

A COST-SAVING METHOD FOR DETERMINING AIR DENSITY IN WEIGHING

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Abstract:

This paper promotes the artefact method for determining the air density for weighing and mass calibrations in industrial mass calibration laboratories. Artefact prototypes were manufactured and air density was determined using these artefacts for a proof of concept. Uncertainties are in an acceptable size. Comparison to the traditional method (measurements of air pressure, temperature, and humidity) shows agreement within uncertainties. Substantial economic advantages of the artefact method are identified compared to the traditional method.

Keywords: air density; mass calibration; artefact method for air density determination; uncertainty

1. SUMMARY

In the calibration of weight pieces of higher accuracy, the air buoyancy correction is important or even obligatory depending on the laboratory location and/or the desired accuracy. For performing the air buoyancy correction, the determination of air density is a prerequisite [1]. This paper describes an alternative to determining the air density from measurements of air pressure, temperature and humidity which appears to be the most widely used method when calibrating weight pieces [2]. The motivation is that this method involves expensive instruments which additionally are costly to maintain. The use of the artefact method for calibration of weight pieces is investigated regarding usability, cost-effectiveness and metrological comparability.

2. INTRODUCTION

The artefact method is one possible method of determining air density [3], [4]. It makes use of evaluating the buoyant force that is exerted on a physical body, an artefact. Usually, when calibrating weight pieces, the weighing value is corrected using the measured air density and the density of the artefact to yield its mass or its conventional mass. But the reverse is also possible.

If the volume and the mass of the artefact are known, and its weighing value is measured, the air density can be calculated. The method has been used in high accuracy calibrations, namely the transition from 1 kg PtIr prototypes to stainless steel pieces using special hollow artefacts made of highly polished stainless steel for assessing the air density. But this method has not been adapted yet to its use in industrial calibration laboratories which calibrate weights of OIML classes E₁ (and less accurate).

In this paper, we review the theoretical background of the method. Furthermore, we have manufactured artefacts of different nominal mass values made of an aluminium alloy (examples see Figure 1, left side) for testing the method in its intended application. With these we have executed the method and we present results of air density measurements performed with the artefacts of two sizes.



Figure 1: Artefacts (left) and regular weights (right) of same nominal mass (500 g and 2 g)

3. THE MEANING OF BUOYANCY IN WEIGHING

As can be seen from the general weighing equation, the resulting force on any physical body immersed in a gravitational field and in an atmosphere (or liquid), is the directional sum of the gravitational force and the buoyant force. The gravitational force is dependent on the local gravitational acceleration and the buoyancy force is,

according to Archimedes' law, the product of the volume of the body and the gravitational acceleration and the density of the surrounding liquid (in our case, the atmospheric gas), see equation (1).

$$F = G - F_{\text{buoy}} = g \cdot m - \rho \cdot g \cdot V \quad (1)$$

As the buoyancy force is a consequence of the atmosphere being attracted by gravitation, both forces are in exactly opposite directions. This justifies equation (1) to be a scalar difference.

Both gravitational force and atmospheric air density have, strictly considered, a gradient with height (due to the gradient of the gravitational field and due to the gradient of hydrostatic pressure), however, the effects of this gradient will be neglected for this publication since the bodies considered here are small enough.

Considering usual bodies, made assumedly from materials with a density in the magnitudes from "water" to "steel", buoyancy forces in air range from approximately one thousandth to one ten-thousandth of their weight force. This is why this force is usually not perceived and there is little awareness of its existence in everyday life.

However, in weighing disciplines, the buoyancy force may become meaningful. Equation (2) (with volumes V_i , air density ρ_a and masses m_i) is the general weighing equation for comparative weighing.

$$m_t = m_r + \rho_a(V_t - V_r) \quad (2)$$

Here we see that the difference between the volume of the weighed body (subscript "t" for "test") and the volume of the reference (subscript "r" for "reference") plays the important role. If it approaches zero, the effect of air buoyancy vanishes.

4. SCOPE OF THIS WORK

This publication aims at the evaluation of air buoyancy effects in weighing and calibration of conventional mass to an accuracy level of "E₁" weight piece calibration (accuracy class E₁ according to [1]). It does not aim at higher accuracy levels.

Reference [1], section 10.2 requests the correction of air buoyancy effects for certain situations and especially for calibrating E₁ class weight pieces. Additionally and implicitly, air buoyancy effects are limited by prescribing certain materials or material density ranges for certain weight classes.

The provisions given in [1] are meant to assure an uncertainty small enough to perform conformity statements using the decision rule given in section 5.2 of [1], consisting of the conditions $TUR \geq 3$ and $\text{guardband} = U$.

5. USUAL APPROACH: CIPM FORMULA

Air density cannot easily be measured directly. Thus, indirect procedures are applied.

Usually, for comparative calibration of weight pieces and, if applicable, for weighing, the CIPM-81/91 formula, equation (3) (derived by Giacomo [5] and modified by Davis [6]), to calculate the air density is applied.

$$\rho_a = \frac{p \cdot M_a}{Z \cdot R \cdot T} \cdot \left[1 - x_v \cdot \left(1 - \frac{M_v}{M_a} \right) \right] \quad (3)$$

For the application of weights calibration as discussed here, also an approximation formula ([1], Annex E.3) is suggested.

Both the CIPM formula as well as the approximation formula require the measurement of:

1. atmospheric air pressure,
2. air temperature,
3. humidity.

Hence it is called the "3-quantity-method" below (in the scope of this paper, CO₂ content is not measured, but estimated).

Apart from being an indirect measurement procedure, the formulae used are empirical ones. Thus, they are subject to revisions (CIPM 81-91) and to criticism (effect of argon content [3] and of the assumed accuracy of the equation [7]). This leads to an uncertainty of the resulting air density evaluated with the CIPM formula of not better than 1×10^{-4} ($k = 1$ [7]) considering best available equipment.

6. MEASUREMENT OF DENSITY OF AIR IN AN E₁ LABORATORY

Kochsiek, Glaeser et al. suggest in their work ([2], Table 34.3), the uncertainties for air density components, when performing E₁ class calibrations, given in Table 1.

Table 1: Measurands and suggested uncertainties

Measurand	Measurement uncertainty ($k = 2$)
Atmospheric air pressure	0.3 mbar
Temperature	0.1 K
Humidity	2 %RH

This results in the distribution of uncertainties for the resulting air density, according to the formulae given in Annex E.3 of [1], given in Figure 2.

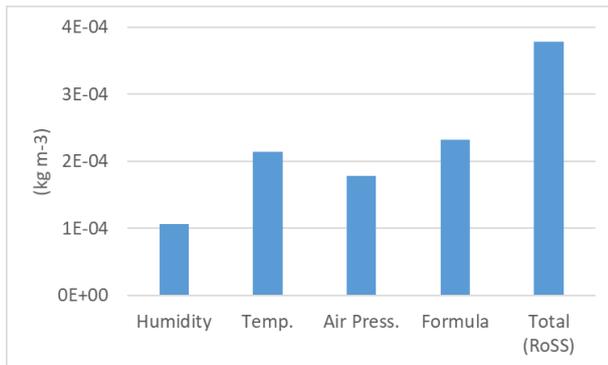


Figure 2: Uncertainty contributions to air density uncertainty for an E_1 laboratory (standard uncertainties)

The chart shows a quite even distribution of components and a resulting standard uncertainty in the region of $4 \times 10^{-4} \text{ kg} \cdot \text{m}^{-3}$ corresponding to approximately 4×10^{-4} in relative numbers ($k = 1$).

As can be concluded from Table 1 above, the measurements require advanced (thus expensive) electronic measuring instruments to reach the suggested uncertainties.

A question that has not been thoroughly analysed in the literature is the “spatial validity” of these ambient measurements: an E_1 laboratory may usually be equipped with one barometer per building floor (assuming same pressure on the same floor, but neglecting different heights of balances), one hygrometer per room (assuming an equal distribution of humidity, but neglecting the influence of transpiring humans in the room) and one temperature sensor per balance weighing chamber (assuming same temperature in the room of interest, but neglecting temperature gradients and/or differences of air and weight pieces). For an E_2 laboratory, one temperature sensor “per room” might suffice.

Of course, the gradients of the respective measurands from the point of measurement (sensor) to the point of interest (balance chamber) have to be assessed and accounted for (e.g. in the uncertainty budget for the respective measurement value). The contributions as indicated in Figure 2 increase accordingly.

The selection of air temperature sensors requires special considerations: air temperature is usually measured with greatest accuracy in a “forced convection” situation (e.g. by ventilation of air against the sensor). Therefore, in traditional ambient temperature measurements (e.g. in meteorology), vented sensors are used and due to the forced convection, self-heating effects can often be neglected. The situation in a high-accuracy weighing environment is quite the opposite: there is no (and shall not be any) air movement. So, sensors with a high self-heating effect would give erroneous temperature values. This needs to be considered for the type of sensors as well as for the control and

readout device. For mass laboratories, sensors of the NTC-thermistor type seem to offer small dimensions and low self-heating effects if they are combined with appropriate readout devices applying small measuring currents.

7. CONSEQUENCES OF THE METHOD

The laboratory equipment used for these measurements of air density comprising a barometer, a hygrometer and several points to measure temperature, is an investment of many thousands of EUR (USD).

Additionally, the devices are subject to periodic re-calibration. This again is a burden of one thousand or more EUR (USD) per year. Moreover, the re-calibration is causing considerable downtime of the mass laboratory. The fact that the three disciplines (pressure, humidity, temperature) might not be calibrated in the same laboratory, complicates the logistics of the lab to keep downtime at an acceptable level.

Thirdly, it would be “Good Metrology” to evaluate and assign calibration intervals for the three disciplines separately. However, this would mean a “sequential” downtime of the mass lab, each time when one of the three disciplines is due for calibration. Therefore, most laboratories will tend to calibrate all disciplines at the same time while the discipline with the shortest interval takes the lead. This results in a costly over-calibration of the other disciplines.

8. USING THE ARTEFACT METHOD FOR CALIBRATION OF CONVENTIONAL MASS

The artefact method has been used by NMI laboratories for the Pt-Ir to stainless steel transition calibration [7]. It is important to understand that the requirements in a laboratory for calibration of conventional mass of weight pieces (up to OIML class E_1) are different:

The weight pieces to be compared do usually not have a large volume difference (unlike the Pt-Ir to stainless steel transition). Consequently, the air buoyancy correction is usually a small contributor to uncertainty, so in this situation, there is no need to perform air buoyancy corrections with utmost accuracy of air density measurements.

With this in mind, it is justified to adapt the artefact method to its intended use:

Air density in a mass calibration laboratory can in principle be measured using two bodies of known (conventional) masses and densities. Equation (4) applies, as it is the re-arranged equation for the determination of conventional mass (the nomenclature of the variables follows [2] and refers to conventional masses).

$$\rho_a = \frac{\rho_t \times (\Delta m_w - m_{c,t}) \times (\rho_r - \rho_0) + \rho_r \times m_{c,r} \times (\rho_t - \rho_0)}{(\Delta m_w - m_{c,t}) \times (\rho_r - \rho_0) + m_{c,r} \times (\rho_t - \rho_0)} \quad (4)$$

For the determination of air density in a mass calibration laboratory, two artefacts are necessary with the same nominal conventional mass, but with differing volumes (densities), and for both artefacts, the conventional masses and their densities need to be known.

The determination of air density then follows the usual substitution procedure for mass calibration and the air density is calculated from the measured difference of conventional mass Δm_w of the buoyancy artefact (subscript “t”) and the reference weight piece (subscript “r”).

An obvious advantage of this method is that it measures the buoyancy effect in exactly the location of interest. Considerations of spatial distribution of temperature, humidity or pressure, need not be taken into account.

Furthermore, the method requires equipment available in the laboratory: apart from a special artefact, only a calibrated reference weight and a suitable mass comparator are needed.

Thirdly, the method employs procedures that are common to the personnel of a mass calibration laboratory (comparison of artefacts by substitution on a mass comparator). Just for the calculation of the result, the usual equation for determining conventional mass is re-arranged to yield ambient air density.

9. DESIGN OF THE ARTEFACTS

The signal of the artefact method is large when the difference in volume between the reference weight piece and the artefact is large at the same nominal mass value. Given that the reference weight piece is made of stainless steel, easy to obtain materials which would offer a significantly different density were e.g. glass or aluminium alloys. Since glass is prone to unwanted electrostatic charges, this work was done with artefacts made of an aluminium alloy. Since aluminium is suspected to show surface corrosion, a special corrosion resistant alloy (AlSi1MgMn) was used (this alloy is sold under the trade name “Anticorodal”).

Since pieces of aluminium are about three times in volume compared to stainless steel pieces of the same mass, it is not feasible – or even impossible considering available space – to use the largest nominal mass for a given comparator. We manufactured two sizes of pieces for this work:

- 500 g pieces with the standard diameter of a 1 kg piece (to fit into a 1 kg comparator)
- 2 g pieces with the standard diameter of a 5 g piece (to fit into a 5 g comparator)

For easy manufacture, these were made in a cylindrical shape. However, for regular use in the laboratory, a design with a lifting knob will be chosen.

10. CALIBRATION OF THE ARTEFACTS

The conventional mass was pre-determined based on an estimated density value. This is a prerequisite for a subsequent density measurement which we did on a Mettler-Toledo VC1005 volume comparator. With the density data available, a final and more accurate determination of the conventional mass could be done. The density values are shown in Table 2.

Table 2: Density values of two artefacts

Nominal mass	2 g	500 g
Density / kg·m ⁻³	2 717.2	2 746.4
	$u = 0.4$	$u = 0.04$

11. UNCERTAINTY EVALUATION

The target audience of this work are calibration laboratories which usually work in the domain of “conventional mass” (and not of mass). Therefore, all measurements were based on the mathematics for conventional mass determination.

The reference uncertainty was determined using the traditional method measuring air pressure, temperature and humidity and estimating carbon dioxide and argon content when applying the CIPM formula. The uncertainty of the artefact method was evaluated with the help of the software “GUM Workbench Professional” [8]. The model equation (4) for the air density using the artefact method was based on conventional mass measurements. It was derived using the equations as given in [2].

12. TESTING THE METHOD

We applied the method in different geographical locations, but mainly in our own accredited calibration laboratory (the accredited scope covers E₁ class calibrations) and at natural air pressures. Therefore, there are some rare data points at the lower end of the range investigated and a larger cluster at the typical conditions of our own laboratory.

For assessing the proof of concept, we compared the artefacts to reference weights using a usual calibration procedure [1], [2]. As a result, we calculated the air density from the artefact method and compared it to the values of the traditional method which were obtained during the comparison procedure.

The results of our measurements are shown in Figure 3.

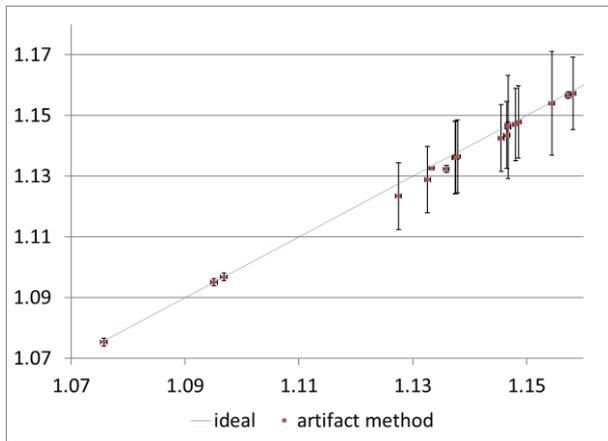


Figure 3: Comparison of the artefact method (y-axis) to the 3-quantity method (x-axis) (units of both axes are $\text{kg}\cdot\text{m}^{-3}$; uncertainties at 95 %)

The graph shows the results of determination the air density by artefacts (y-axis) in comparison to the air density measured for the same situation by traditional means (x-axis). The points have the corresponding uncertainty bars ($k = 2$) in both directions. The data is from different geographical heights and for two different nominal values and for different qualities of artefact density values. The ideal curve is also shown. One can conclude that all data points (except one outlier) coincide with the expected ideal curve within uncertainties. As expected, the uncertainties from the artefact method are larger than from the “3-quantity method”. The relative expanded uncertainties reached with this proof of concept are in the range of 1×10^{-3} to 1×10^{-2} (or two to twenty times larger than with the 3-quantity method). The largest contributor to the uncertainty is the uncertainty in the conventional mass of the aluminium artefacts $m_{c,t}$ (which is mostly affected by the air buoyancy correction when calibrating its conventional mass value).

13. FINDINGS

Our proof of concept showed that the air density can be determined in a mass calibration laboratory with the artefact method using calibrated pieces made of an aluminium alloy and calibrated reference weights made of stainless steel.

The uncertainties obtained for the air density (when using the described aluminium artefacts) are larger than with the “3-quantity method”.

Yet, the uncertainties are still small enough for supporting the calibration of E₁ class weight pieces (see section 8 above, the air density is usually no relevant contributor).

The artefacts need an initial calibration of density and conventional mass. Assuming a constant density over time, the monitoring of the stability of the artefacts can be done by a simple calibration of conventional mass. This can usually

be done by the mass calibration laboratory owning the artefacts with its own equipment and procedures.

Over a period of many months, our artefacts made of AlSi1MgMn showed no sign of wear and tear and their masses did not show any significant change and no visible surface corrosion.

Provided that the achieved air density uncertainty is considered small enough, the proposed method offers vast advantages compared to the “3-quantity method” with respect to investment costs, maintenance costs and downtime considerations.

It seems favourable to use one artefact for every comparator. It is also favourable to use large pieces to achieve small uncertainties. Having in mind the large volume of the artefacts and the limited space in the comparators, it might be a good idea to use a 2 g aluminium artefact for a 5 g comparator, a 50 g artefact for a 100 g comparator, a 500 g artefact for a 1 kg comparator etc.

The results we have obtained for air densities show good agreement with results of the “3-quantity method” within their uncertainties.

Considerations about spatial validity are the same that apply for the “3-quantity-method” and are likewise the responsibility of the laboratory.

Since the ambient conditions in a laboratory usually do not change rapidly with time, an air density determination done with an artefact can be declared valid for a certain period of time (e.g. hours) while estimated changes of air density during this period can be included as a contribution in the uncertainty budget.

The method requires far less initial investment than the “3-quantity method”.

The maintenance (re-calibrations) of the artefacts can usually be done by the laboratory itself minimising both downtime and costs: the material density may be assumed constant over time and the conventional mass value can be calibrated by the laboratory.

However, in contrast to the “3-quantity method”, the results from the artefact method are not provided instantly and electronically. Instead, the values have to be gained by a procedure which has to be executed (manual comparators) or at least triggered (automated comparators) by human interaction.

This proof of concept shows that the method is working and is easy to perform and the resulting uncertainties are appropriate for the intended use. Furthermore, the results are consistent to the other method within their uncertainties (see Figure 3).

In comparison to the stainless steel/PtIr transition calibration, the requirements of the same method applied for the industrial calibration of weight pieces according to OIML R 111-1 at ambient air, are less strict. Therefore, the use of

solid pieces made from an aluminium alloy appears to be possible.

The great advantages of the artefact method compared to the method using measurements of pressure, temperature and humidity are:

- far less initial investment cost
- far smaller maintenance effort and cost
- shorter downtime compared to the calibration of the electronic instruments for pressure, temperature and humidity

14. REMAINING OPEN QUESTIONS

The aim of this work was to provide a proof of concept of the artefact method in its intended use to support mass calibrations of class E₁. This generally succeeded. However, effects of possible covariances in the calculation of the air density values using the artefact method in a mass laboratory should be better understood and included in the calculations before the method is widely applied. Hints and considerations can be taken from [9].

15. CONCLUSION

This paper re-investigates the artefact method (according to Archimedes' principle) for determining air density for the calibration of weight pieces rather than using the traditionally applied "3-quantity method" (measuring air pressure, temperature and humidity). For the proof of concept, we employed aluminium artefacts of two different nominal values. The air density values obtained with these artefacts are consistent with the air density of the "3-quantity method" within uncertainties. The advantages of the artefact method lie in much lower costs and efforts with respect to maintenance compared to the traditional method. This paper intends to encourage mass calibration laboratories to investigate and use the artefact method to determine air density.

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Certain commercial equipment, instruments, and materials are identified in this paper in order to adequately describe the experimental procedure. Such identification does not imply recommendation or endorsement by the authors, nor does it imply that the materials or equipment identified are the only or best available for the purpose.