IMEKO 2010 TC3, TC5 and TC22 Conferences
Metrology in Modern Context
November 22–25, 2010, Pattaya, Chonburi, Thailand

# NON AUTOMATIC WEIGHBRIDGES SCALES INTERCOMPARISON IN SPAIN: A DIFFERENT APPROACH

*Nieves Medina* [1], *José Ángel Robles* [2]
[1] Spanish Centre of Metrology (CEM), Tres Cantos, Spain, mnmedina@cem.mityc.es
[1] Spanish Centre of Metrology (CEM), Tres Cantos, Spain, jarobles@cem.mityc.es

**Abstract** – This paper shows an interesting case for a national comparison where two transfer standards have been used. Participating laboratories have calibrated one transfer standard or the other, but one that calibrated both, so that can be used as a link to make a joint evalution for all the participant laboratories.

**Keywords**: weighbridges scale, comparison

## 1. INTRODUCTION

The non automatic weighbridges scales are very important for trade and shipping transport. It is clear the many commercial transactions are based on the use of this kind of instruments. As a consequence the economical consequences of their bad metrological performance are very important. On the other hand, they are also very important for security in transportation by truck.

The aim of this comparison is the calibration of non automatic weighbridges scales to support the scope of accreditated laboratories in this field. This calibration was performed according to the EURAMET cg-18 guide [1] and errors with their corresponding uncertainties were determined for 500 kg, 5 000 kg, 10 000 kg, 20 000 kg, 30 000 kg and 40 000 kg as nominal loads.

A total of 14 laboratories participated in the comparison. Only one of them calibrated both transfer standards. CEM acted as a coordinator and was in charged of the results evaluation. The calibration of this kind of instruments is very hard and time consuming and, as consequence, it is very expensive. It is remarkable to achieve such a big comparison with a high number of participants.

Two transfer standards have been used. Both transfer standards had the same features (Max 60 000 kg and e = 20 kg). The practical reason is the fact that this calibration is more expensive and time consuming depending on how far you have to move your standard weights. That is the reason why two transfer standards in two distant different locations where used, one in the northeast of Spain and the other one in the centre of Spain. The reason for that was convenience according to the participating laboratories geographical distribution.

## 2. EVALUATION PROCEDURE

The results evaluation has been performed according to [2], and it is based on a least squared fit.

As a starting point we had 14 results $y_1$, $y_2$, …$y_{14}$ for each nominal load, which is evaluated independently. 11 laboratories calibrated one weighbridges scale and 4 laboratories calibrated the other one. As a result two reference values $a_1$ and $a_2$, one for each weighbridges scale, were obtained for each nominal load.

If there was no uncertainty the model would be (1)

$$\begin{pmatrix} E(y_1) \\ E(y_2) \\ E(y_3) \\ E(y_4) \\ E(y_5) \\ E(y_6) \\ E(y_7) \\ E(y_8) \\ E(y_9) \\ E(y_{10}) \\ E(y_{11}) \\ E(y_{12}) \\ E(y_{13}) \\ E(y_{14}) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{1}$$

or written as a matrix (2)

$$E(\hat{y}) = \hat{X} \cdot \hat{a} \tag{2}$$

where $E(y_i)$ is he expected value for the mean $y_i$ obtained by laboratory $i$ for one nominal load.

The uncertainties will be given in the following variance covariance matrix, where correlation between measurements $y_{11}$ and $y_{10}$ is included, because they have been performed by the same laboratory.

$$\hat{\Sigma} = \begin{pmatrix}
u^2(y_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & u^2(y_2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & u^2(y_3) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & u^2(y_4) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & u^2(y_5) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & u^2(y_6) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & u^2(y_7) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & u^2(y_8) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u^2(y_9) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u^2(y_{10}) & u(y_{10},y_{11}) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u(y_{10},y_{11}) & u^2(y_{11}) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u^2(y_{12}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u^2(y_{13}) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u^2(y_{14})
\end{pmatrix}$$

For the least squared fit the function to minimise is given by expression (3)

$$\left(\hat{y}-\hat{X}\cdot\hat{a}\right)^{T}\hat{\Sigma}^{-1}\left(\hat{y}-\hat{X}\cdot\hat{a}\right) \quad (3)$$

which solution is given by equation (4)

$$\hat{a}=\left(\hat{X}^{T}\cdot\hat{\Sigma}^{-1}\cdot\hat{X}\right)^{-1}\left(\hat{X}^{T}\cdot\hat{\Sigma}^{-1}\cdot\hat{y}\right) \quad (4)$$

The uncertainty for this solution (the variance-covariance matrix for the reference values) will be given by equation (5)

$$u^{2}(\hat{a})=\hat{X}\cdot\left(\hat{X}^{T}\cdot\hat{\Sigma}^{-1}\cdot\hat{X}\right)^{-1}\cdot\hat{X}^{T} \quad (5)$$

The equivalence degrees (deviations from the reference values) are given by equation (6)

$$\hat{d}=\hat{y}-\hat{a} \quad (6)$$

and their uncertainties are given by equation (7)

$$u^{2}\left(\hat{d}\right)=u^{2}(\hat{y})-u^{2}(\hat{a}) \quad (7)$$

The minus sign is a consequence of correlation between measurements and the obtained reference values.

The results will be consistent if condition (8) is fulfilled

$$\left|\hat{d}\right|<U\left(\hat{d}\right) \quad (8)$$

A chi-squared test will be used to demonstrate the measurement equivalence. The observed chi-squared function in this case will be given by equation (9)

$$\chi^{2}=(\hat{y}-\hat{a})^{T}\cdot\hat{\Sigma}^{-1}\cdot(\hat{y}-\hat{a}) \quad (9)$$

and the degrees of freedom will be the measurements number minus the reference values number, in our case, 14 – 2 =12.

There is consistency if the condition (10) is fulfilled

$$Pr\{\ \chi^{2}(12)>\chi^{2}\ \}<0{,}05 \quad (10)$$

In case the previous condition is not fulfilled the condition is evaluated again excluding, one by one, the laboratories which did not fulfil the consistency condition (10).

The normalised degrees of equivalence between laboratories *i* and *j* is given by equation (11).

$$d_{i,j}=\frac{y_{i}-y_{j}}{2\sqrt{u^{2}(y_{i})+u^{2}(y_{j})}} \quad (11)$$

It has to be less than 1 for every pair of laboratories in order to demonstrate that their measurement results are consistent.

## 3. RESULTS

In this section the results for each nominal value are shown.

### 3.1. 500 kg

The reference values are $a_1$ = -3,6 kg with u ($a_1$) ($k$=1) = 3,2 kg and $a_2$ = -3,0 kg with u ($a_2$) ($k$=1) = 3,2 kg.

Fig 1. Results for 500 kg

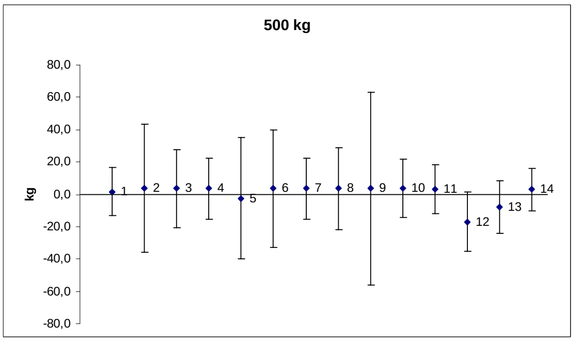



Table 1. Results for 500 kg

| Lab | $y_i$ (kg) | $u_c(y_i)$ (kg) ($k$ = 1) | $d_i$ (kg) | U($d_i$) (kg) ($k$=2) |
|---|---|---|---|---|
| 1 | -2,0 | 8,0 | 1,6 | 14,7 |
| 2 | 0,0 | 20,0 | 3,6 | 39,5 |
| 3 | 0,0 | 12,5 | 3,6 | 24,2 |
| 4 | 0,0 | 10,0 | 3,6 | 19,0 |
| 5 | -6,0 | 19,0 | -2,4 | 37,5 |
| 6 | 0,0 | 18,5 | 3,6 | 36,5 |
| 7 | 0,0 | 10,0 | 3,6 | 19,0 |
| 8 | 0,0 | 13,0 | 3,6 | 25,2 |
| 9 | 0,0 | 30,0 | 3,6 | 59,7 |
| 10 | 0,0 | 9,5 | 3,6 | 17,9 |
| 11 | 0,0 | 8,0 | 3,0 | 15,1 |
| 12 | -20,0 | 9,5 | -17,0 | 18,2 |
| 13 | -11,0 | 8,5 | -8,0 | 16,1 |
| 14 | 0,0 | 7,0 | 3,0 | 13,0 |

It can be checked that condition (8) is always fulfilled.

The chi-squared value for the data is $\chi^2=5{,}0$.

As we have 12 degrees of freedom, then $\chi^2(\upsilon)=21$ for a 0,05 probability, so condition (10) is also fulfilled.

The normalised degrees of freedom are less than 1 in all cases.

### 3.2. 5 000 kg

The reference values are $a_1$ = -2,5 kg with u ($a_1$) ($k$=1) = 2,9 kg and $a_2$ = -22,9 kg with u ($a_2$) ($k$=1) = 2,9 kg.

It can be checked that condition (8) is always fulfilled.

The chi-squared value for the data is $\chi^2=5{,}0$.

As we have 12 degrees of freedom, then $\chi^2(\upsilon)=21$ for a 0,05 probability, so condition (10) is also fulfilled.

The normalised degrees of freedom are less than 1 in all cases but laboratory 12, which is not consistent with other five laboratories.

Fig 2. Results for 5 000 kg

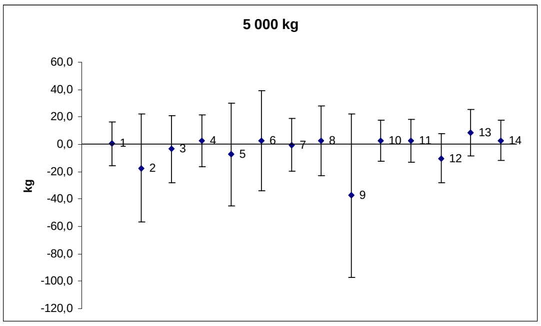



Table 2. Results for 5 000 kg

| Lab | $y_i$ (kg) | $u_c(y_i)$ (kg) ($k$ = 1) | $d_i$ (kg) | $U(d_i)$ (kg) ($k$=2) |
|---|---|---|---|---|
| 1 | -2,0 | 8,5 | 0,5 | 15,9 |
| 2 | -20,0 | 20,0 | -17,5 | 39,6 |
| 3 | -6,0 | 12,5 | -3,5 | 24,3 |
| 4 | 0,0 | 10,0 | 2,5 | 19,1 |
| 5 | -10,0 | 19,0 | -7,5 | 37,5 |
| 6 | 0,0 | 18,5 | 2,5 | 36,5 |
| 7 | -3,0 | 10,0 | -0,5 | 19,1 |
| 8 | 0,0 | 13,0 | 2,5 | 25,3 |
| 9 | -40,0 | 30,0 | -37,5 | 59,7 |
| 10 | 0,0 | 8,0 | 2,5 | 14,9 |
| 11 | -20,0 | 8,5 | 2,7 | 15,8 |
| 12 | -33,0 | 9,5 | -10,3 | 17,9 |
| 13 | -14,0 | 9,0 | 8,7 | 16,9 |
| 14 | -20,0 | 8,0 | 2,7 | 14,7 |

### *3.3. 10 000 kg*

The reference values are $a_1$ = -1,0 kg with u ($a_1$) ($k$=1) = 3,0 kg and $a_2$ = -21,2 kg with u ($a_2$) ($k$=1) = 3,0 kg.

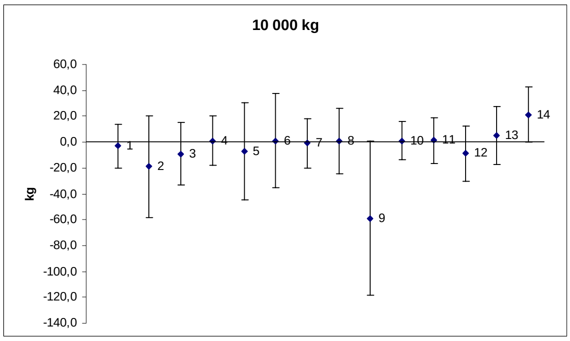
Table 3. Results for 10 000 kg

| Lab | $y_i$ (kg) | $u_c(y_i)$ (kg) ($k$ = 1) | $d_i$ (kg) | $U(d_i)$ (kg) ($k$=2) |
|---|---|---|---|---|
| 1 | -4,0 | 9,0 | -3,0 | 17,0 |
| 2 | -20,0 | 20,0 | -19,0 | 39,5 |
| 3 | -10,0 | 12,5 | -9,0 | 24,3 |
| 4 | 0,0 | 10,0 | 1,0 | 19,1 |
| 5 | -8,0 | 19,0 | -7,0 | 37,5 |
| 6 | 0,0 | 18,5 | 1,0 | 36,5 |
| 7 | -2,0 | 10,0 | -1,0 | 19,1 |
| 8 | 0,0 | 13,0 | 1,0 | 25,3 |
| 9 | -60,0 | 30,0 | -59,0 | 59,7 |
| 10 | 0,0 | 8,0 | 1,0 | 14,8 |
| 11 | -20,0 | 9,5 | 1,2 | 17,6 |
| 12 | -30,0 | 10,0 | -8,8 | 21,2 |
| 13 | -16,0 | 10,5 | 5,2 | 22,2 |
| 14 | 0,0 | 10,0 | 21,2 | 21,2 |

It can be checked that condition (8) is always fulfilled.
The chi-squared value for the data is $\chi^2 = 12{,}2$.

As we have 12 degrees of freedom, then $\chi^2(\upsilon) = 21$ for a 0,05 probability, so condition (10) is also fulfilled.
The normalised degrees of freedom are less than 1 in all cases but laboratory 12, which is not consistent with other two laboratories.

Fig 3. Results for 10 000 kg




### *3.4. 20 000 kg*

The reference values are $a_1$ = -3,7 kg with u ($a_1$) ($k$=1) = 3,2 kg and $a_2$ = -25,7 kg with u ($a_2$) ($k$=1) = 3,2 kg.

Fig 4. Results for 20 000 kg

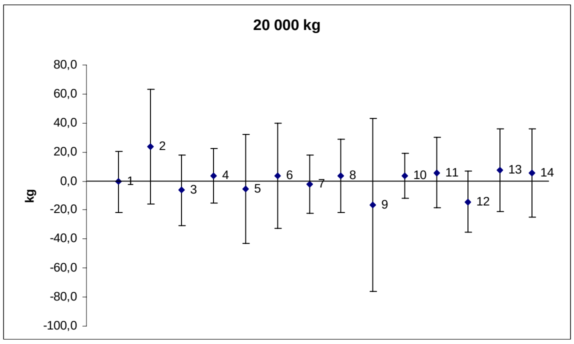



Table 4. Results for 500 kg

| Lab | $y_i$ (kg) | $u_c(y_i)$ (kg) ($k$ = 1) | $d_i$ (kg) | $U(d_i)$ (kg) ($k$=2) |
|---|---|---|---|---|
| 1 | -4,0 | 11,0 | -0,3 | 21,1 |
| 2 | 20,0 | 20,0 | 23,7 | 39,5 |
| 3 | -10,0 | 12,5 | -6,3 | 24,2 |
| 4 | 0,0 | 10,0 | 3,7 | 19,0 |
| 5 | -9,0 | 19,0 | -5,3 | 37,5 |
| 6 | 0,0 | 18,5 | 3,7 | 36,5 |
| 7 | -6,0 | 10,5 | -2,3 | 20,0 |
| 8 | 0,0 | 13,0 | 3,7 | 25,2 |
| 9 | -20,0 | 30,0 | -16,3 | 59,7 |
| 10 | 0,0 | 8,5 | 3,7 | 15,8 |
| 11 | -20,0 | 13,0 | 5,7 | 24,2 |
| 12 | -40,0 | 11,5 | -14,3 | 20,9 |
| 13 | -18,0 | 15,0 | 7,7 | 28,5 |
| 14 | -20,0 | 16,0 | 5,7 | 30,6 |

It can be checked that condition (8) is always fulfilled.
The chi-squared value for the data is $\chi^2 = 4{,}5$.

As we have 12 degrees of freedom, then $\chi^2(\upsilon)=21$ for a 0,05 probability, so condition (10) is also fulfilled.
The normalised degrees of freedom are less than 1 in all cases but laboratory 12, which is not consistent with other two laboratories.

### 3.5. 30 000 kg

The reference values are $a_1$ = 1,4 kg with u ($a_1$) ($k$=1) = 4,3 kg and $a_2$ = -17,6 kg with u ($a_2$) ($k$=1) = 4,3 kg.

Fig 5. Results for 30 000 kg

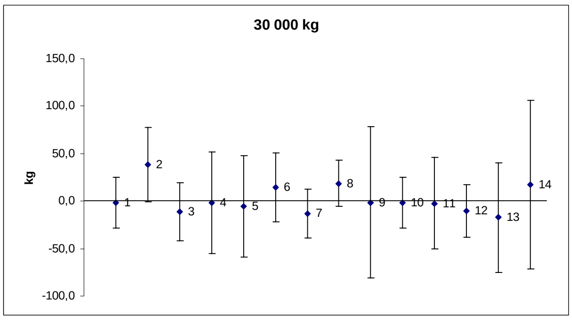



Table 5. Results for 30 000 kg

| Lab | $y_i$ (kg) | $u_c(y_i)$ (kg) ($k$ = 1) | $d_i$ (kg) | $U(d_i)$ (kg) ($k$=2) |
|---|---|---|---|---|
| 1 | 0,0 | 14,0 | -1,4 | 26,6 |
| 2 | 40,0 | 20,0 | 38,6 | 39,1 |
| 3 | -10,0 | 16,0 | -11,4 | 30,8 |
| 4 | 0,0 | 27,0 | -1,4 | 53,3 |
| 5 | -4,0 | 27,0 | -5,4 | 53,3 |
| 6 | 16,0 | 18,5 | 14,6 | 36,0 |
| 7 | -12,0 | 13,5 | -13,4 | 25,6 |
| 8 | 20,0 | 13,0 | 18,6 | 24,5 |
| 9 | 0,0 | 40,0 | -1,4 | 79,5 |
| 10 | 0,0 | 14,0 | -1,4 | 26,6 |
| 11 | -20,0 | 24,5 | -2,4 | 48,2 |
| 12 | -28,0 | 14,5 | -10,4 | 27,7 |
| 13 | -35,0 | 29,0 | -17,4 | 57,4 |
| 14 | 0,0 | 44,5 | 17,6 | 88,6 |

It can be checked that condition (8) is always fulfilled.
The chi-squared value for the data is $\chi^2 = 9{,}0$.

As we have 12 degrees of freedom, then $\chi^2(\upsilon)=21$ for a 0,05 probability, so condition (10) is also fulfilled.
The normalised degrees of freedom are less than 1 in all cases but laboratories 2, 7, 8, 12 y 13.

### 3.6. 40 000 kg

The reference values are $a_1$ = 13,8 kg with u ($a_1$) ($k$=1) = 4,4 kg and $a_2$ = -12,1 kg with u ($a_2$) ($k$=1) = 4,4 kg.
It can be checked that condition (8) is not fulfilled by laboratory 2.
The chi-squared value for the data is $\chi^2 = 10{,}8$.

As we have 12 degrees of freedom, then $\chi^2(\upsilon)=21$ for a 0,05 probability, so condition (10) is also fulfilled.

The normalised degrees of freedom are less than 1 in all cases but laboratories 2, 3, 7, 12 y 13

Fig 6. Results for 40 000 kg

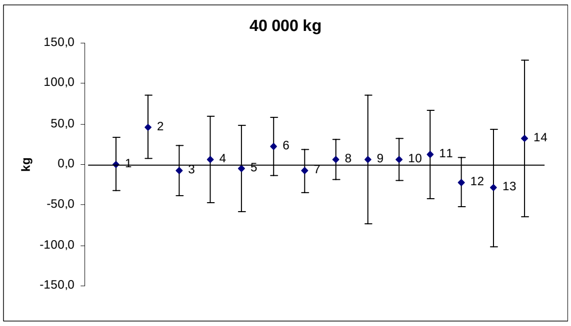



Table 6. Results for 40 000 kg

| Lab | $y_i$ (kg) | $u_c(y_i)$ (kg) ($k$ = 1) | $d_i$ (kg) | $U(d_i)$ (kg) ($k$=2) |
|---|---|---|---|---|
| 1 | 14,0 | 17,0 | 0,2 | 32,8 |
| 2 | 60,0 | 20,0 | 46,2 | 39,0 |
| 3 | 6,0 | 16,0 | -7,8 | 30,8 |
| 4 | 20,0 | 27,0 | 6,2 | 53,3 |
| 5 | 9,0 | 27,0 | -4,8 | 53,3 |
| 6 | 36,0 | 18,5 | 22,2 | 35,9 |
| 7 | 6,0 | 14,0 | -7,8 | 26,6 |
| 8 | 20,0 | 13,0 | 6,2 | 24,5 |
| 9 | 20,0 | 40,0 | 6,2 | 79,5 |
| 10 | 20,0 | 14,0 | 6,2 | 26,6 |
| 11 | 0,0 | 27,5 | 12,1 | 54,3 |
| 12 | -34,0 | 16,0 | -21,9 | 30,8 |
| 13 | -41,0 | 36,5 | -28,9 | 72,5 |
| 14 | 20,0 | 48,5 | 32,1 | 96,6 |

## 3. CONCLUSIONS

The obtained degree of equivalence is very high because there was only one discrepant laboratory and only for 40 000 kg.

There are also some inconsistencies between laboratories especially for higher loads. The reason for that may be some instability in the standards that cannot be taken into account by the evaluation procedure.

This is the second comparison for this kind of instruments in Spain. The first one was a theoretical comparison and the obtained results where highly non-consistent. The reason for the observed discrepancies was the fact that the measurement procedures and uncertainty evaluations were very different. Nowadays this problem has been solved with the document EURAMET cg-18 [1].

## REFERENCES

[1] *EURAMET/cg-18/v 0.2,* (January 2009).
[2] L. Nielsen *Evaluation of measurement intercomparisons by the method of least squares,* Internal document DFM.