A MAP BASED ALTERNATIVE FOR THE DETERMINATION OF THE LOCAL GRAVITY

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ABSTRACT

For the determination of the local gravity high precise mobile gravimeters are in use since more than a hundred years. As a result of the basic measurement principle the determination is carried out point-by-point with the consequence that the procedure is time consuming and expensive. Thus in many countries the results of several measurement campaigns have been collected over decades.

Whereas the measured values are available to the public only to some extent, the data were often processed to gravity anomaly maps for the purpose of geological ground surveys.

In this research two samples of gravity anomaly maps of Northern Germany and Italy are introduced with a detailed description of the rebuild of a gravity value from an anomaly map. For the single steps a theoretical budget of the uncertainty is given which has been verified by independent control points.

1. INTRODUCTION

Since the development of mobile gravimeter instruments ín the middle of the 19th century the value of the local gravity was recorded for a multitude of places. Despite the technical innovation und the decrease of the uncertainty to 10^{-9} the pointwise data acquisition still remains a time consuming and expensive procedure. Several countries carried out measurement campaigns over some decades to actualise and densify their gravity networks and nivellements profiles. A further part of measurements were taken systematically within in the scope of scientific targets and in view of the exploration of mineral resources by private companies. The raw results of the point measurements are provided to the public only to some extent unless commercically.

Admittedly the location of the surveys and mathematically reduced values were drawn into local maps on large scale as a basis for the later development of gravity anomaly maps with a contineous field illustration [3]. Thematic maps with the focus on gravity anomalies are manufactured in numerous countries and can be obtained for a comparatively small budget.

By means of two map samples this paper demonstrates, in which way anomaly maps can be used for the deduction of the local gravity and which uncertainty can be expected.

2. INTERPRETATION OF PRINTED GRAVITY MAPS

The raw measurements of the gravity are rarly illustrated in maps since their strong correlation with height would cause an appearance that is quite similar to a topographical map. This would constrain the room for interpretations of the surrounding gravity field. A further disadvantage results from a low interpolation accuracy due to the heterogeneity of the data unless a high point density is available [2].

Previous to the map production the measured data are transformed by a uniform procedure: A model gravity value is subtracted from the measured one and afterwards reduced to the zero altitude on the sea level. The outcome, the gravity anomalies, is of the magnitude $10^{-4} – 10^{-5}\ ^{m}/s^{2}$ and can differ in its physical meaning as a result of the reduction method. For gravity maps usually so-called Bouguer anomalies are employed, that are reduced by the gravity effect of the topographic masses. By this means Bouguer anomalies are only influenced by the impact of deep rock formations so that they are strongly correlated with density steps and therewith ideal for the detection of petroleum deposits. For this reason gravity maps are edited in particular by national geological or geophysical institutes.

The thematic focus of the gravity maps is the presentation of contour lines at the same anomaly level. Similarly to topographical maps two neighboured lines differ in a predefined equidistance, that is usually an integer given in the old unit *mGal* (= $10^{-5}\ {}^{m}/{s^2}$): Often used equidistances in gravity maps are 1, 2, 5 and 10 mGal. The spacing between the lines can be filled by a scaled color scheme to ease the map interpretation. Further signatures are largely omitted except for some exclusive cities, rivers and border lines.

In the present examination the following map sheets were used:

Table 1: Describing details of the Bouguer anomaly maps of Northern Germany and Italy

Area	Map series	Date	Map projection	Scale	Equidistance	Bouguer Reduction	Density ρ
Northern Germany	3 parts	1987	Conic (Lambert)	1:500.000	$1\cdot10^{-5}\ {}^{m}/{s^2}$	Spherical: r=166.7 km	2.67 cm $\cdot g^{-3}$
Italy	1 part	2004	Rectangular (UTM)	1:1.250.000	$10\cdot10^{-5}\ {}^{m}/{s^2}$	Spherical: r=166.7 km	2.67 cm $\cdot g^{-3}$

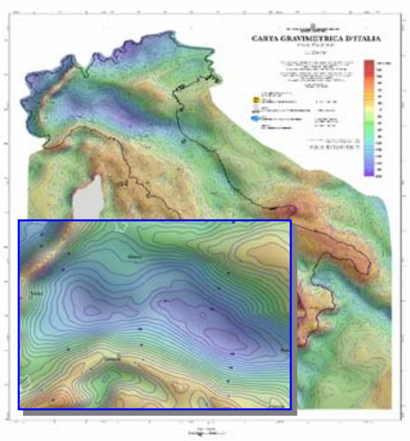


Figure1: "Carta Gravimetrica d'Italia", Bouguer-gravity anomalies of Italy [7]

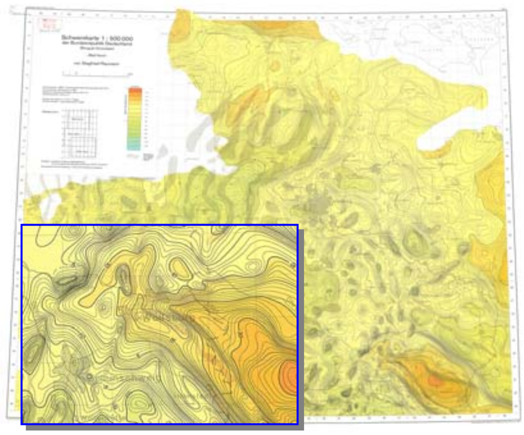


Figure2: "Schwerekarte 1:500000", [3], Bouguer-gravity anomalies of Northern Germany

2.1 Interpolation in the map

From the picture of the contour lines one get a contineous information about the projected area so that points between two lines can be interpolated without a breakage – except for the margins.

A linear interpolation between the neighboured lines is normally sufficient, see eq.3 and fig. 3 for details:

$$L_P = L_1 + {}^{a}\!/_{a+b} \cdot (L_2 - L_1) \qquad (1)$$

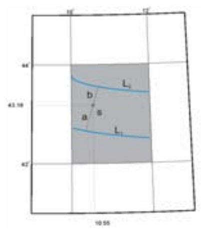


Figure 3: Interpolation between two niveau lines in a map grid

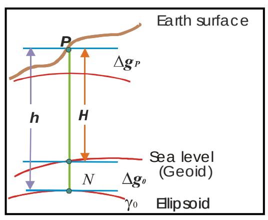


Figure 4: Definition of Bouguer anomalies at the sea level

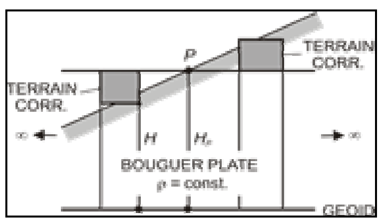


Figure 5: Reduction of Bouguer plate and terrain corr. (TC)

The procedure can be accelerated if the wanted value is simply estimated which is possible for most cases without a big loss of uncertainty (see 4. Uncertainty budget).

The location of a point itself has to be detected with more care since a wrong position may lead to a large error. The geographical coordinates (φ,λ) can be found by following the curves of the grid lines to the map frame.

2.2 Determination of the acceleration due to gravity

The computation of the acceleration due to gravity at the Earth's surface from an interpolated Bouguer anomaly corresponds to the inversion of the original reduction process for the map creation. Therefor it is indispensable to have look for further information in the map legend or from an additional documentation about the required reduction parameters.
The local gravity in the point P results from the simple linear equation :

$$g_{loc} = A_P + \gamma_0 + R_{Bouguer} - TC \qquad (2)$$

wherein A_P is the interpolated anomaly, $\gamma_0(\varphi,H_P)$ is the normal gravity from a reference model, $R_{Bouguer}(H_P)$ is the reduction of the so-called spherical Bouguer-plate and TC is the terrain correction as the difference between the topography and the spherical plate (see fig. 5).
The reduction can be completly performed by solving the following equations :

$$\gamma_0^{GRS80} = 9.780327 \cdot \left(1 + 0.0053024 \cdot \sin^2(\varphi) - 0.0000058 \cdot \sin^2(2\varphi)\right) - 3.086 \cdot 10^{-6} \cdot H_P \qquad (3)$$

$$R_{Bouguer} = 2\pi \cdot G \cdot \rho \cdot \left(H_P - \sqrt{H_P^2 + r^2} + r \right) \qquad (4)$$

$$TC = G \cdot \rho \cdot \iiint \frac{H_i - H_P}{s_i} d\varphi d\lambda dH \qquad (5)$$

with $G = 6.672 \cdot 10^{-11}$ Gravity constant
ρ = const. Rock density (map information table)
r = const. Radius of the spherical Bouguer plate (map information table)
φ,λ geographical coordinates: latitude, longitude
H altitude above sea level

The eq.(3) and (4) can be solved with the help of the geographical coordinates and the parameters given in the map whereas for the computation of the terrain correction eq.(5) a digital elevation model (DEM) becomes necessary. Generally TC is integrated up to a distance of 25 km from the center point P. In the flatlands the infuence of TC is small and can often be neglected (see 6. Test Results). Eq. (3) may differ for some older maps if the reference model GRS67 instead of GRS80 were used [2].

3. DIGITAL MAP DATA

The evaluation of a printed anomaly map normally remains limited to a few points, since the manual effort for a large quantity would become uncomfortable. The interactive part of the user can be lessend effectively if the determination of a point within the graticule together with the interpolation between the contour lines would be automised by a computer program. That presupposes that the contour lines and their geographical location would be available as digital data. Since anomaly maps basically are obtainable as printed sheets a special procedure has been developed in this framework to digitise the map content with a minimum loss of information. The procedure profits from the instance that the focus of the map picture lies on the contour lines with only few distortions by additional signatures.

(1) In the first step the analogue template was transduced to a digital raster image with the aid of a common flat bed scanner. Due to the map size multiple parts with a small overlap were generated. The resolution for scanning has to be selected in that manner that the accuracy of the digital evaluation is not diminished compared to the original. Therfor the mini-

mum spacing between two lines in the printed map that can be seperated by the human eyes serves as the criteria. This spacing is given in the literature with 0.2 – 0.3 mm depending on the contrast [1]. Transformed to the resolution of the scanner which is measured in dots per inch (dpi) this means:

DPI = 25.4 mm / 0.2 mm = 127 dpi , with 1 inch = 2.54 cm

Modern scanners enable resolutions of at least 300dpi for colored tamplates.

(2) In the following working steps several different digital filters and image algorithms were applied to the raster image to transform it into a pure black-and-white only consisting of lines. On this basis a contiguous line could be assign with an anomaly value which is stored automatically for each pixel of the line. Thereby the high information density of the printed original persists in the digital complement. The result of the digitised map is still a raster image, that contains an information at each line elemant.

(3) Finally the local pixel coordinates (x,y) were transduced to geographical coordinates (φ,λ) with the aid of the crosses of the graticule using an affine transformation and the instruction of the map projection.

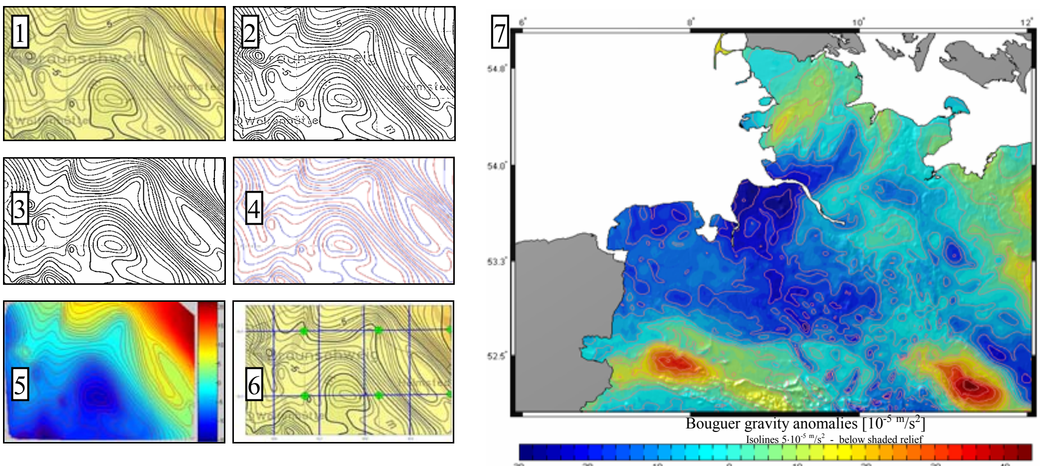


Figure 6: Single steps of the vectorizing process: 1. Scanning of the original 2. Sharpening and color filtering 3. Cleaning of undesired parts 4. Assigning the polygon lines with anomaly values 5. Controlling 6. Georeferencing 7. Result of the digitised Bouguer map of Northern Germany

3D-Interpolation

For the interpolation of arbitrary values the geostatistical estimation procedure *Kriging* was chosen because it characterises best the spatial behaviour of an area by the statistical neighbourhood relation of the known points [6]. At the same time it gives an information about the uncertainty of the interpolation result. From the spacings of the surrounding points it is possible to find a statistical dependency how the magnitude at the points influence each other. This dependency can be interpreted as a variance, that can be displayed in form of a variogram. To extend this known neighbourhood relation to new points an analytical function has to be found that rebuilds the empirical behaviour adequately. By experience one knows that only certain types of positive definite functions come into question [6].

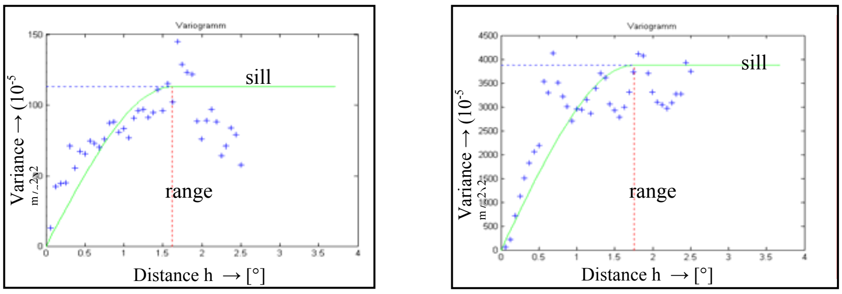


Figure 7: Variogram plots for new stations in the test areas of Northern Germany (left) and Italy (right) related to the surrounding contour line points

Figure 7 shows two typical variograms for the test areas where the function curve runs asympticto an upper limit (sill). This sill represents the variance of the anomalies in the neighbourhood of the new point. The distance where the variogram reaches the sill assigns the range (red line) at which a statistical correlation is to be expected. From the left diagram one can read off the fact that the surrounding anomalies vary with about $\pm 10 \cdot 10^{-5}$ $^{m}/s^{2}$ and influence the new point up to a spherical distance of 1.5° (~150 km). In the neighbourhood of the point in Italy (right diagram) the anomaly field shows an explicit stronger variation with $\pm 60 \cdot 10^{-5}$ $^{m}/s^{2}$.

4. UNCERTAINTITY BUDGET

4.1 Uncertaintity of the source data

To assess the uncertainty of the map content some additional information about the map creation procedure and the utilised source have to be found out. Normally one should assume that the location of the contour lines and the presented intervall is based upon a sufficient point density.
In an accompanying documentation about the gravity map of Northern Germany it is stated that the point density varies from about 7 points per 100 km^2 in the North up to 40 and more points in the South. Furthermore some areas were densified by additional measurements of torsion balances. If one still considers that the majortiy of the measurements were carried out in the years 1934 –1940 when the instruments were of lower accuracy and other parts were added several decades later it is not possible specify a uniform uncertainty for the map area.

A uniform uncertainty U for contour lines can be derived assuming that the standard deviation of the source data is less than half of a contour line interval within a probability of 95%.

$$k_{95\%} \cdot U_{Countour} = \text{contour interval} / 2 \quad \text{with} \quad k_{95\%} = 2$$

$$U_{Countour} = \text{contour interval} / 4$$

4.2 Uncertainty of the interpolation

The linear interpolation between two contour lines in the map is normaly uncritical since it is not worse than 0.3·contour interval. The more problematic part comes from the exact location of a point in the map. If the point is located into a square of a very high line density, an unaccurate positioning may lead to an error of a few contour intervals. With an adequate accurateness the error should not exceed half of a contour interval.
The interpolation of the digital data with the Kriging method depends on the number of surrounding points and their equally distribution. Accordingly it suffers in areas with a low point density which is expressed in the kriging-variance. This is the case if the anomaly gradient is low, that means, the distance between two neighboured contour lines is large.

4.3 Uncertainty of the digitising process

The uncertainty of the digitising process can be kept small if it is done with care and if the preconditions are in the limits.
If the resolution of the raster scanning was set high enough the determination of the contour line points can be done with subpixel accuracy.

4.4 Uncertainty of the anomaly reductions

The uncertainy contributions of the reductions to the gravity value can be derived from the differentials of the eq. (3) – (5):

$$\frac{\partial R_{Bouguer}}{\partial H} = 0.12 \cdot 10^{-5} - \frac{2H}{\sqrt{H^2 + r^2}} \approx 1.2 \cdot 10^{-6}\ {}^{m}\!/_{s^2} / m \qquad (6)$$

$$\frac{\partial \gamma_0}{\partial H} = 3.086 \cdot 10^{-6}\ {}^{m}\!/_{s^2} / m \qquad (7)$$

$$\frac{\P g_0}{\P j} = 9.78 \times (0.0106 \times \sin(dj) \cos(dj) - 0.0000232 \times \sin(2dj) \cos(2dj))\ \ {}^{m}\!/_{s^2} / [°] \qquad (8)$$

The contribution of the terrain correction varies speciallly between flatlands and alpine regions .
Table 2 gives an outline about the important contributions to an over-all uncertaintity for the map derived gravity value:

	Error type	**Error magnitude**
A	Source data	Contour interval / 4
B	Linear Interpolation	< Contour interval / 2
C	Interpolation by Kriging	< Contour interval / 3
D	Bouguer plate reduction (d*H*)	$1.2 \cdot 10^{-6}$ $^{m}/s^2$
E	Normal gravity (d*H*)	$3.1 \cdot 10^{-6}$ $^{m}/s^2$
F	Normal gravity (dφ)	$1.8 \cdot 10^{-7}$ $^{m}/s^2$ with dφ=0.0001°(~10m) $1.8 \cdot 10^{-6}$ $^{m}/s^2$ dφ=0.001° (~100m)
G	Terrain correction	0 $_{(flat)}$ ….. $5 \cdot 10^{-4}$ $^{m}/s^2$ $_{(mountaineous)}$

Table 2: Contributions to the error budget

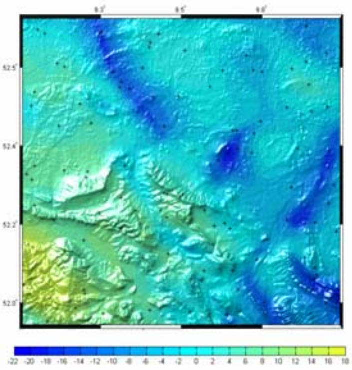
5. TEST RESULTS

The theoretical considerations to the error budget has been proved practically in two steps:
At first the quality of the interpolation with the Kriging method was analysed by a stepwise decrease of the contour interval. From the neglected contour lines several control points have been extracted that are not part of the Kriging points. Like this way the interpolation result is uncorrelated and independent from the control value. Table 3 gives the mean deviation of the interpolation at the control points for the contour intervals 2, 5 and $10 \cdot 10^{-5}$ $^{m}/s^2$. The results of the two areas behave quite similar and keep both below the sill of (contour interval / 3) for each contour step.

Test area	Mean relative deviation at Control points [10^{-6}]			mean Kriging-Variance [10^{-6}]		
Contour interval in [10^{-5} $^{m}/s^2$]	2	5	10	2	5	10
Braunschweig	0.33	0.65	1.73	4.3	7.6	18.3
Hannover	0.32	0.85	2.30	1.3	2.3	5.4

Table 3: Mean relative deviation of Kriging-interpolations at independent control points

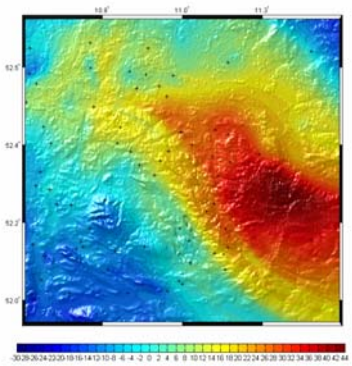
In a second research step the over-all permance of the gravity maps was tested at control points from a point database of superior accuracy. Two German test areas, *Hannover* and *Braunschweig*, were chosen since they are both important economical areas, they are close together (50 km), but the behavior of the anomalies shows a different face (see figures 8 +9). From the Italian map the Northern part were taken including the alps, the industrial region of Milano und the flatlands of river Po.



Test area[1] *Hannover*							
Relative error [10^{-6}]	< 0.3	0.3-0.6	0.6-1.0	1-5	5-10	>10	U
1	40	35	15	9	0	0	0.4
1 (without TC)	43	29	15	12	0	0	0.5
5	17	27	19	36	0	0	0.8
10	9	10	20	53	7	0	1.7
Free air anomalies	2	6	2	24	26	39	11.8

Table 4: % of points within one error class; Contour interval in 10^{-5} m/s^2; mean uncertainty U of all points in 10^{-5} m/s^2

Figure 8: Test area 1 ‚*Hannover*', Coloured Bouguer anomalies [10^{-5} m/s^2] with a shaded relief; 100 control points with known gravity



Test area[2] *Braunschweig*							
Relative error [10^{-6}]	< 0.3	0.3-0.6	0.6-1.0	1-5	5-10	>10	U
1	44	24	17	10	4	0	1.2
1 (without TC)	46	17	23	10	4	0	1.2
5	30	23	19	24	4	0	1.3
10	23	16	11	43	6	2	2.2
Free air anomalies	2	3	3	17	13	63	15.9

Table 5: % of points within one error class; Contour interval in 10^{-5} m/s^2; mean uncertainty U of all points in 10^{-5} m/s^2

Figure 9: Test area 2 ‚Braunschweig', Coloured Bouguer anomalies [10^{-5} m/s^2] with a shaded relief; 100 control points with known gravity

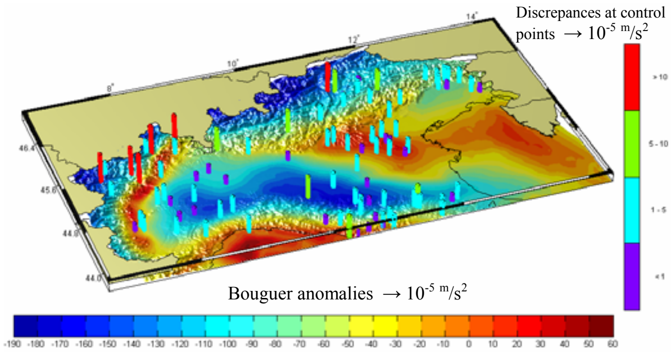


Figure 10: Test area 3 ‚Italy North', Coloured Bouguer anomalies [10^{-5} m/s^2] with a shaded relief; 100 control points with known gravity

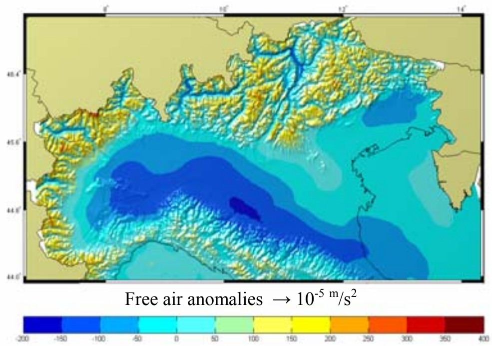


Figure 11: Free air anomalies F_A [10^{-5} m/s^2]
$F_A = g_{loc} - g_0$

	Test area[3] *Italy*							
	Relative error [10^{-6}]	< 1	1 - 2	2 – 10	10-50	50 - 100	>100	U
Interval	10	27	28	38	7	0	0	4.4
	10 (without TC)	20	22	38	19	1	0	10.4
	Free air anomalies	1	2	10	49	25	13	50.7

Table 6: Number of points per error class, mean uncertainty U of all points [10^{-5} m/s^2]

For the gravity reductions the coordinates of the control points were used to avoid misleading

errors.The deviations between map and control values were classified into error groups to stress the distribution of the errors.
From the tables 4–6 it can be seen that 90% of the deviations in area[1] 1, 85% in area[2] and 93% in area[3] are inside of the original contour interval. With a decrease of the contour interval to $5\cdot10^{-5}$ $^{m}/s^{2}$ and $10\cdot10^{-5}$ $^{m}/s^{2}$ the uncertaintity grows with the factor 2. In area[2] the uncertainty appears higher than in area[1] which depends on few points in the Eastern part that are not validated by the control point data base and are assumed to be outliers.
The terrain correction TC has hardly effect in the German areas and can be neglected. In the Italian alps the omission of TC denotes a severe degradation of the uncertainty.
In contrast to the devations of the map calculations, the tables include also the free air anomalies in the last rows, which can be considered as the absolute error of the simple normal gravity (eq. 3).
The uncertainty of the free air anomalies in the sense of absolute errors is ten times higher than a Bouguer map with an contour interval of $10\cdot10^{-5}$ $^{m}/s^{2}$.

6. DISCUSSION

In this research gravity maps are introduced as a powerful source for the acquisition of gravity values at the surface of the Earth. Since the gravity is not accessible directly from the map some further reductions have to applied that can be lessend depending on the demands. A more comfortable solution is given by the evaluation of the digital map data since the procedure is controlled by a program.
It could be shown that a map with a small contour line equidistance of $1\cdot10^{-5}$ $^{m}/s^{2}$ has the potential for the computation of a gravity value with an uncertainty of $<1\cdot10^{-6}$ with 90% probability. For a map with an equidistance of $10\cdot10^{-5}$ $^{m}/s^{2}$ the uncertainty remains still in the range of $5\cdot10^{-6}$. If the laborious computation of the terrain correction is neglected only a few points in the Italian alps were affected seriously with maximum deviation of $5\cdot10^{-5}$.

As a conclusion of the analysis we can state that the demands of most applications in physics and metrology concerning the contribution of the uncertainty of the gravity will be fullfilled by gravity maps easily.

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