

The unit kilogram

needs and experiments for a new definition

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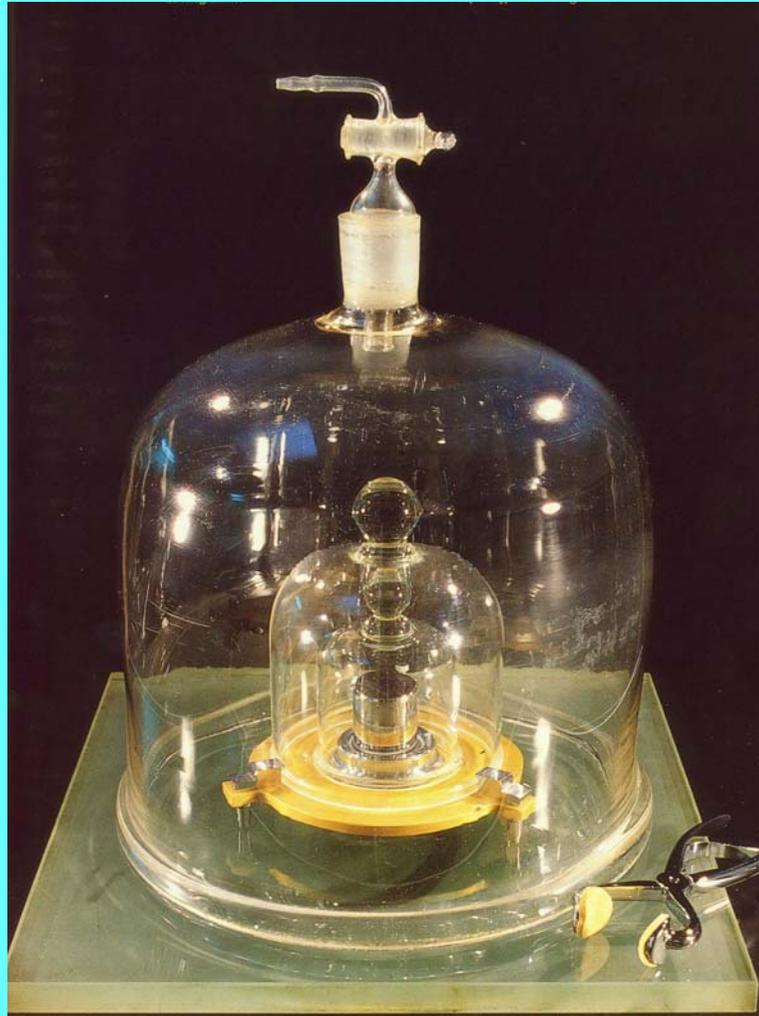
- **The present kilogram**
- **Different approaches for a new definition**
- **Ion accumulation in detail**

Definition of the kilogram

**The kilogram is the unit of mass;
it is equal to the mass of the
international prototype of the kilogram**

(1st CGPM 1889 and 3rd CGPM 1901)

The international prototype of the Kilogram **PTB**

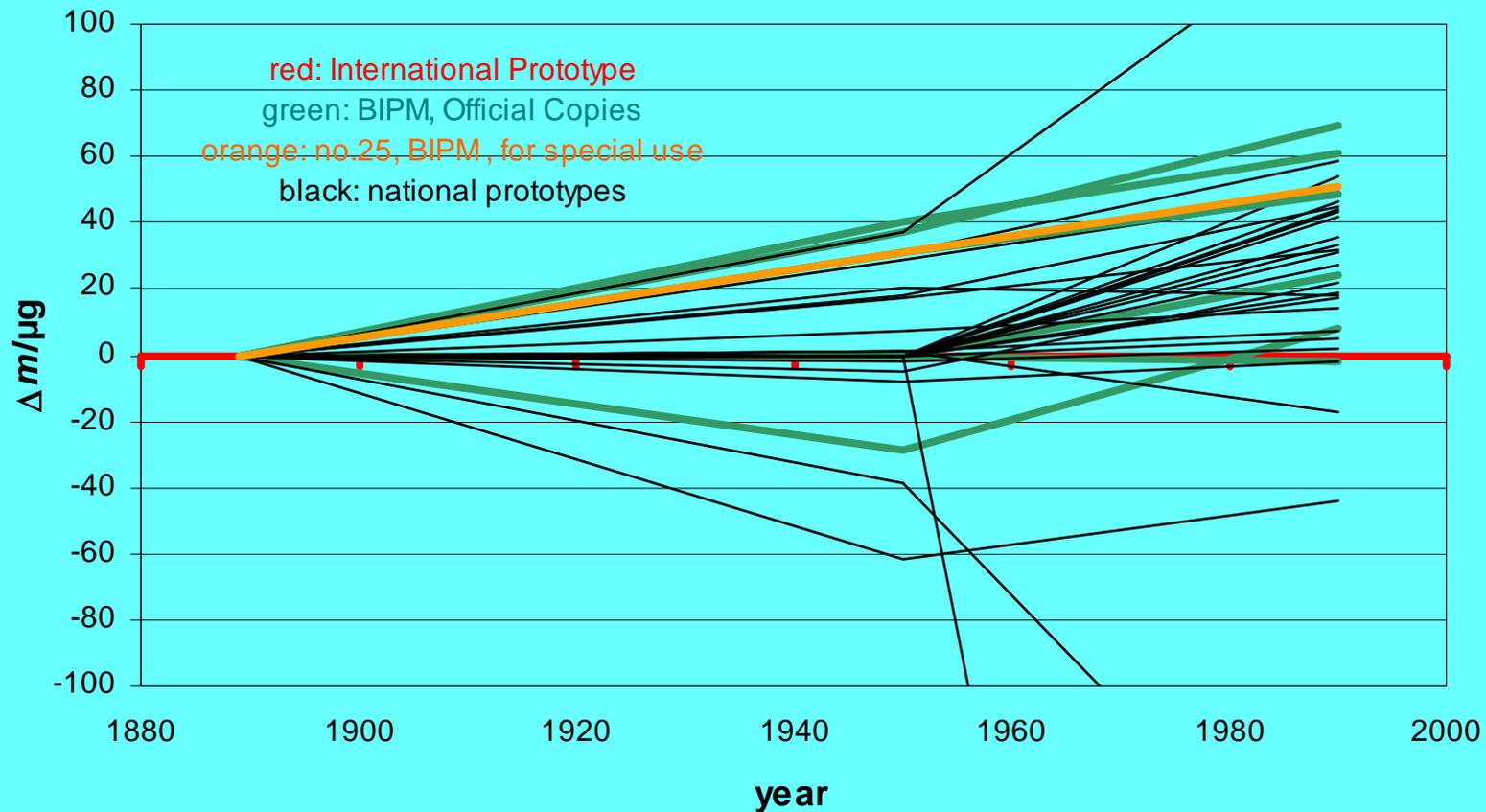


BIPM



Michael Gläser, PTB, 19th IMEKO TC 3, Cairo 2005

Mass changes of the prototypes 1889, 1950, 1990



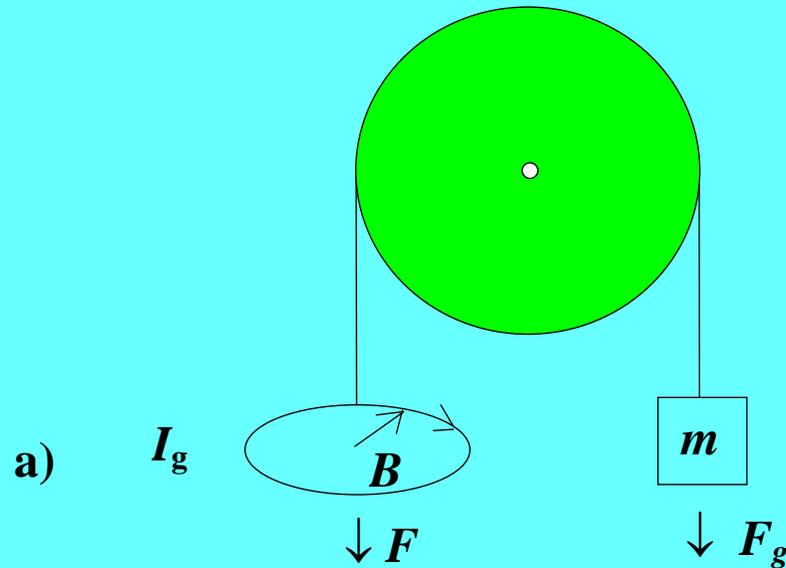
Why is the kilogram to be re-defined?

- **The International Prototype may become damaged**
(it happened to some national copies, e.g. no.1,11,22,29,33,42)
- **It could get lost**
(the prototype of Argentina, no.45 has been stolen)
- **It has probably been changed already**
(40 other prototypes became heavier by about 40 μg within 100 years)
- **The other base units are defined by constants in nature**
(fundamental constants of physics or material constants)

Approaches for a new Definition

- **Watt balance experiment (Planck constant)**
two modes of operation in the same magnetic field:
 - gravitational force equilibrated by electromagnetic force
 - induced voltage by a moving coil
- **Silicon crystal (Avogadro constant)**
ratio between the molar volume and the atomic volume
- **Ion accumulation (Atomic mass unit)**
mass and charge accumulated from an ion beam
mass/charge ratio measured = mass/charge ratio of a single ion
- **Magnetic levitation (Planck constant)**
gravitational force equilibrated by electromagnetic force
differential change of superconducting current at different height

Watt Balance



Measuring the coil current in a magnetic field ~ weight force



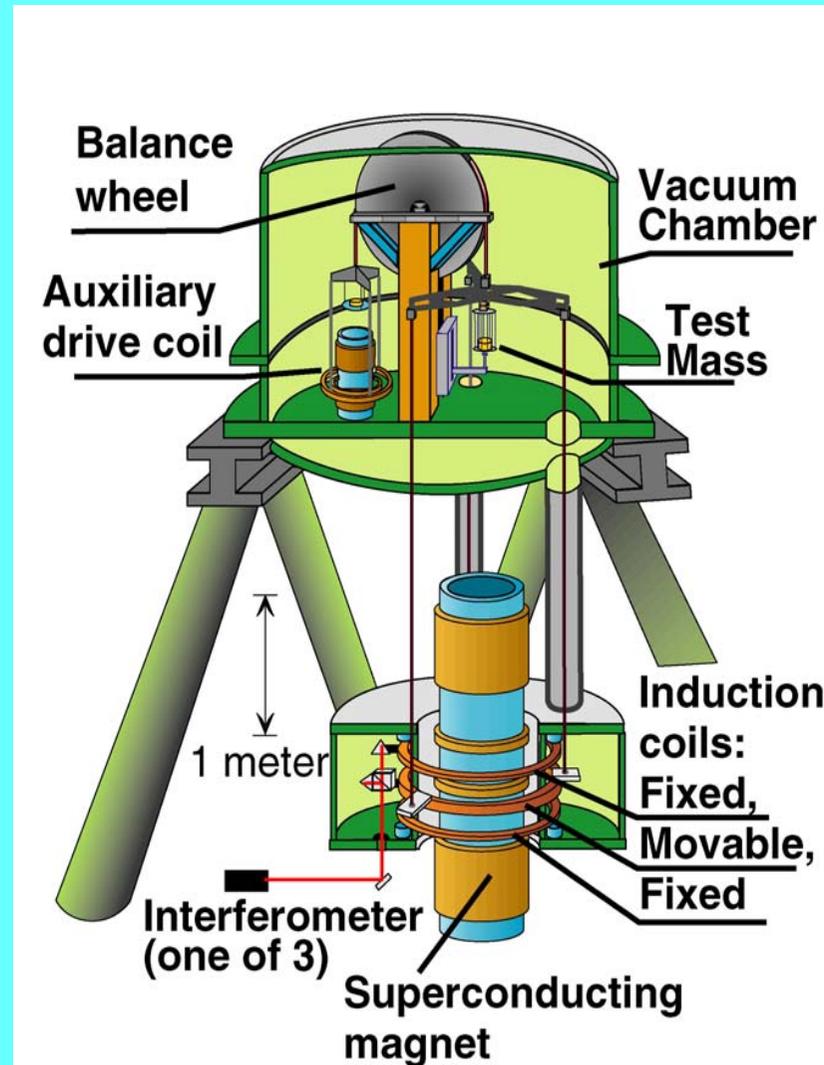
Measuring the induced voltage at the coil moved in the same magnetic field

$$UI = mgv$$

$$UI = h f_g f_{\text{ind}} / 4$$

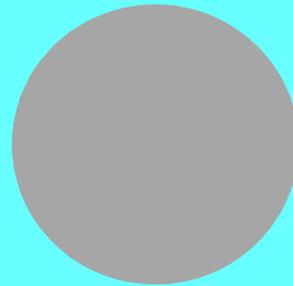
Planck constant $h = 6,626\ 068\ 91\ (58) \times 10^{-34}\ \text{Js}$ (NIST, 1998)

The new NIST Watt Balance



Avogadro Constant

Silicon sphere

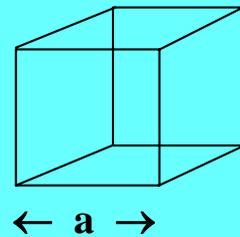


mass m

volume V

molar mass M_{Si}

unit cell of a silicon crystal



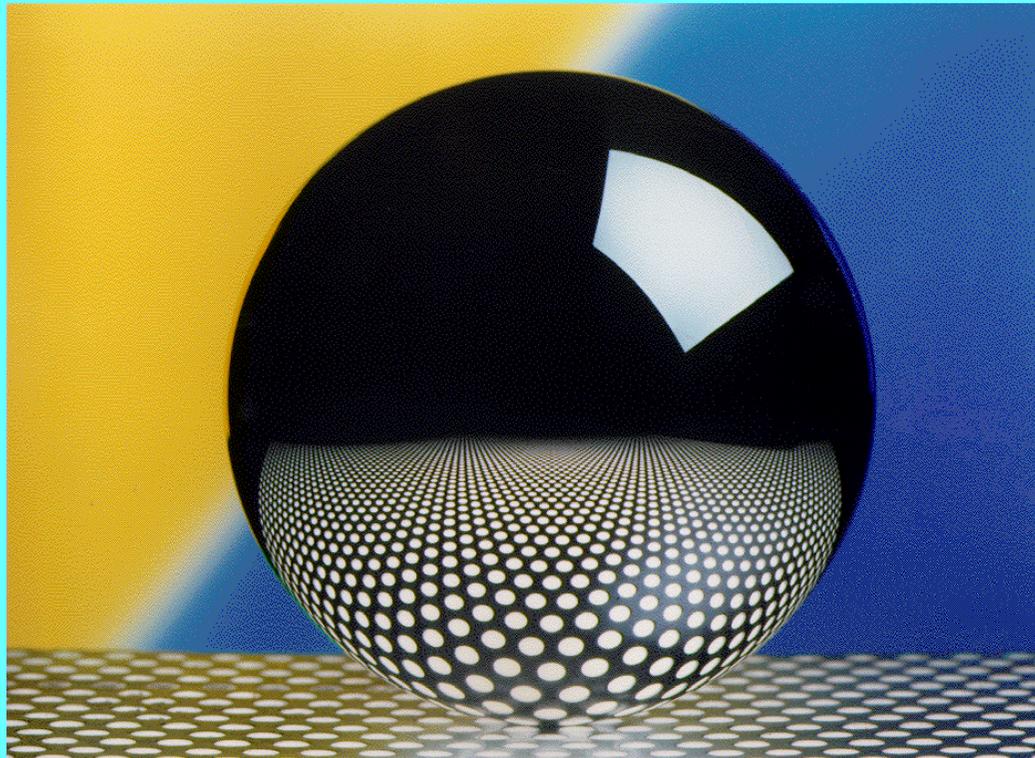
volume $v_0 = a^3$ (a lattice parameter)

contains $n = 8$ silicon atoms

$$N_A = \frac{V}{v_0 / n} \cdot \frac{M_{\text{Si}}}{m}$$

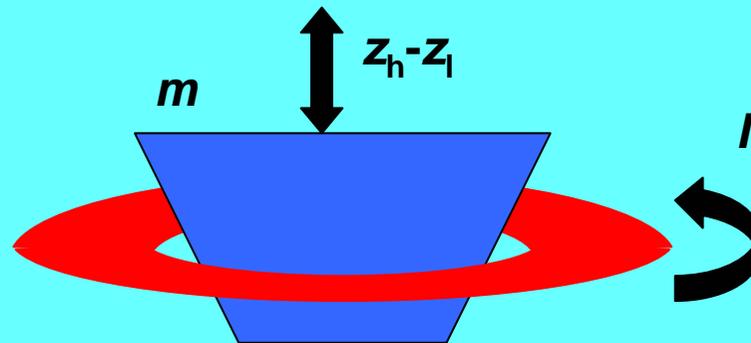
$$M_{\text{Si}} = f_{28} m_{\text{Si-28}} + f_{29} m_{\text{Si-29}} + f_{30} m_{\text{Si-30}}$$

The Silicon Sphere



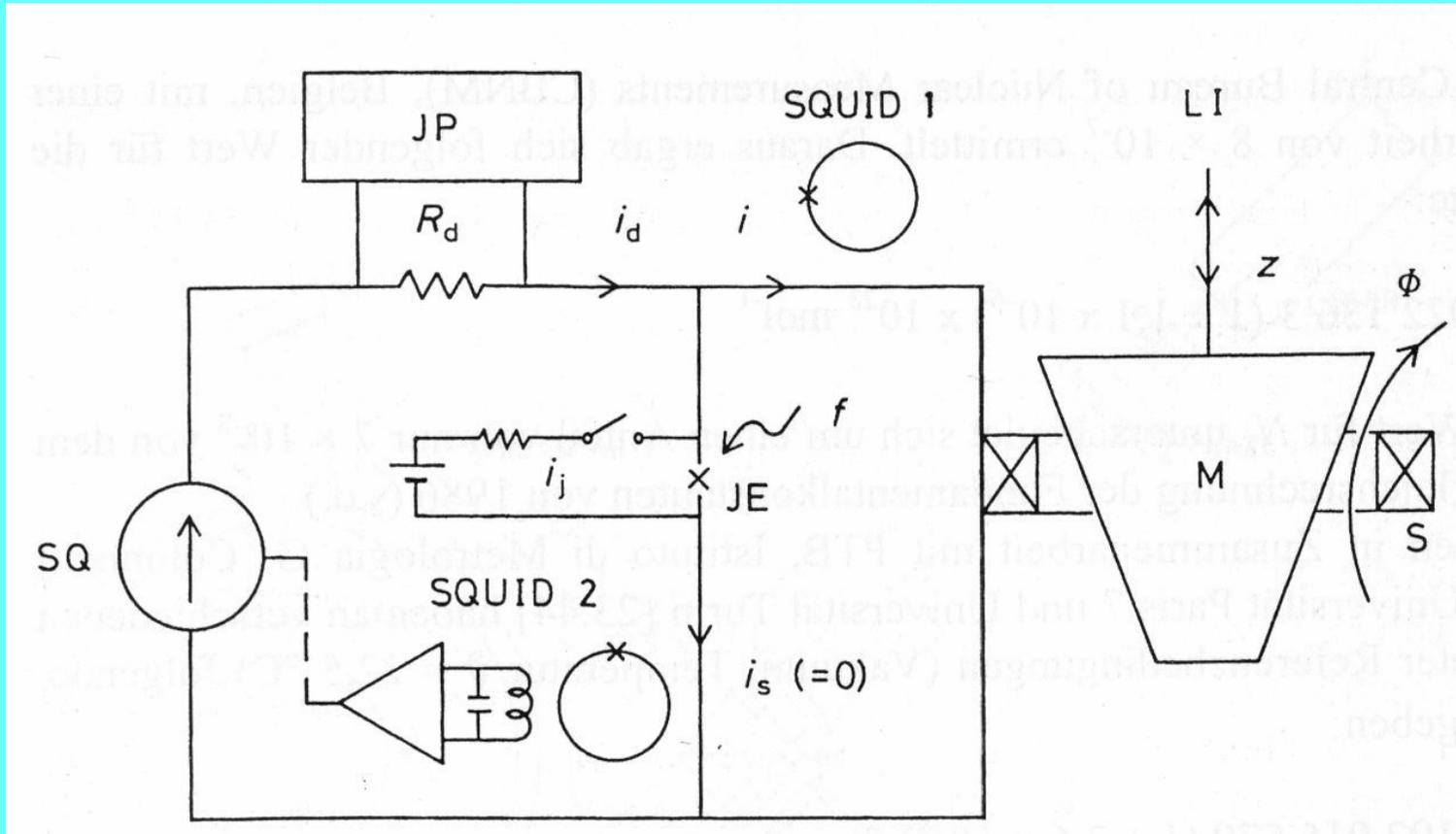
Made from a single crystal, machined to a few nm

Magnetic Levitation

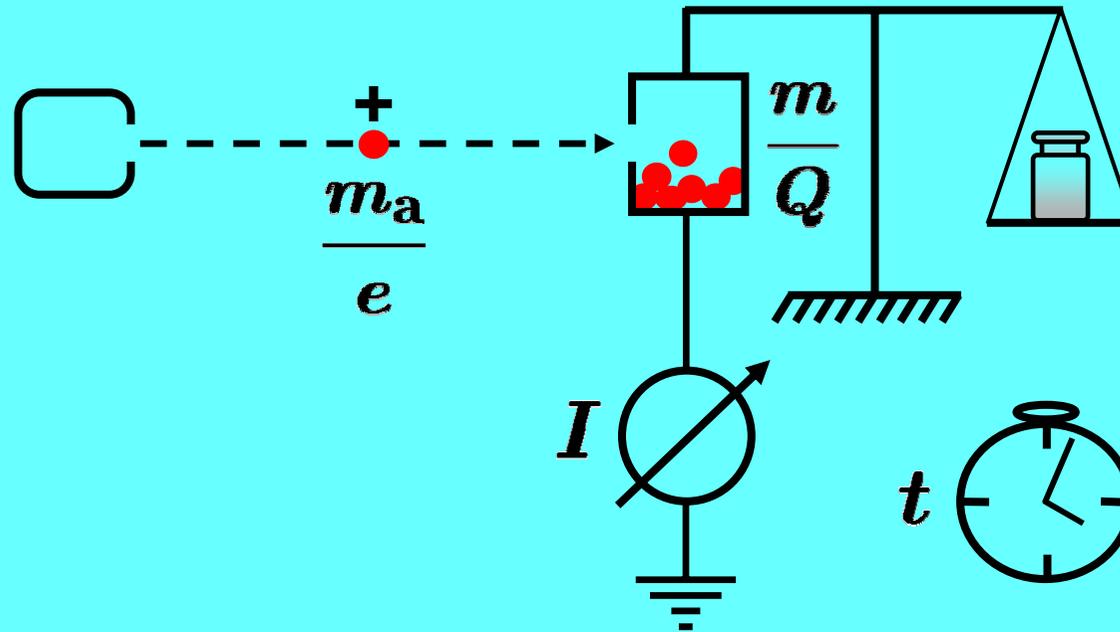


$$h = \frac{4mg(z_h - z_l)}{\int_{N_l}^{N_h} f dN - \frac{1}{2}(N_h f_h - N_l f_l)}$$

Magnetic Levitation (NMIJ/AIST)



Ion Accumulation



$$\frac{m_a}{e} = \frac{m}{Q}$$

$$m_a = A_R \cdot m_u$$

$$Q = \int I(t) \cdot dt \quad m_u = m \frac{e}{A_R} \left[\int I(t) \cdot dt \right]^{-1}$$

Elimination of „e“ by quantum standards

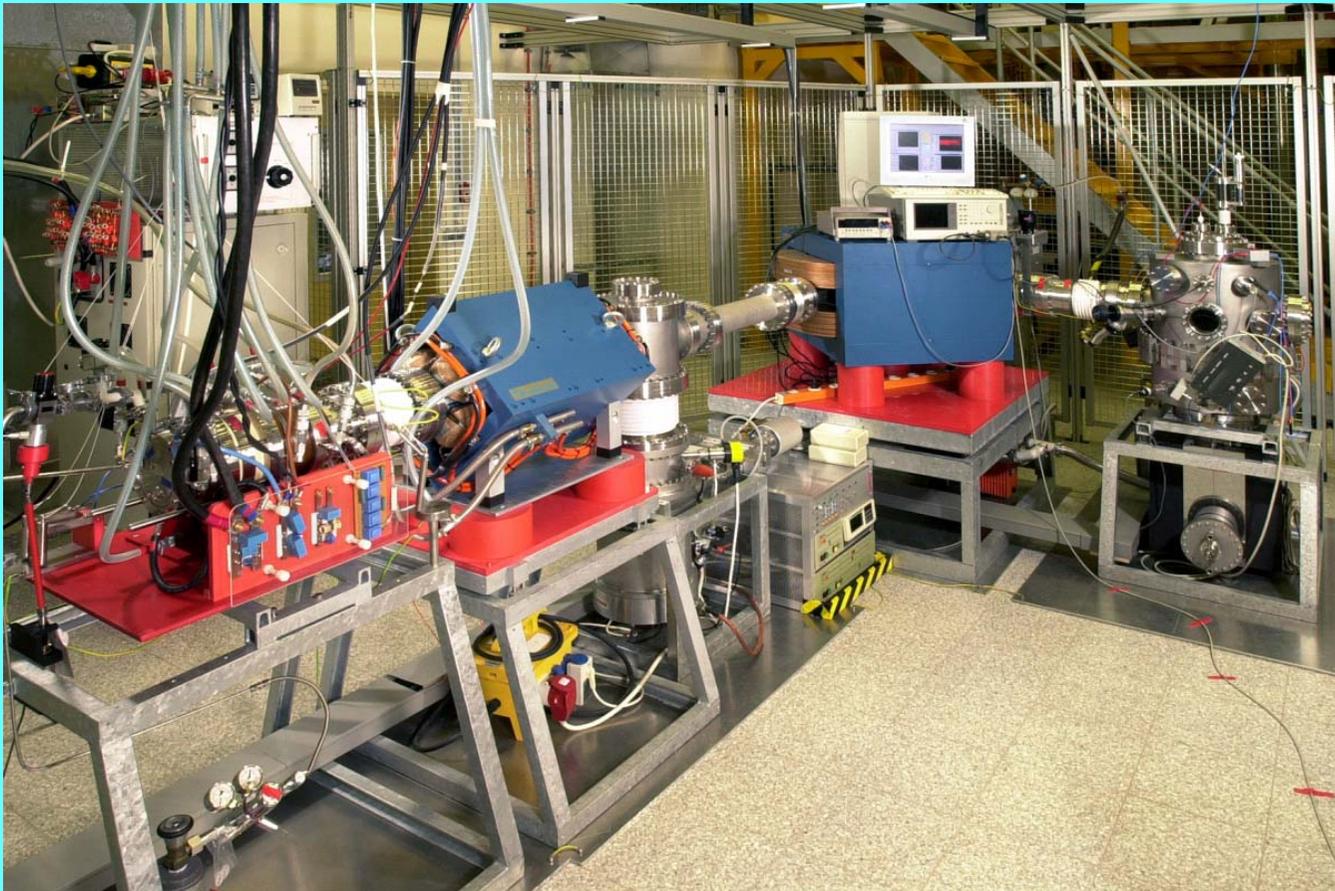
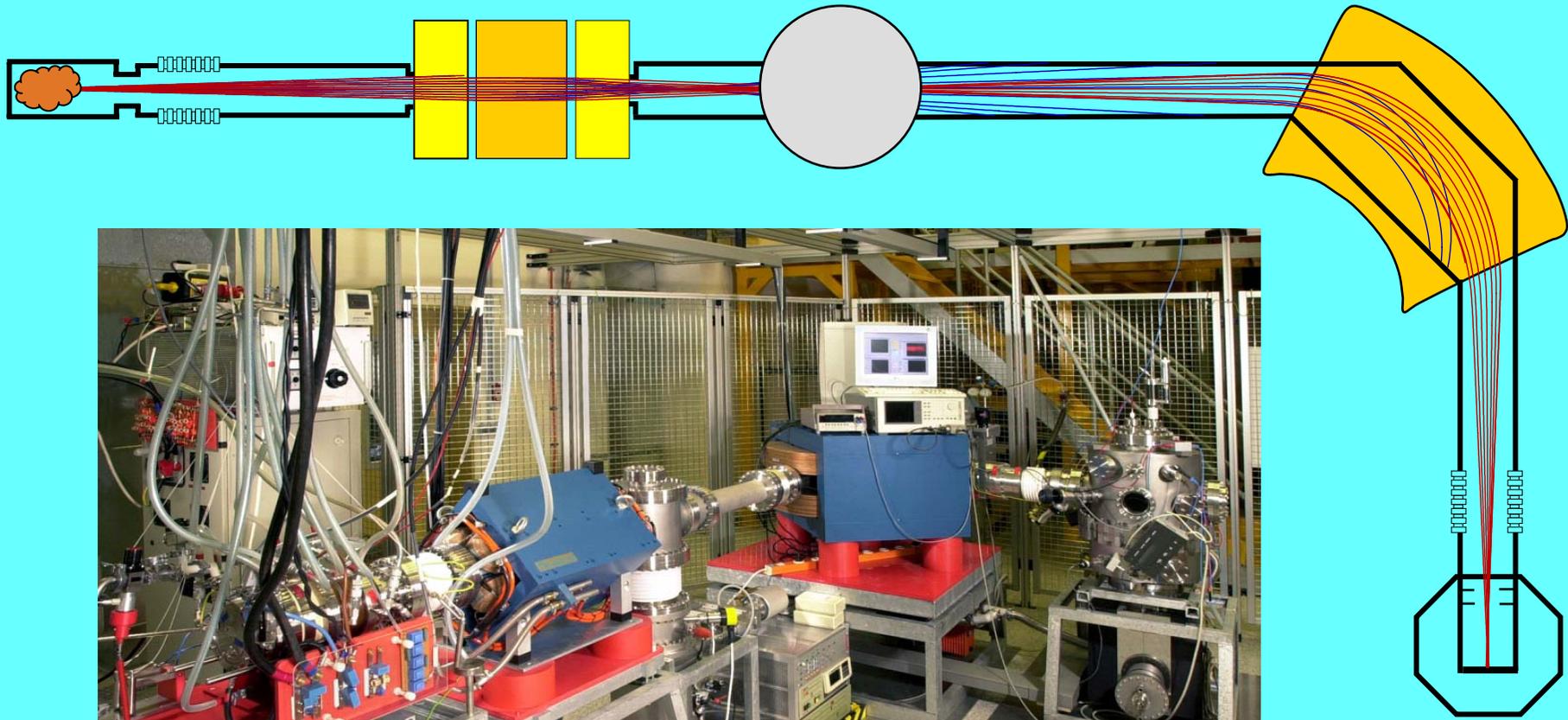
$$R = \frac{h}{n_1 e^2} \quad \text{Quantum Hall resistance } R$$

$$\frac{f}{U} = \frac{2e}{n_2 h} \quad \text{Josephson voltage } U$$

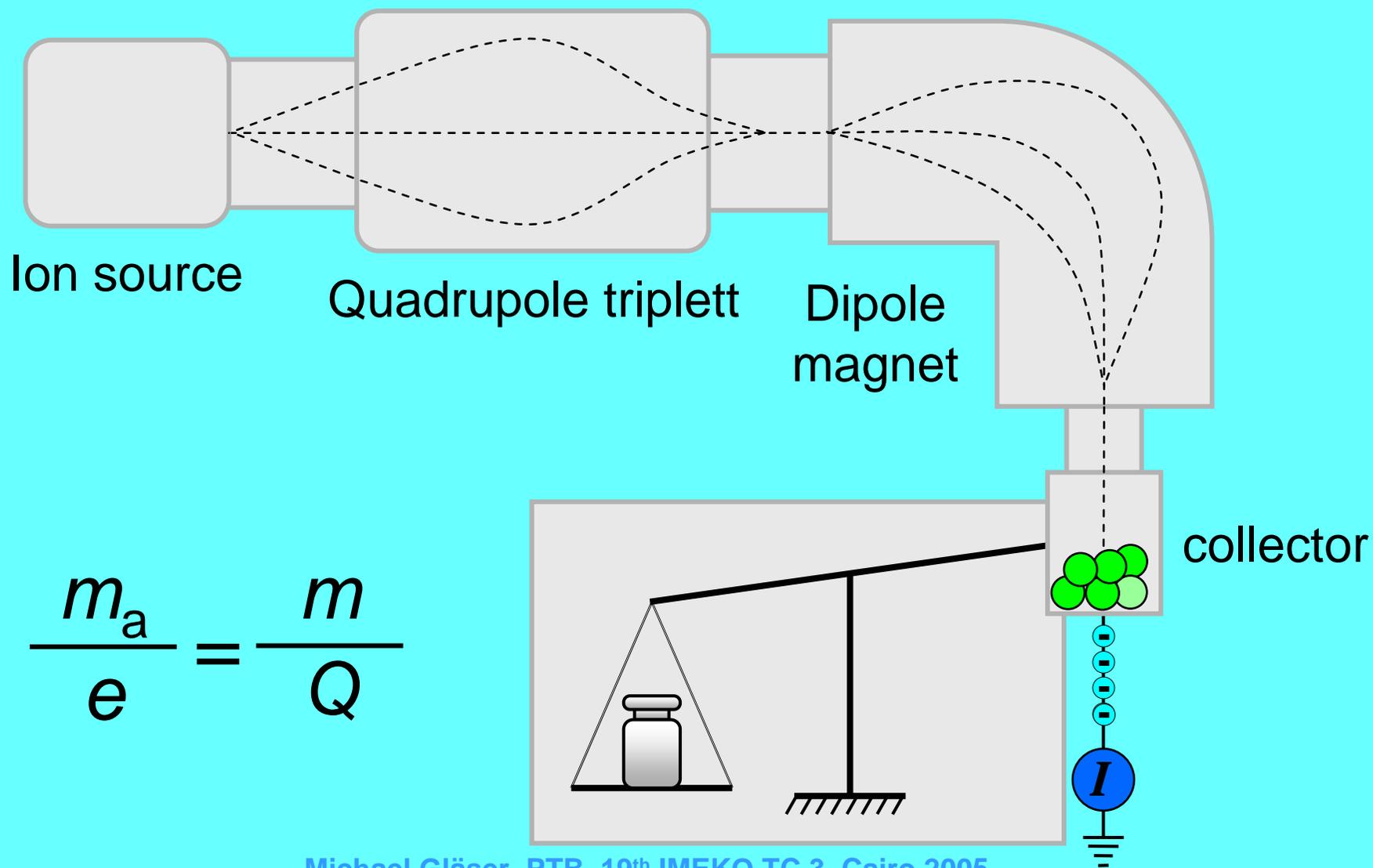
$$I = \frac{U}{R} = \frac{1}{2} n_1 n_2 e f$$

$$m_u = \frac{2m}{n_1 n_2 A_r} \left(\int_{t=0}^{t=t_m} f(t) dt \right)^{-1}$$

Experimental set-up



Ion trajectories



$$\frac{m_a}{e} = \frac{m}{Q}$$

Some mono-isotopic elements

Element	Z	A	Ionis.en./eV	$t_m/^{\circ}\text{C}$	$t_b/^{\circ}\text{C}$	$T/^{\circ}\text{C}$ at $p=10^{-14}\text{mbar}$
Be	4	9	9,32	1278	2970	444
Al	13	27	5,99	660	2467	444
Sc	21	45	6,54	1539	2831	533
Mn	25	55	7,44	1244	1962	335
Co	27	59	7,86	1495	2870	633
Y	39	89	6,38	1526	3338	651
Nb	41	93	6,76	2468	4744	1239
Rh	45	103	7,46	1966	3727	888
Au	79	197	9,23	1065	2807	540
Bi	83	209	7,29	271	1564	167

Accumulation time

$$t = m \cdot \frac{e}{I \cdot m_A}$$

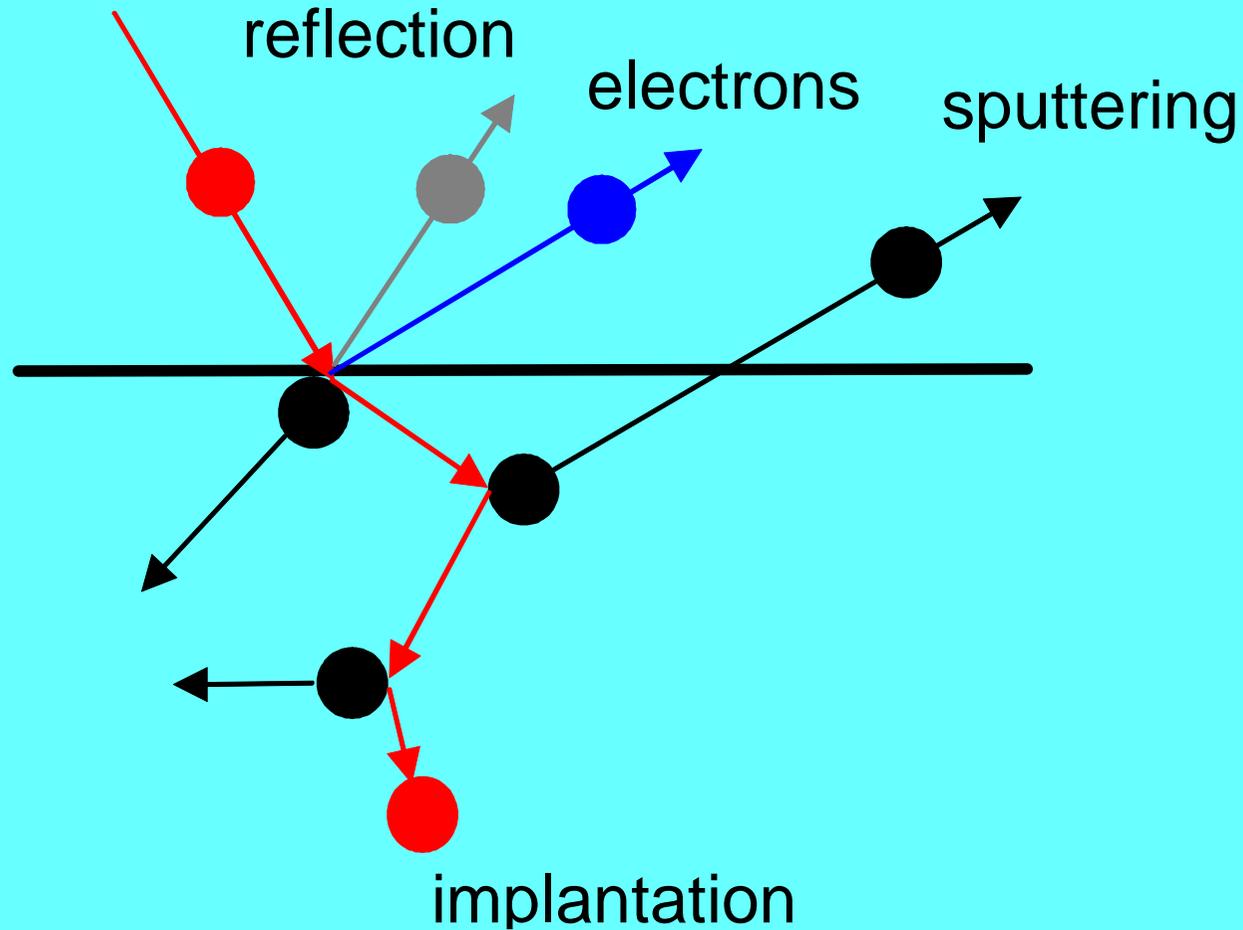
ion current in mA	particle flow in part/s	mass flow in g/day	accumulation time for 10 g bismuth in days
1	$6,24 \times 10^{15}$	0,187	53,44
6	$3,74 \times 10^{16}$	1,123	8,91
10	$6,24 \times 10^{16}$	1,871	5,34

Problems

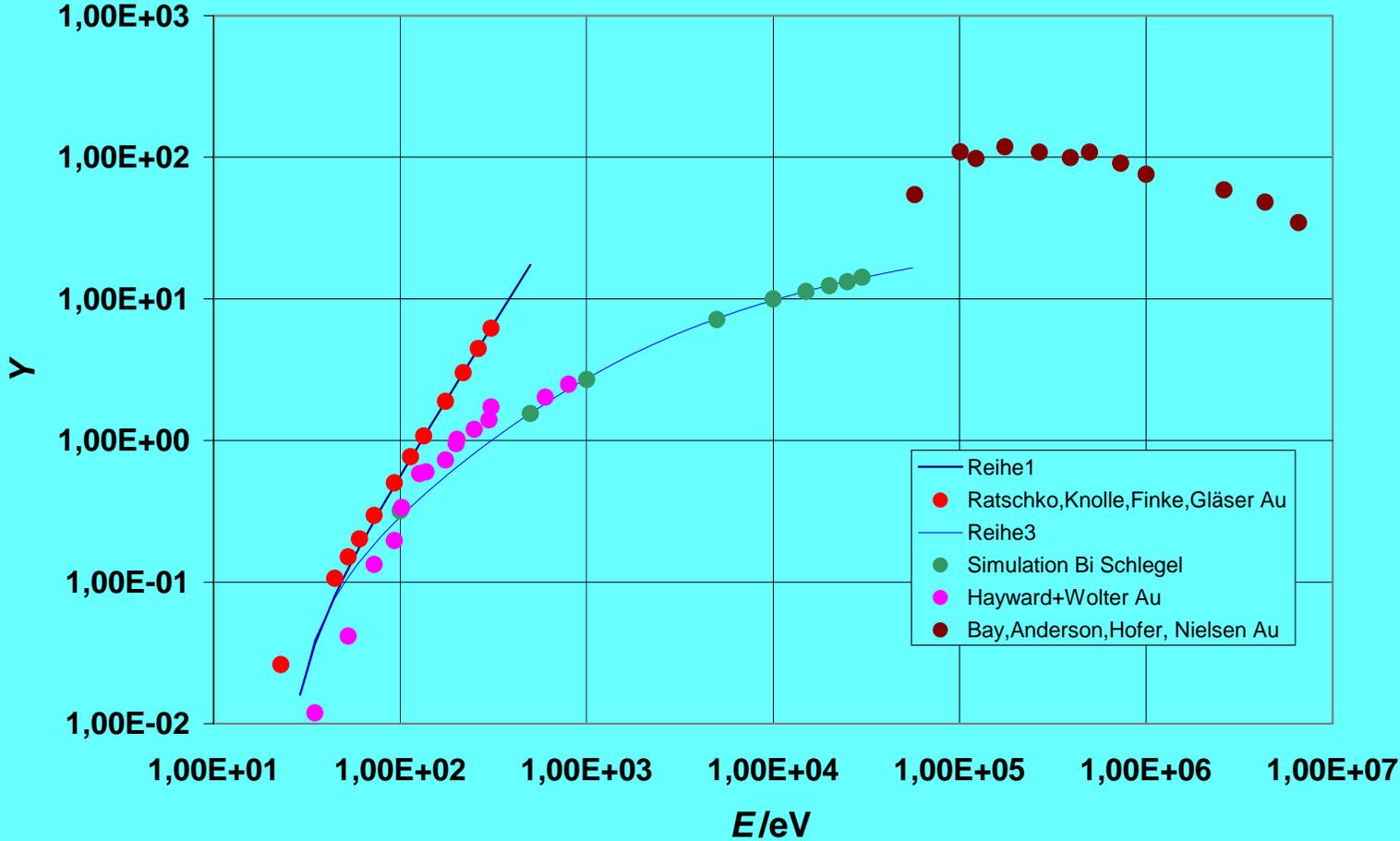


- Lost particles due to sputtering and reflection
→ deceleration from 25 keV to < 1 keV
- Foreign particles (electrons, residual gas molecules, aperture material)
- Focusing the decelerated beam
- Sufficient accumulated mass
→ high ion current of several mA
- Current accuracy
- Weighing accuracy

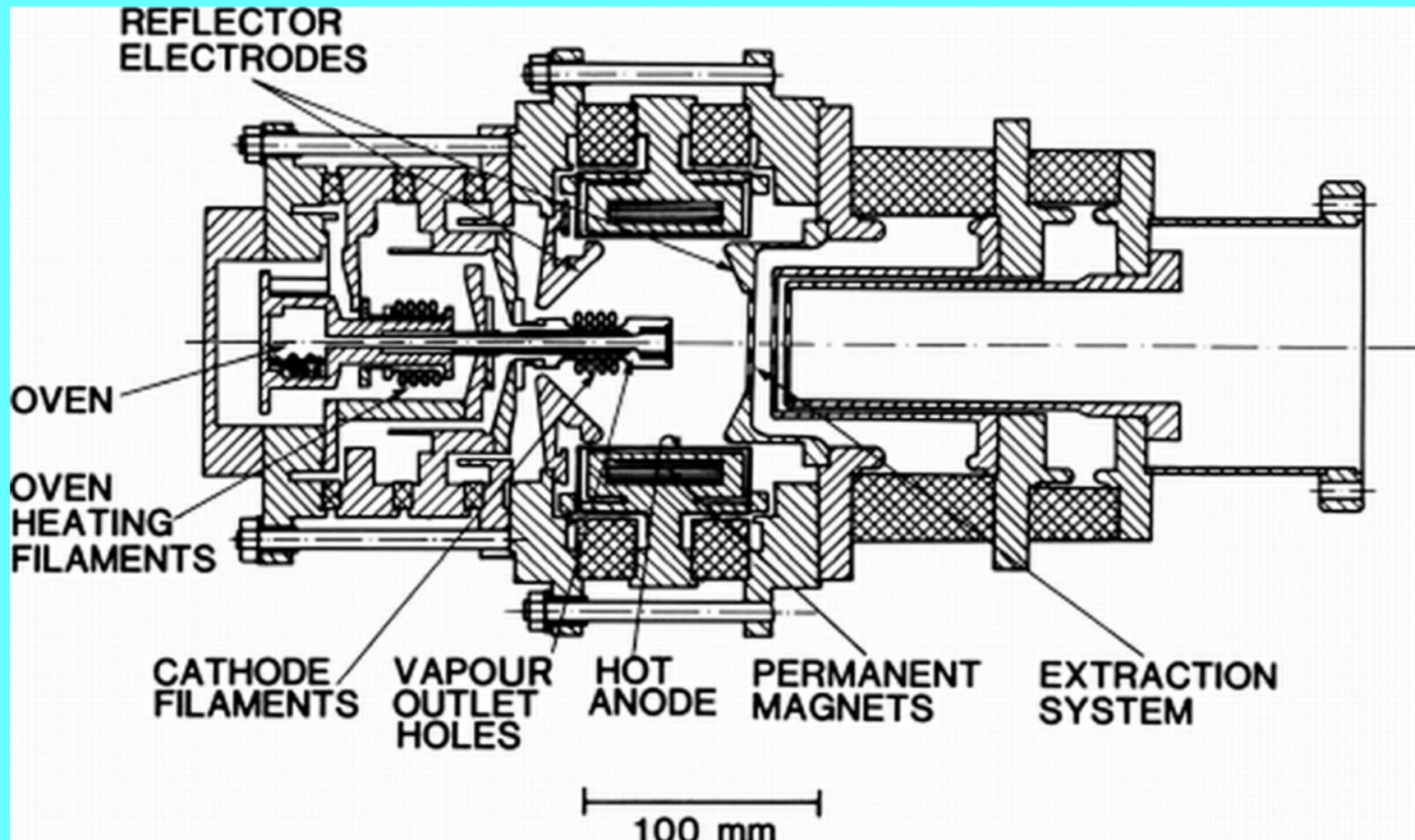
Ion surface effects



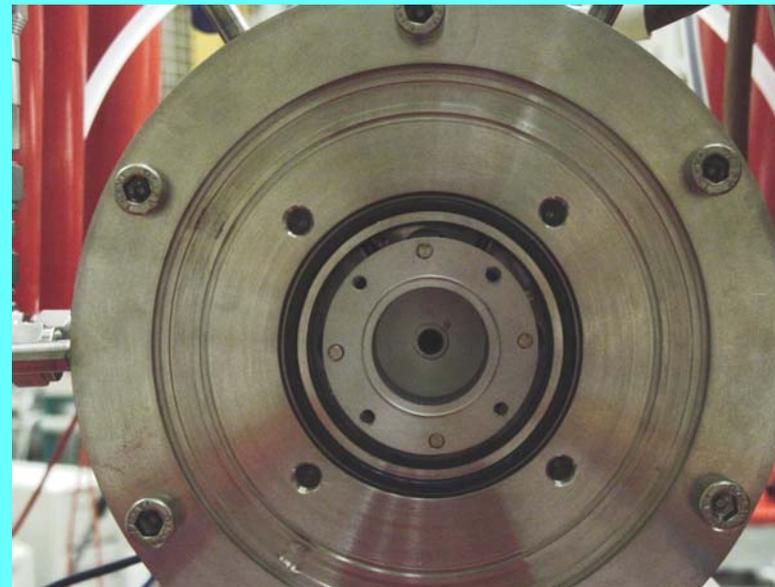
Sputter yields



Ion source



Oven



Bismuth

Source

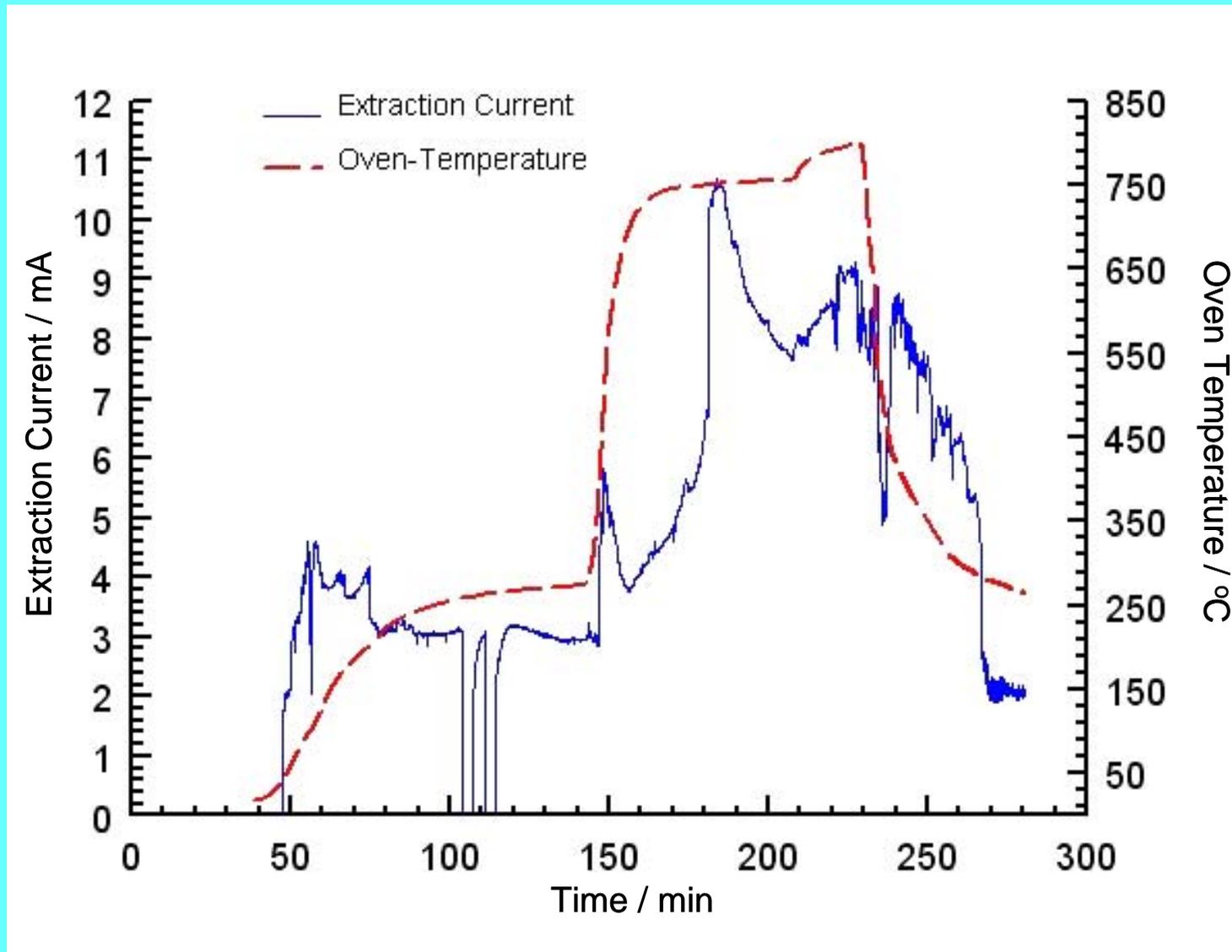


Target



1 g Bismuth has been evaporated within about 100 minutes

Ion current with bismuth oven

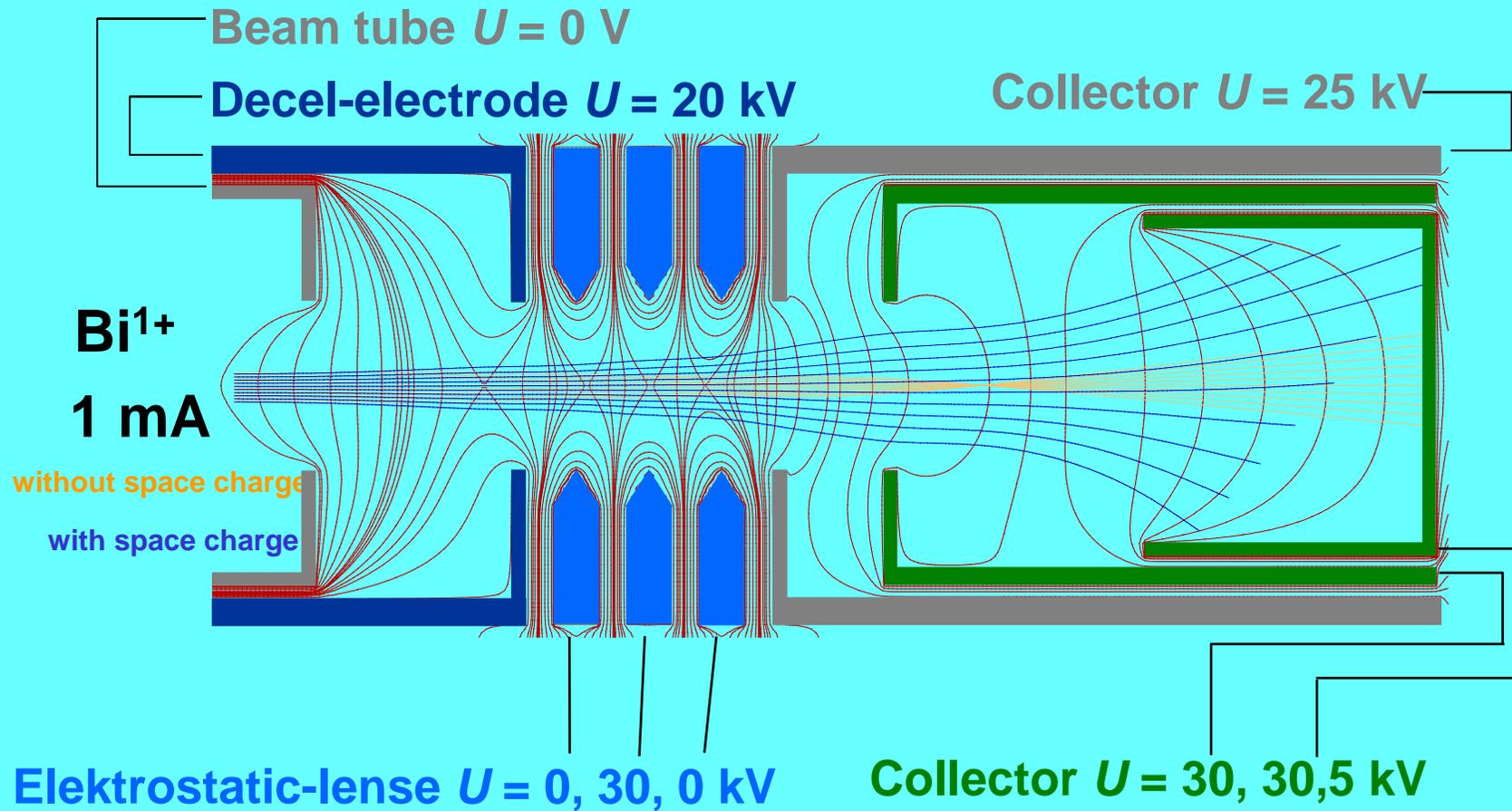


Deceleration system and collector

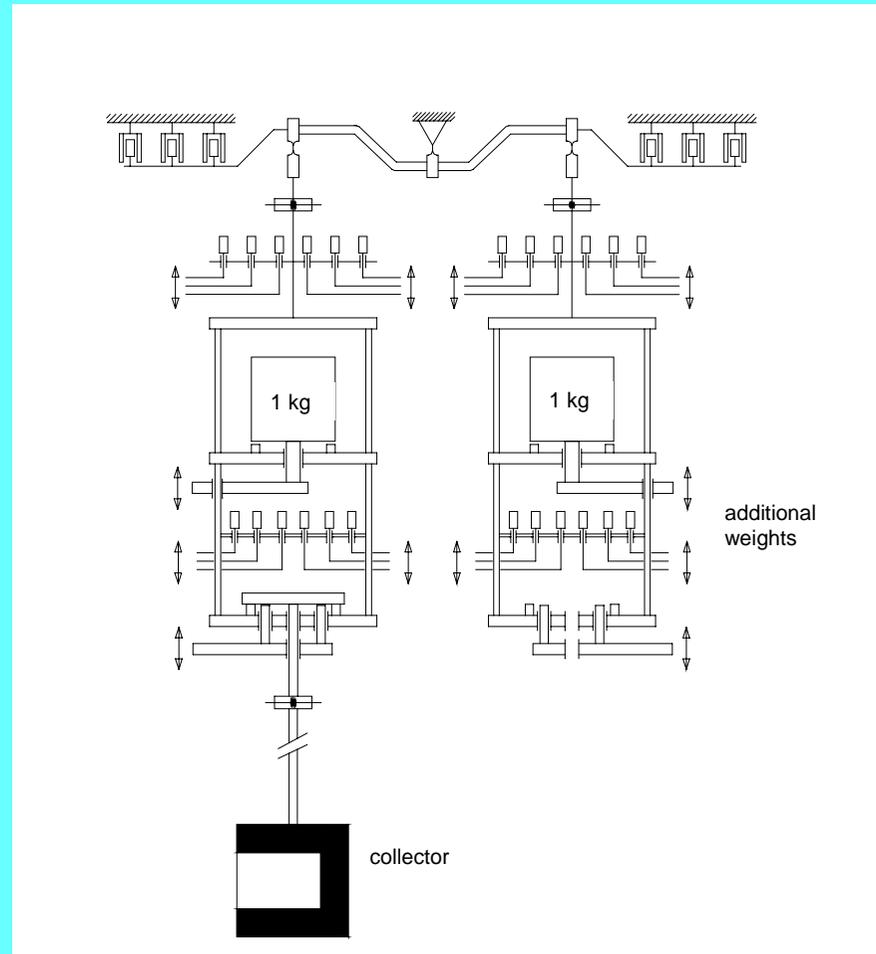
decelerate

focus

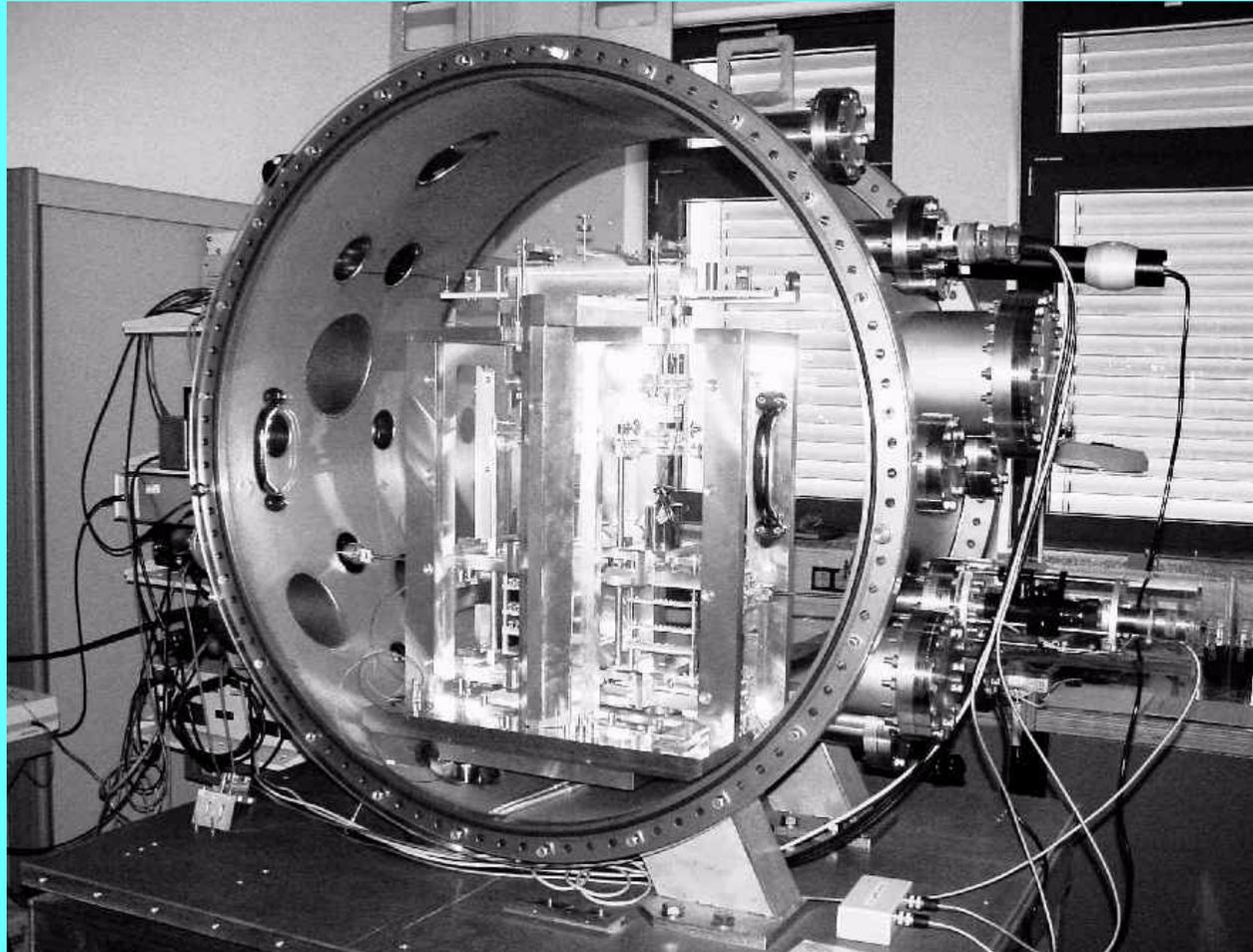
collect



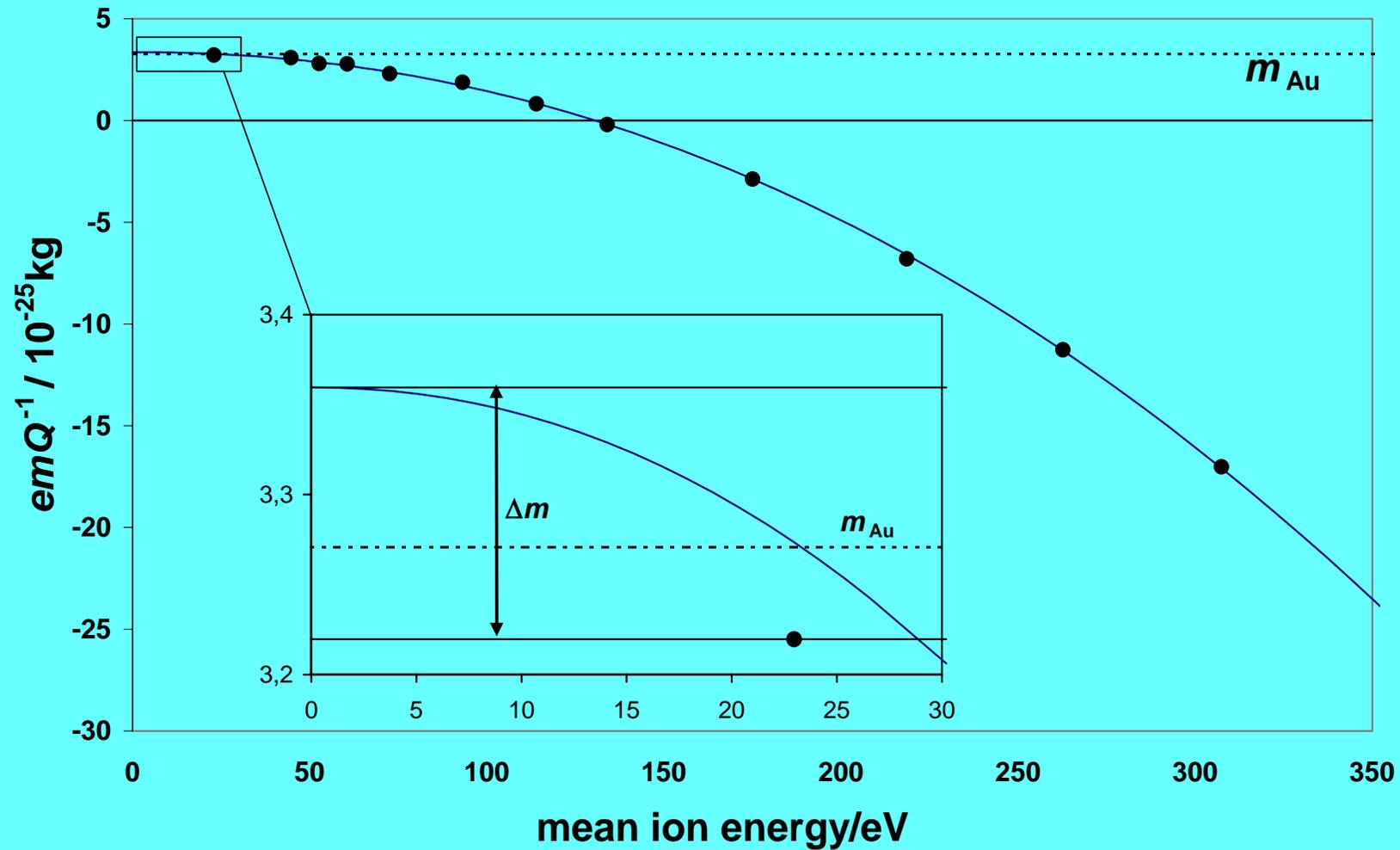
Ion accumulation balance



Ion accumulation balance



Measurement of the mass of a gold atom



Current results of the experiments



Experiment	Fundamental constant	Relative uncertainty	Institute
Avogadro constant (Silicon)	N_A	$3,4 \times 10^{-7}$	PTB, NIST, IMGC, NMIJ/AIST, CSIRO, IRMM
Ion accumulation	m_u	$1,5 \times 10^{-2}$	PTB
Watt balance	h	$8,7 \times 10^{-8}$	NPL, NIST, OF-MET, BNM, BIPM
Magnetic levitation	h	$>1 \times 10^{-6}$	NMIJ/AIST, MIKES, VNIIM

New definitions under discussion



- The kilogram is the mass of X_C unbound ^{12}C (or $X_{\text{Si}} \text{}^{28}\text{Si}$ or $X_{\text{Bi}} \text{}^{209}\text{Bi}$) atoms at rest and in their ground state.
- The kilogram is the mass of a body at rest such that the value of the Avogadro constant N_A is exactly $X_N \text{ mol}^{-1}$.
- The kilogram is the mass of a body at rest such that the value of the Planck constant h is exactly $X_P \text{ Js}$.

A new definition just now?



- **Proposal:** redefine the kilogram based on the **present CODATA value (2002) of h or of N_A**
(Mills, Mohr, Quinn, Taylor, Williams, to be published in Metrologia)
- **Consequence:** the mass of the international prototype of the kilogram has an **uncertainty of 170 μg** (relative: 1.7×10^{-7}) in terms of the new SI kilogram.
- **Compromise:** adoption of a **conventional value** of exact 1 kg for the mass of the international prototype (analogous to the Josephson and von Klitzing constants).
- **Advantage:** The uncertainty for several fundamental constants is reduced (by factors of about 10 to 20)
- **Disadvantage:** The present small uncertainties of mass comparisons ($<1 \mu\text{g}$ for Pt-Ir standards and $12 \mu\text{g}$ for stainless-steel standards) are related to a unit being not coherent with SI kilogram.
- **Remark:** At present, the experimental **discrepancy** between h and N_A is about 1×10^{-6} (relative). The conventional and SI kg may differ by the same value.