposes. Ultrasonic power is one of the most relevant quantities for safety use for human beings [1]. The most widely used method to assess ultrasonic power level is through the radiation force balance (RFB), even though other methods have been implemented [2], [3]. The RFB method is relatively easy to implement. An absorber target is used to absorb the ultrasonic radiation. A weighing scale measures the radiated force absorbed by the absorber target. The fundamental equation for the absorption scheme of the RFB method can be expressed as equation (1).

$$P = c F \quad (1)$$

where P represents the ultrasonic power, c is the speed of sound in the water, dependent on temperature, and F is the time average radiation force.

F can be determined as $F = Mg$, where $M = |m_{\text{on}} - m_{\text{off}}|$ is the difference in the mass readings of the weighing balance when turning the

ultrasonic equipment on and off and g is the acceleration of local gravity.

1.2. Kibble Balance Background

The Kibble balance was invented at the National Physical Laboratory (NPL) by Bryan Kibble in 1975 and indirectly compares electrical power and mechanical power, measured in units of watts [4], [5], [6].

The Kibble balance is one of the methods that can be used to determine the relationship between the mass and the Planck constant [4], [5], [7]. It was fundamental to the redefinition of the kilogram by the Comité International des Poids et Mesures (CIPM) in 2018 and implemented in 2019. Before the SI revision, the kilogram was the last of the seven base SI units defined by a material artefact known as the international prototype of kilogram (IPK) [8].

The Kibble balance is an active measurement system, and it consists of a wire coil suspended by the arm and placed in a strong magnetic field. A precisely adjusted electromagnetic force compensates for the weight of an unknown mass.

The device has two modes: moving and weighing [4].

In the moving mode, the coil (wire length L) is moved in the magnetic field (flux density B) with a vertical velocity v to induce a voltage V in the coil. The induced voltage is related to the velocity through the flux integral $B L$ [5].

In weighing mode, the weight ($m g$) of a mass m is opposed by the vertical force $B L I$ generated by a current I flowing in a wire coil length L at a magnetic flux density B [5].

Combining the moving mode and the weighing mode equations, the $B L$ factor, common to both, is eliminated from the equation, and rearranging the variables, expressions for electrical and mechanical power are equated, and a solution for mass is obtained according to equation (2) [5].

$$m = \frac{v I}{v g} \quad (2)$$

The above equation relates mechanical power to electrical power and mass to electrical quantities. That way, achieving extremely low global uncertainty (of the order of 1 part in 10^8) is possible due to the use of the electrical quantum standards [5].

1.3. Using a Kibble Balance to Measure Ultrasonic Power

Ultrasonic power (USP) derives from ultrasonic radiation force. Any conventional balance or microbalance can be used for a measurement of that quantity. Therefore, the most common way to measure USP is with the aid of a radiation force balance (RFB) using conventional weighing technology. However, it could be of interest to develop a method integrated with a measuring system in which the USP is derived directly from a more stable quantity, namely electrical quantities. Fortunately, it is fully available from the theoretical background for the Kibble balance.

Using a Kibble balance to measure USP could make it easier to calibrate ultrasonic emitting equipment even in a Point-of-Care (POC). The obvious advantage is that if the measurement system used to measure ultrasonic power employs a Kibble balance instead of a conventional balance, it is not necessary to know the local gravity acceleration. Another important motivation is that electrical quantities are much more easily spreadable with relatively lower uncertainty. So, when combining the RFB method equation (1) and the Kibble balance equation (2), equation (3) is obtained, which allows measuring ultrasonic power by the Kibble balance principle.

$$P = c m g \rightarrow P = \frac{c V I g}{v g} \rightarrow P = \frac{c V I}{v} \quad (3)$$

where the ultrasonic power P is a function of the variables c (speed of the ultrasound in the water), V (voltage induced in the coil), I (current applied to the coil) and v (vertical speed of the coil).

In equation (3), one can observe that the variable g (local acceleration of gravity) is eliminated from the mathematical model. Therefore, this allows an ultrasonic power measurement to be carried out in any part of the earth without knowing the local gravity acceleration.

A bench-top Kibble balance was constructed using additive manufacturing to validate that concept. The electronic was as simple as feasible to be easily scalable. An in-house application was developed in Python to control all measuring instruments. The uncertainty budget was set following the Guide to the Expression of

Uncertainty in Measurement (GUM) published by the Bureau International des Poids et Mesures (BIPM) [9].

2. MATERIALS AND METHODS

2.1. Mechanical Project

The Kibble balance can be constructed in different ways. For the Kibble balance to measure the ultrasonic radiation force, it must be able to measure the mass range of approximately 100 mg to 1500 mg to measure an ultrasonic power range of 1 W to 20 W.

For that purpose, a bench-top Kibble balance was constructed using parts, such as the arm-beam and the knife-edge, from a balance of the twentieth century. All other balance parts were designed using computer-aided design (CAD) to be printed in 3D. Figure 1 shows the final prototype of the bench-top Kibble balance developed.

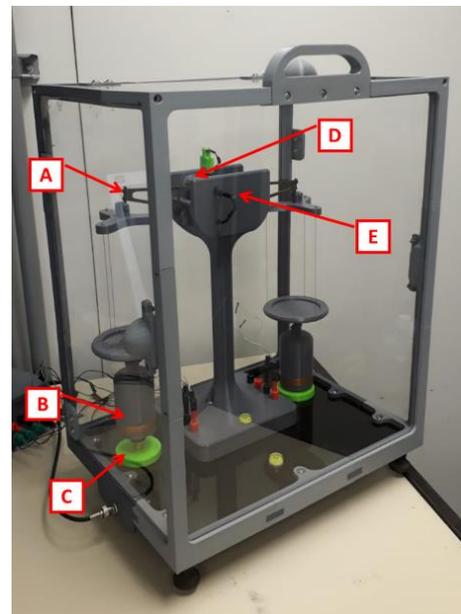


Figure 1: Prototype of a Kibble balance designed and built for measuring ultrasonic power. A – equal-arm beam; B – coil; C - magnets assembly; D – shadow sensor; E – line laser

A pair of neodymium magnets (N35) are fixed vertically, on both sides of the balance, by brass screws supported by a 3D printed base to generate the magnetic field.

The coils have an outside diameter of 56 mm and were wound by hand with an electric screwdriver. Each coil has approximately 2500 turns of AWG-36 copper wire, obtaining a final resistance of roughly 500 Ω .

To monitor the balance movement as well the position of the coil, a line laser and a shadow sensor (Brand First Sensor, model PC50-7-TO8) were used. On one side of the equal-arm beam of the balance, a line laser is directed toward the shadow sensor, fixed to the other side of the balance equal-

arm beam, in a way that the arm partially obstructs about 50 % of the light when the balance is in zero position (in balance). When the balance arm-beam is tilted, the shadow sensor receives more or less light depending on the tilt direction. Hence, it is possible to correlate the shadow sensor indication with the coil position by using a micrometer to calibrate the shadow sensor, for instance.

2.2. Electronic Project and Automation

For the electronic design of control and automation of the balance, it was decided to use a Raspberry Pi 3 microcontroller, model B+ in conjunction with an oscilloscope model DSO-X 3012A (Agilent Technologies, USA) and two 4-channel reels.

The Raspberry was used to control the entire measurement system by controlling the oscilloscope and the relays. The oscilloscope channels are used to obtain the position of the balance through the shadow sensor indication, measure the voltage induced in the coil in moving mode or the current applied to the coil in weighing mode, and measure the water temperature by a thermocouple. The oscilloscope also has a waveform generator used to apply the signals needed to control the balance.

For the control and monitoring of all measuring instruments, an in-house application was developed in Python v3.7 (see Figure 2). The application can perform the moving and weighing modes, calibrate the shadow sensor, assess the results, and so forth.

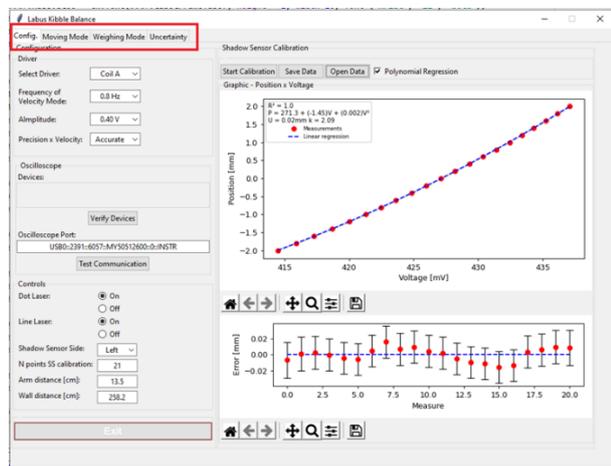


Figure 2: Front end of the in-house application

3. RESULTS AND DISCUSSION

3.1. Moving Mode

In moving mode it is possible to determine the $(BL)_v$ factor, which is the ratio of the voltage induced in the coil V to the coil velocity v .

It was arbitrarily decided to use coil A to generate the oscillatory movement by applying a sinusoidal signal on it and coil B as the measuring coil. A sinusoidal signal of 0.8 Hz frequency and 400 mV_{pp} amplitude was used for approximately

30 s. This configuration produced the best results, which means a lower measurement uncertainty for the $(BL)_v$ factor.

So, with the coil velocity signals and the coil's induced voltage, it is possible to calculate the factor $(BL)_v$.

A graph can be generated by plotting the signal of the voltage induced in coil V against coil velocity v , as shown in Figure 3. The factor $(BL)_v$ can be determined by the slope of the line obtained by linear regression ($R^2 = 0.9987$). In the case of this work $(BL)_v = (15.34 \pm 0.57) \text{ V} \cdot \text{s} \cdot \text{m}^{-1}$.

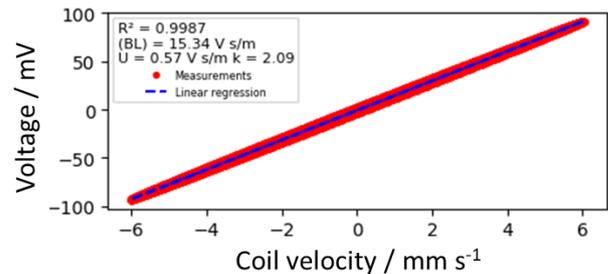


Figure 3: Graph of voltage induced in the coil versus coil velocity

3.2. Weighing Mode and Ultrasonic Power Measurement

After performing moving mode, the ultrasonic power measurement can be carried out in weighing mode, using equation (3).

To measure ultrasonic power, a container with water is placed on each side of the balance. On the side where the measurements will be performed, an absorber target is fixed to the bottom of the container, to absorb the ultrasonic beam, as shown in Figure 4.

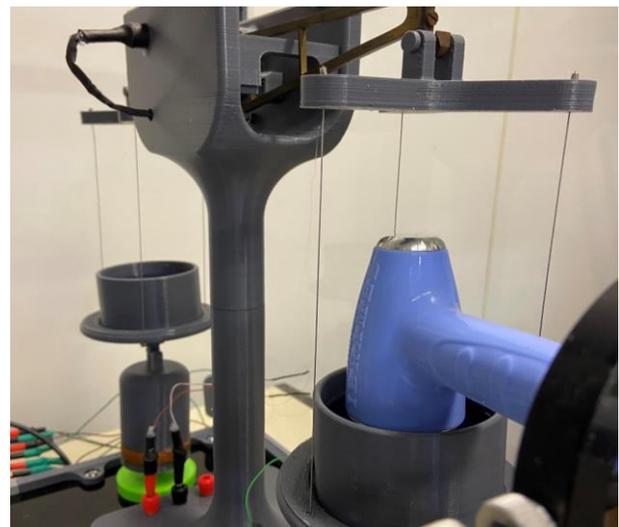


Figure 4: Kibble balance mounted for ultrasonic power measurement

Then, the transducer is introduced to a certain depth in the water. It was observed that the alignment of the transducer with respect to the absorber target is a critical factor for the

measurements. The balance indicates a low power value when the system is not well aligned. So, through a mechanical system composed of micrometers and goniometers, the transducer can be tilted in different directions until it reaches the highest power indication.

Ten measurements were performed under repeatable conditions of measurement, which means the transducer was reassembled and realigned in the system, and the water in the containers was changed for each repetition, considering each repetition as the mean of ten sequential measurements.

Due to the low volume of water in the container, fast heating of the water was observed for powers above 14.5 W. It was necessary to wait for the water to cool down to continue the measurements. All measurements were performed between 18 °C and 32 °C, and the ultrasound speed was corrected automatically by the application according to the temperature.

To validate the bench-top Kibble balance, an ultrasound therapy equipment, model Sonopulse 1 MHz (IBRAMED, Brazil), calibrated according to the Brazilian national metrology institute (NMI) primary standard [10], [11], was used as a reference (Table 1). The uncertainties were assessed following the GUM [9].

Table 1: Measurements obtained by the Brazilian NMI primary standard for ultrasonic power

Brazilian NMI primary ultrasonic power measurement system (reference)		
Measured value	Expanded uncertainty ($p = 95\%$)	Coverage factor k
1.405 W	0.076 W (5.4 %)	2.36
4.05 W	0.11 W (2.6 %)	1.99
9.57 W	0.37 W (3.9 %)	2.20
14.5 W	1.1 W (7.8 %)	2.45
21.7 W	2.8 W (12.7 %)	2.78

Due to some constructional limitations of the balance, such as friction between the knife edge and its base, non-uniformity of the magnetic field or misalignment of the coil and the magnet assembly, the measurement system has a systematic error. Thus, the results obtained from the reference ultrasound equipment calculated a calibration curve using linear regression to correct the bias identified.

In addition, the detection limit (LD) and quantification limit (LQ) were calculated to determine the ideal working range of the measurement system. The LD is the lowest value detected but not necessarily quantified by the method under the established conditions. The LQ is the smallest quantity that can be quantitatively determined with acceptable precision and accuracy.

Figure 5 shows the correction equation obtained for power measurement (corrected value = $-0.3903 + 1.1276 \times$ measured value), the expanded uncertainty (shaded in grey) and the values of LD and LQ as well.

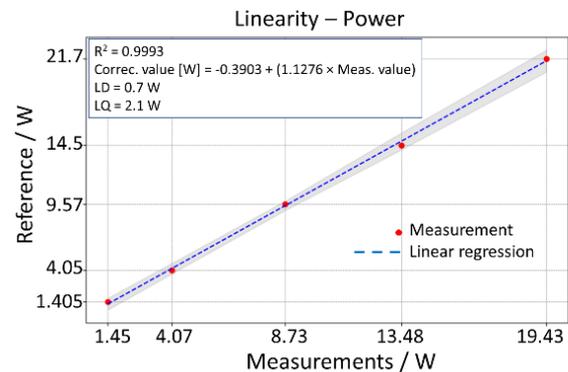


Figure 5: Calibration curve for power measurement by the Kibble balance

The detection limit obtained was 0.7 W, and the quantification limit was 2.1 W, which means the smallest value that the balance can detect is approximately 0.7 W. The smallest value that can be accurately measured at acceptable accuracy is 2.1 W.

The correction equation was used to correct the measurements obtained by the bench-top Kibble balance with respect to the NMI primary standard. The results with the expanded uncertainty can be seen in Table 2. One could argue that the corrections are very significant, which is true. However, it is worth noting that the corrections are stable within the stated uncertainty as it was determined over a long-term study (> 6 months).

Table 2: Measurements obtained by the bench-top Kibble balance

Bench-top Kibble balance		
Measured value	Expanded uncertainty ($p = 95\%$)	Coverage factor k
1.25 W	0.47 W (37.6 %)	2.78
4.20 W	0.48 W (11.4 %)	2.36
9.46 W	0.41 W (4.3 %)	2.26
14.81 W	0.67 W (4.5 %)	2.20
21.51 W	0.96 W (4.5 %)	2.20

Figure 6 shows indicative expanded uncertainties according to the measurement range from both methods, the Brazilian NMI primary standard and the bench-top Kibble balance. The results shown are the uncertainty of a device calibrated by the bench-top Kibble balance and the primary standard for ultrasonic power. The primary standard did not calibrate the Kibble balance. That is why the Kibble balance uncertainties can be smaller than the primary standard in some power ranges. However, it is important to note that the for

the results reported the Kibble balance has its ultrasonic power traceability linked to the primary standard through the correction fit.

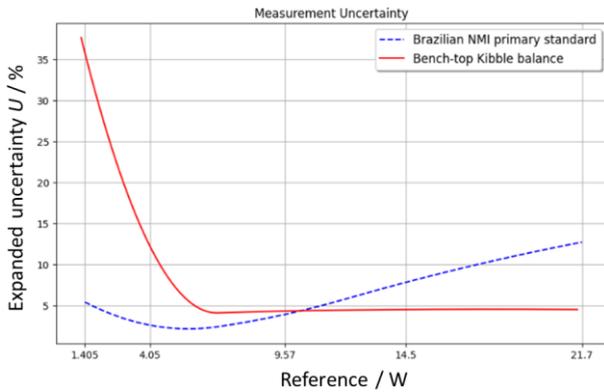


Figure 6: Uncertainty contribution according to the measurement range

It can be seen in Table 2 and in Figure 6 that, when using the bench-top Kibble balance, the measurement uncertainty decreases significantly for powers above 4.1 W, indicating a possible measurement limitation for ranges below this value. The detection limit also highlighted this, and the quantification limit was obtained.

It also observed that the expanded uncertainty obtained using the Kibble balance principle is higher than the Brazilian NMI primary ultrasonic power measurement system for values below 9.46 W. However, the bench-top Kibble balance can reach a lower expanded uncertainty for values above this value.

The main sources of uncertainty identified were the calibration curve uncertainty, the repeatability of the measurements (type A), and the type B uncertainty, which includes all the other sources, such as the equipment resolution, the uncertainty declared on the calibration certificate and so forth. Figure 7 shows the contribution of uncertainty sources according to the reference power.

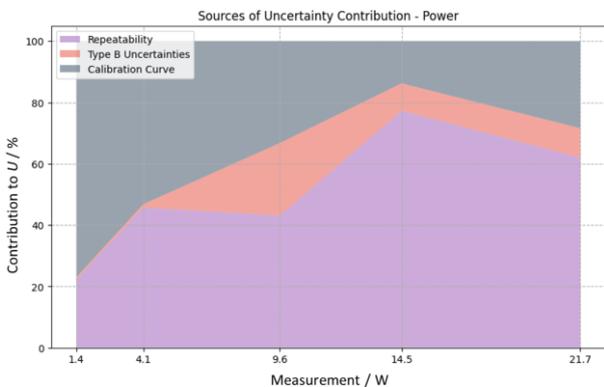


Figure 7: Contribution of sources to the standard uncertainty, according to the reference power

Note that the contribution of the calibration curve is greater for the power range below 4.1 W, contributing more than 50 % to the final uncertainty.

For the range above 4.1 W, there is a reduction in the contribution of the calibration curve and an increase in the contribution of the repeatability uncertainty (type A).

In Table 3, the results were compared with the Brazilian primary standard [10], [11] using the Normalised Error as statistics [12], [13].

Table 3: Results compared using the Normalised Error as statistics.

Reference	Measured value	Normalised Error E_n
1.405 W	1.25 W	0.33
4.05 W	4.20 W	0.31
9.57 W	9.46 W	0.20
14.5 W	14.81 W	0.23
21.7 W	21.51 W	0.06

It can be seen in Table 3 that the normalised error obtained was satisfactory for all points ($E_n \leq 1$), which means that the measurements cannot be considered statistically different between the different systems.

4. SUMMARY

The Kibble balance method to measure mass has gathered a vast international effort as a more accessible and straightforward way to derive a mechanical measurement from electrical quantities. Ultrasonic power is a quantity that could be benefited from that approach. This paper describes the development of a bench-top Kibble balance to measure ultrasonic radiation force. The results confirmed that the method is metrologically valid, as the statistical comparison with a primary standard for ultrasonic made that clear. The ultrasonic frequency for that proof of concept was limited to 1 MHz, and the ultrasonic power quantification range was between 1.4 W and 21.7 W. Within the range of 9.6 W and 21.7 W, the expanded uncertainty ($p = 0.95$) was smaller than 5 % for ultrasonic power. Given that the prototype developed was built using a 3D printer and that the entire electronic, magnetic, control and automation system was designed manually, with low-cost items, it is considered that the results presented were consistent and served to demonstrate the viability of using the Kibble balance principle for ultrasonic power measurement.

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