SARTORIUS SUSCEPTOMETER FOR PRECISE MEASUREMENT OF SUSCEPTIBILITY AND MAGNETIZATION OF WEIGHTS

Thomas Fröhlich[1], Thomas Fehling[1], Benno Gatzemeier[1], Detlef Heydenbluth[2]

[1]Sartorius AG, Göttingen, Germany
[2]Technische Universität Ilmenau, Germany

ABSTRACT

The Sartorius Susceptometer is a commercially available susceptometer as described in the Draft revision of International Recommendation OIML R111. It comes with a calibration sheet. The procedure of the factory calibration is described. A measurement comparing five calibrated susceptibility standards is discussed. The magnets used for calibration are monitored concerning the stability of the magnetic dipole moment over about one year.

1. INTRODUCTION

The Sartorius Susceptometer [6] allows the magnetization and the magnetic susceptibility of weights to be determined as recommended in the Draft revision of International Recommendation OIML R 111. It is available with a readability of the mass comparator of 1µg or 10µg.

Traceable measurements require the vertical distance Z_0 between the mid-height of the magnet and the bottom of the weight under test to be determined using one or more traceable susceptibility standards. Furthermore, the magnetic dipole moment m_d of the test magnet has to be known. Various methods for determining Z_0 and m_d are described in [3]. The following section describes the factory calibration.

2. CALIBRATION

Prior to delivery of each susceptometer, the distances of the five selectable vertical positions were determined by Sartorius using a combination of a so called cathetometer method (method a in [3]) and the method c in [3]. Method c requires a susceptibility standard to be used. At Sartorius, we use one cylindrical stainless steel weight calibrated at the PTB, Braunschweig. Detailed data are given below in Table 3 in the PTB2419 column. The susceptibility value is given for different calibration field strengths between 5 and 30 kA/m, and the change is insignificant. The smallest calibration field strength 5 kA/m is greater than the highest field strength of the susceptometer at the lowest vertical position Z1, which is 2.7 kA/m.

The height steps between the five vertical positions are determined after the milling and grinding process using a 3D coordinate measuring machine. The vertical distance of the lowest position is determined with the known susceptibility of the susceptibility standard. As a result of the extremely stable mechanical structure of the unit, the vertical distances on the Sartorius susceptometer are practically immune to changes. The values for the vertical distances determined by Sartorius are indicated in the calibration certificate that is given for each susceptometer unit. The distance of the lowest position is known from the mounting and adjusting process (cathetometer method). This additional knowledge is used for checking the value from the calibration with the susceptibility standard.

The magnetic dipole moments of the test magnets used in Sartorius susceptometers are determined and recorded in the calibration certificates, too. The method is based on measuring the force between pairs of the four magnets and solving the resulting system of equations with 6 equations and the 4 unknown magnetic dipole moments in least square sense as described in [3]. The change of the indication of the mass comparator as a result of the force between each pair (i,j) of the magnets is expressed as $m_{ij}=F_{ij}/g$. We always use the same 3 additional magnets and a spacer of height λ=60mm. The distance between the mid-points of the magnets is $Z_0+\lambda+L/2$ where L=4.6mm is the height of the magnet.

As explained in [3], we have to solve the over determined equation system K log X = Y with

$$K = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \qquad Y = \begin{pmatrix} \log(F_{12}) \\ \log(F_{13}) \\ \log(F_{14}) \\ \log(F_{23}) \\ \log(F_{24}) \\ \log(F_{34}) \end{pmatrix} + \log\left(\frac{4\pi(Z_0+\lambda+L/2)^4}{6\mu_0}\right)$$

and $X = (m_{d1}, \ldots, m_{d4})^T$ the column vector of 4 unknown magnetic dipole moments. The solution is

$$X = \exp(LY)$$

with the solution matrix

$$L = (K^T K)^{-1} K^T = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{6} & -\frac{1}{6} & -\frac{1}{6} \\ \frac{1}{3} & -\frac{1}{6} & -\frac{1}{6} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} & \frac{1}{3} \\ -\frac{1}{6} & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}.$$

From the first line, one obtains the magnetic dipole moment m_{d1}

$$m_{d1}^2 = (Z_0+\lambda+L/2)^4 \left(\frac{4\pi}{6\mu_0}\right) \sqrt[3]{\frac{F_{12}^2 F_{13}^2 F_{14}^2}{F_{23}F_{24}F_{34}}}.$$

Plugging this into equation (B.6.2) of [2]

$$\chi = \frac{F_a}{I_a \frac{3\mu_0 m_d^2}{64\pi Z_0^4} - 0.4F_a}$$

with the known susceptibility and dimensions of the standard and F_a from our measurement, we obtain

$$f(Z_0) = \frac{F_a}{I_a \frac{(Z_0+\lambda+L/2)^4 \sqrt[3]{\frac{F_{12}^2F_{13}^2F_{14}^2}{F_{23}F_{24}F_{34}}}}{32Z_0^4} - 0.4F_a} - \chi = 0.$$

Unfortunately, we found no explicit solution of this one-dimensional nonlinear function. It is solved numerically using the cathetometer Z_0 as a staring value.

In summary the steps of factory calibration are as follows: First, we measure F_a with the PTB2419 susceptibility standard and m_{ij} for each combination of the magnet of the susceptometer and the three additional magnets. Then Z_0 and m_d are calculated as described above. The vertical distance from the cathetometer method and the historical data of the 3 calibration magnets are used to check the results.

Redetermination of the vertical distances using one or more traceable susceptibility standards is supported by the included susceptometer software [6]. It is also possible to redetermine the magnetic dipole moment using the procedure described in [3]. The 3 additional magnets and the spacer are available from Sartorius as a so-called calibration kit (see figure 1).

Figure1: Calibration kit: 3 magnets and a spacer in a wooden box with sufficient dimensions in order to ensure a magnetic field strength that is less than 200A/m outside the box

3. CALIBRATION EXAMPLE AND COMPARISON

We measured the distance steps between the vertical positions Z1,. . . ,Z5 as given in Table 1. The distance of the lowest position Z1 was determined in the mounting and adjusting process. Table 2 gives the readings for the magnet measurement.

Table 1: Vertical distance measured with 3D coordinate measuring machine

	Position				
	Z1	Z2	Z3	Z4	Z5
Z_0 in mm	18.36	20.44	27.43	35.45	43.43
Difference to Z1 in mm	0	2.08	9.07	17.09	25.07

Table 2: Readings for the magnet measurement at each vertical position, using a spacer of height λ=60mm

Position	Readings in µg					
	M1-M2	M1-M3	M1-M4	M2-M3	M2-M4	M3-M4
Z1+λ+L/2	10667	10483	11005	11222	11772	11538
Z2+λ+L/2	9646	9491	9958	10140	10632	10426
Z3+λ+L/2	6978	6846	7183	7314	7669	7521
Z4+λ+L/2	4946	4863	5101	5192	5444	5333
Z5+λ+L/2	3630	3561	3738	3810	4000	3920

On addition, we purchased four iron acrylic susceptibility standards (see [5]) from the NPL. All standards are cylindrical. The geometrical dimensions are relatively small due to the diameter restrictions during manufacture. The nominal values were chosen near the maximum acceptable limits given by the draft revision of the OIML R111 for the different error classes. Each iron acrylic susceptibility standard was calibrated at the NPL. The calibration field strength agrees with the vertical positions of the susceptometer given in the last line of Table 3. With the iron acrylic susceptibility standards, we can check the susceptometer calibration over the full range of use. Table 4 gives the readings for the susceptibility standards for all 5 vertical positions.

Table 3: Susceptibility standards: susceptibility, expanded uncertainty and magnetic field strength from calibration certificate, dimensions and vertical position used for calibration

	NPL1005	NPL 1024	NPL11	NPL16	PTB2419
χ	0.0055	0.02657	0.1173	0.693	0.004012
$U(\chi)$ k=2	0.00005	0.000205	0.00056	0.0034	0.000035
H in kA/m	2.7	2.0	0.8	0.2	5.0
Diameter in mm	39.67	39.81	39.60	25.00	59.20
Height in mm	26.60	25.25	27.50	25.00	45.30
Used at position	Z1	Z2	Z3	Z5	Z1

Table 4: Readings for the susceptibility standards at each vertical position

Position	Readings m_1/m_2 in µg				
	NPL1005	NPL 1024	NPL11	NPL16	PTB2419
Z1	-610/-568	-2904/-2572	-12353/-11318	-40263/-36910	-513/-475
Z2	-378/-347	-1837/-1550	-7666/-6880	-23913/-21234	-318/-297
Z3	-97/-80	-483/-330	-1939/-1546	-5352/-4077	-94/-80
Z4	-32/-22	-151/-72	-571/-369	-1463/-831	-31/-26
Z5	-10/-4	-62/-19	-215/-103	-526/-191	-17/-8

Calibration can be performed with each of the susceptibility standards as well as with the cathetometer measurements. The susceptometer calibration gives a value for the vertical distance Z_0 and the magnetic dipole moment m_d. With these results the susceptibility can be calculated from readings for the susceptibility standards. We perform each susceptometer calibration at the vertical position belonging to the field strength where the standard was calibrated (last line in Table 3). Therefore, we have a comparison of the calibration values of the susceptibility standards. Table 5 gives the results of this comparison. For convenience, the relative deviation of the calculated susceptibility values from the calibration data is given in Table 6. The last line in both Tables relates to the question as to which errors will occur if we perform regular factory calibration using the PTB2419 susceptibility standard.

Table 5: Resulting susceptibility values for the standards at the vertical position given in Table 3 depending on different calibration methods

Calibration with	Z_0 in mm	m_d in A/m^2	χ NPL1005	NPL 1024	NPL11	NPL16	PTB2419
Method a	18.360	0.0832	0.00594	0.02798	0.1203	0.696	0.00444
Method c, NPL1005	17.974	0.0825	0.00550	0.02608	0.1139	0.665	0.00413
Method c, NPL1024	18.076	0.0827	0.00561	0.02657	0.1156	0.673	0.00421
Method c, NPL11	18.181	0.0829	0.00573	0.02708	0.1173	0.682	0.00430
Method c, NPL16	18.323	0.0831	0.00590	0.02779	0.1197	0.693	0.00441
Method c, PTB2419	17.810	0.0822	0.00532	0.02530	0.1112	0.653	0.00401

Table 6: Relative deviation of the susceptibility values given in Table 5 from the calibration data (Table 3)

Calibration with	$\Delta\chi/\chi$				
	NPL1005	NPL 1024	NPL11	NPL16	PTB2419
Method a	8.0%	5.3%	2.6%	0.4%	10.7%
Method c, NPL1005	0.0%	-1.9%	-2.9%	-4.0%	3.1%
Method c, NPL1024	2.1%	0.0%	-1.5%	-2.8%	5.1%
Method c, NPL11	4.3%	1.9%	0.0%	-1.6%	7.1%
Method c, NPL16	7.3%	4.6%	2.1%	0.0%	10.0%
Method c, PTB 2419	-3.3%	-4.8%	-5.2%	-5.8%	0.0%

The PTB2419 and NPL1005 values (where χ<<1 holds) agree at a 3% level. For the higher values NPL1024, NPL11 and NPL16 the approximation of the demagnetization factor as explained in section 4.1.2. of [4] becomes more important. Obviously, the cathetometer method shows a considerable deviation from the PTB2419 and NPL1005 calibration. The cause of this is still under investigation.

4. LONG TERM STABILITY OF m_d

As mentioned above, we always use the same set of three additional magnets for the susceptometer factory calibration. Therefore, we have historical data on our magnets compiled over about one year. Figure 2 shows the magnetic dipole moment m_d of one of these magnets. The data were collected from some different units from our production and calibration process as well as from two susceptometers at our research laboratory. Good repeatability and no significant drift could be observed in Figure 2. The magnets used for the Sartorius factory calibration are stored in a wooden box as shown in Figure 1 under normal laboratory and office conditions.

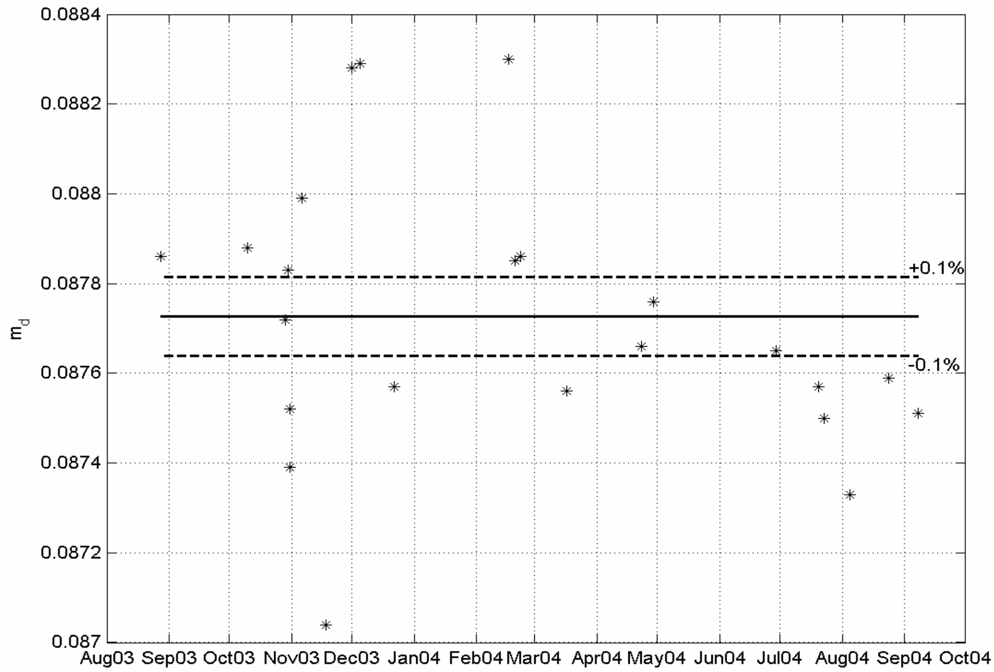

Figure2: Several measurements of m_d for one magnet over approx. 1 year; the mean value and the ±0.1% deviation are indicated by the lines

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Addresses of the Authors:

Dr. Thomas Fröhlich, Mechatronic Division, Development of Mass Comparators, Sartorius AG, Weender Landstrasse 94-108, 37075 Göttingen, Germany.
thomas.froehlich@sartorius.com

Thomas Fehling, Mechatronic Division, Development, Manufacturing and Service of MassComparators, Sartorius AG, Weender Landstrasse 94-108, 37075 Göttingen, Germany.
thomas.fehling@sartorius.com

Benno Gatzemeier, Mechatronic Division, Marketing and Sales of Mass Comparators and Weights, Sartorius AG, Weender Landstrasse 94-108, 37075 Göttingen, Germany.
benno.gatzemeier@sartorius.com

Dr. Detlef Heydenbluth, Institut for Process- and Sensortechnics, Technical University of Ilmenau, Technische Universität Ilmenau, P.O. Box: 100565, 98694 Ilmenau, Germany.
detlef.heydenbluth@tu-ilmenau.de