CORRECTIONS OF AERODYNAMIC LOADINGS MEASUREMENT ON AIRFOIL CASCADE AT BENDING-TORSION VIBRATIONS

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ABSTRACT

The paper describes improvement of the unsteady aerodynamic forces and moments measurements in an airfoil cascade. These loads are caused by shifting and angular vibrations of airfoils. The improvement was achieved by taking airfoils and other elements deformations, induced by action of innercia and measured aerodynamic firces, in consideration during the calibration and measurement process. The different variants of airfoils supporting are considered.

1. INTRODUCTION

For the analysis of dynamic stability of turbomachine blades it is necessary to know the unsteady aerodynamic forces and moments induced by their oscillations. Usually these loads are measured on airfoil cascades (fig. 1). The aerodynamic load at arbitrary airfoil oscillations can be described by forces *L*, *K* and moment *M*. It is possible to neglect influence of airfoil oscillations along the *X*-axis, and also force *K* [1]. Therefore, to measure the force *L* and moment *M* in translational *y* and angular α direction of the airfoils oscillations is sufficient.

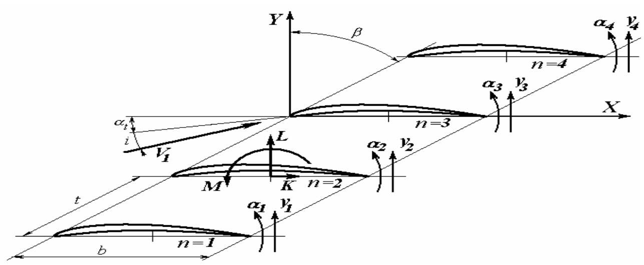


Figure 1: Scheme of a compressor airfoil cascade,
b - airfoil chord, t - cascade pitch, β - stage angle, V_1 - inlet stream velocity

In order to minimize the wind tunnel wall influence on the measured loads, it is necessary to make the airfoils as long as possible, which reduces their rigidity. It causes additional airfoil strains, that affect the measurable loads. The use of more and more thin airfoils in gas-turbine engines aggravates the indicated inconsistency, but such airfoils require a careful research of dynamic stability.

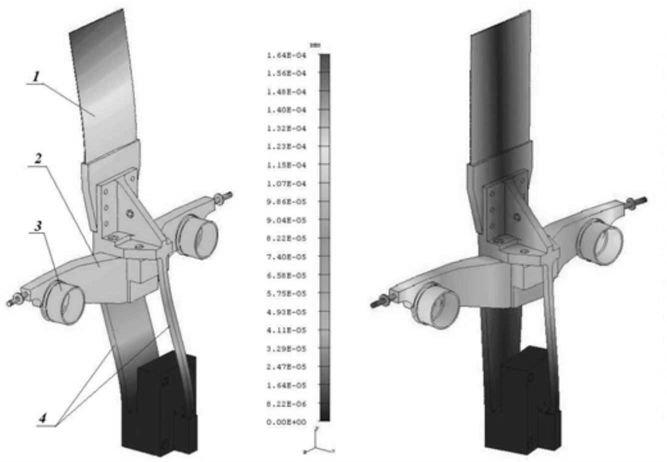
According to technique described in [2], the aerodynamic forces are measured by a strain gauge dynamometer, and the moments evaluated from current magnitude in a mobile coil of an electrodynamic vibrator, with help of which the angular airfoil oscillations are excited. This technique was further developed in [3], where the angular and translational airfoil oscillations were excited by a pair of electrodynamic vibrators, and the electric currents in their mobile coils determined the aerodynamic forces and moment. An attempt was made in [4] to introduce corrections taking into account the deformation of the airfoil and of the vibrator coils supporting elements. In this work the measurement corrections of the unsteady aerodynamic forces for the case of flexural airfoil oscillations in a cascade were obtained. The technique of the vibrator calibration was updated.

The purpose of the present work is to include the flexural deformations of the airfoils in a cascade at the translational - angular oscillations in their fixing place. The different variants of airfoils suporting are analysed.

2. METHOD SPECIFICATION

The airfoils are located in the measuring room of a wind tunnel on individual measuring systems (fig. 2). The elastic elements of a different width form an oscillatory system in the shape of an elastic parallelogram that ensures translational and angular displacements of the airfoil fixing part. The displacements of the airfoil fixing are measured by eddy-current contactless gauges. A pair of electrodynamic vibrators, the mobile coils of which are fixed to a crossbeam, excites the required oscillatory displacements of the airfoil fixing.

If the first natural airfoil frequency is much higher than the exciting vibrators frequency ω, the amplitudes of airfoil displacements will be constant along its length. However, the calculation of the natural modes shows, that at flexural oscillations of the measuring system this condition is defaulted and displacements of the airfoil tip considerably exceed displacements of its root part. Contrary to that, the torsion displacement oscillation amplitudes are practically constant along the airfoil length. Therefore, we shall further take into account influence only of the flexural airfoil displacements on the aerodynamic forces measurements at the flexure-torsion oscillations of the measuring system. The measurement of the aerodynamic moment does not require any specification and is described in [3]. The aerodynamic loads of an airfoil depend not only on its oscillations, but also on oscillations of adjacent airfoils. As an example we shall consider vibrations of two airfoils in a cascade.



flexural oscillation mode torsion oscillation mode

Figure 2: Amplitudes of measuring system displacements,
1 - airfoil, 2 – cross beam, 3 - mobile coil of an electrodynamic vibrator, 4 - main and auxiliary elastic elements

The measuring system is designed so, that the centre of masses lies on the torsion axis. In this case the flexural and torsion oscillation modes are not connected mechanically among themselves, and they can be considered as independent of each other. It is necessary only to take into account the aerodynamic coupling between the flexural and torsion oscillation modes. In this case there is enough to consider merely the flexural mode oscillations of the measuring systems under the action of aerodynamic and of electrodynamic vibrators forces. Simultaneously it is necessary to take into account, that the aerodynamic forces acting on airfoils are induced not only by the flexural oscillations of the measuring systems, but also by the torsion ones. The scheme of the flexural oscillations excitation of two measuring systems is represented in fig. 3.

The following conventional symbols are used in the scheme: m_{b1}, C_{b1}, m_{b2}, C_{b2} – given masses and flexural rigidities of the cross-beams with coils, m_1, C_1, m_2, C_2 – given masses and flexural rigidities of oscillatory systems, y_{b1}, y_1, y_{b2}, y_2 – complex vibration amplitudes of the cross-beams and oscillatory systems displacements related to the fix frame, C – the mechanical coupling rigidity of the oscillatory systems mediated by the structure of the rig frame. We shall consider a pair of vibrators of the oscillatory system as one giving the double force

$$F_{v1} = 2\mu_1 i_1, \qquad (1)$$

where i_1, μ_1 - current and transfer coefficient of an electrodynamic vibrator. Let us consider the oscillatory system to be linear, and with harmonic displacements at feedeng the vibrator by a sinusoidal current with frequency ω. We shall suppose in the first approximation that on an airfoil, the root part of which makes simple harmonic motions with amplitude y_1 and frequency ω, the distributed uniform load Q_1 acts. This load has inertial and aerodynamic component:

$$Q_1 = (m_{p1}\omega^2 y_1 + L_1)/h \;, \qquad (2)$$

where m_{p1} - airfoil mass, h - airfoil length, L_1 – unsteady aerodynamic force.

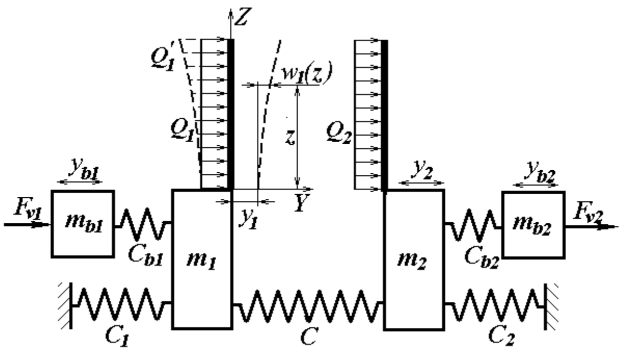


Figure 3: Schemes of two cascade airfoils excitations in flexural oscillation modes

We shall also consider, that at small airfoil vibration amplitudes, the non-stationary aerodynamic forces will be linearly connected to oscillations [1]:

$$L_1/h = qbC_{y1}, \quad L_2/h = qbC_{y2}, \qquad (3)$$

where $C_{y1} = \bar{y}_1 l_{11y} + \alpha_1 l_{11\alpha} + \bar{y}_2 l_{12y} + \alpha_2 l_{12\alpha}$, $C_{y2} = \bar{y}_1 l_{21y} + \alpha_1 l_{21\alpha} + \bar{y}_2 l_{22y} + \alpha_2 l_{22\alpha}$, $\bar{y}_1 = y_1/b$, $\bar{y}_2 = y_2/b$ - dimensionless (related to the airfoil chord) complex amplitudes of translational oscillations, α_1, α_2 - complex amplitudes of angular oscillations in radians, q - dynamic pressure, l_{11y}, $l_{11\alpha}$, l_{12y}, $l_{12\alpha}$, l_{21y}, $l_{21\alpha}$, l_{22y}, $l_{22\alpha}$ - aerodynamic coupling coefficients, representing the complex constants of proportionalities between airfoil oscillations and forces on an airfoil, induced by these oscillations.

Then the distributed load on an airfoil can be noted as

$$Q_1 = \frac{m_{p1} b}{h}\omega^2\left[\bar{y}_1 + \bar{q}_1 C_{y1}\right], \qquad (4)$$

where $\bar{q}_1 = \dfrac{qh}{m_{p1}\omega^2}$ - relative dynamic pressure, m_{p1} - airfoil mass.

Let's mark with k_{f1} relation of the lowest airfoil natural frequency to the operational one ω. The calculation can be limited to the first modes of airfoil oscillations $w_{0i}(z)$, if the value $k_{f1} > 2$ (see [4]). In this case, we shall receive distribution of the airfoil deformations in shares of the airfoil chord with error less than 1%.

$$\overline{w}_1(z) = \frac{\pi}{4}\frac{w_{01}(z)}{k_{f1}^2 - 1}\left(\overline{y}_1 + \overline{q}_1 C_{y1}\right), \qquad \overline{w}_2(z) = \frac{\pi}{4}\frac{w_{01}(z)}{k_{f2}^2 - 1}\left(\overline{y}_2 + \overline{q}_2 C_{y2}\right) \tag{5}$$

Knowing displacements of the constituent airfoil cross-sections $\overline{y}_1^{'} = \overline{y}_1 + \overline{w}_1(z)$, $\alpha_1^{'} = \alpha_1$, $\overline{y}_2^{'} = \overline{y}_2 + \overline{w}_2(z)$, $\alpha_2^{'} = \alpha_2$, we can find the updated distributed load on the airfoil with influnce of its deformation:

$$Q_1^{'}(z) = \frac{m_{p1}b}{h}\omega^2\left[\overline{y}_1^{'} + \overline{q}_1\left(\overline{y}_1^{'}l_{11y} + \alpha_1^{'}l_{11\alpha} + \overline{y}_2^{'}l_{12y} + \alpha_2^{'}l_{12\alpha}\right)\right], \tag{6}$$

After the loading integration along the airfoil length, we shall receive resulting forces:

$$R_1^{'} = m_{p1}b\omega^2\overline{y}_1(1+\theta_1) + qhb\left(\overline{y}_1 l_{11y}^{'} + \alpha_1 l_{11\alpha}^{'} + \overline{y}_2 l_{12y}^{'} + \alpha_2 l_{12\alpha}^{'}\right), \tag{7}$$

$$R_2^{'} = m_{p2}b\omega^2\overline{y}_2(1+\theta_2) + qhb\left(\overline{y}_1 l_{21y}^{'} + \alpha_1 l_{21\alpha}^{'} + \overline{y}_2 l_{22y}^{'} + \alpha_2 l_{22\alpha}^{'}\right). \tag{8}$$

Following substitutions were introduced into the last two formulas:

$$\theta_1 = \frac{0{,}613}{k_{f1}^2 - 1}, \qquad \theta_2 = \frac{0{,}613}{k_{f2}^2 - 1}, \tag{9}$$

and also

$$\begin{aligned}
l_{11y}^{'} &= (1+2\theta_1)l_{11y} + \theta_1\overline{q}_1 l_{11y}^2 + \theta_2\overline{q}_2 l_{12y} l_{21y} \\
l_{12y}^{'} &= l_{12y}(1+\theta_1+\theta_2+\theta_1\overline{q}_1 l_{11y} + \theta_2\overline{q}_2 l_{22y}) \\
l_{21y}^{'} &= l_{21y}(1+\theta_1+\theta_2+\theta_1\overline{q}_1 l_{11y} + \theta_2\overline{q}_2 l_{22y}) \\
l_{22y}^{'} &= (1+2\theta_2)l_{22y} + \theta_2\overline{q}_2 l_{22y}^2 + \theta_1\overline{q}_1 l_{12y} l_{21y}
\end{aligned}, \tag{10}$$

$$\begin{aligned}
l_{11\alpha}^{'} &= (1+\theta_1)l_{11\alpha} + \theta_1\overline{q}_1 l_{11y} l_{11\alpha} + \theta_2\overline{q}_2 l_{12y} l_{21\alpha} \\
l_{12\alpha}^{'} &= (1+\theta_1)l_{12\alpha} + \theta_1\overline{q}_1 l_{11y} l_{12\alpha} + \theta_2\overline{q}_2 l_{12y} l_{22\alpha} \\
l_{21\alpha}^{'} &= (1+\theta_2)l_{21\alpha} + \theta_2\overline{q}_2 l_{22y} l_{21\alpha} + \theta_1\overline{q}_1 l_{21y} l_{11\alpha} \\
l_{22\alpha}^{'} &= (1+\theta_2)l_{22\alpha} + \theta_2\overline{q}_2 l_{22y} l_{22\alpha} + \theta_1\overline{q}_1 l_{21y} l_{12\alpha}
\end{aligned}. \tag{11}$$

Now we write down two movement equations, one for the crossbeam and the other for the vibration system. The airfoil action is replaced by its reaction $R_1^{'}$:

$$\begin{cases} \left(C_{b1} - m_{b1}\omega^2\right)y_{b1} - C_{b1}y_1 = 2\mu_1 i_1 \\ \left(C_{b1} + C_1 + C - m_1\omega^2\right)y_1 - C_{b1}y_{b1} - C\cdot y_2 = R_1^{'} \end{cases} \tag{12}$$

Supposing the vibration excitation of both systems in flowing and in not flowing fluid of identical displacements

$$y_1 = y_{01},\; y_2 = y_{02}, \tag{13}$$

the equations (12) can be transformed to the following forms:

$$\left(y_1 l_{11}^{'} + y_2 l_{12}^{'}\right) = \frac{2\eta_1\mu_1}{qh}\cdot\left(i_{01} - i_1\right), \tag{14}$$

$$\left(y_1 l_{21}^{'} + y_2 l_{22}^{'}\right) = \frac{2\eta_2\mu_2}{qh}\cdot\left(i_{02} - i_2\right), \tag{15}$$

where $\eta_1 = \frac{k_{b1}^2}{(k_{b1}^2 - 1)}$, and k_{b1}^2 - quadrate of the relation of cross-beam natural frequency to operational frequency ω.

If we perform similar measurements in a stream and without a stream at another version of the linearly independent systems oscillations, we shall receive further two equations similar to (14) and (15). The system of these four equations will allow determining the unknown quantities $l_{11}^{'}, l_{12}^{'}, l_{21}^{'}, l_{22}^{'}$. In turn, it allows to find the updated aerodynamic coupling coefficients l_{11}, l_{12}, l_{21}, l_{22} from the equation systems (10) and (11).

The unknown products $\mu_1\eta_1$ and $\mu_2\eta_2$ can be found by dynamic calibration. For this purpose there is possible to apply to the system a known inertial force instead of the aerodynamic force, having attached to the mass m_{m1} an additional mass Δm. More details about deriving the expressions (14) and (15) and about dynamic calibration it is possible to read in [4].

The rigidity and, therefore, the natural frequency of an airfoil can be increased by fastening its ends to two measuring systems, as shown in fig. 4. Considering the airfoil as a beam with turning joints on extremities, we shall receive

$$\theta_1 = \frac{0{,}811}{k_{f1}^2 - 1}, \qquad \theta_2 = \frac{0{,}811}{k_{f2}^2 - 1}. \qquad (16)$$

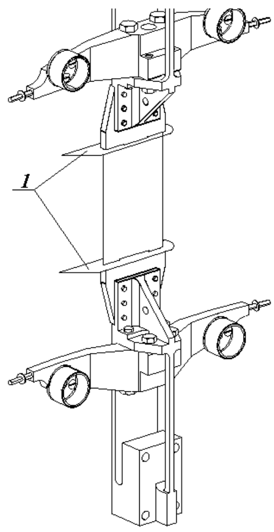


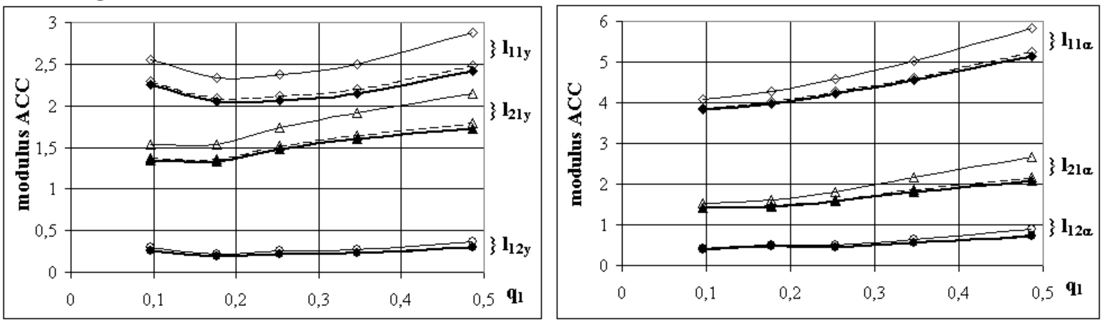
Figure 4: Measuring system with airfoil suspended at both ends. 1 - elastic hinges

3. EVALUATION OF THE AIRFOIL DEFORMATION INFLUENCE ON RESULTS OF FORCES MEASUREMENT

If the airfoils can be considered as absolutely rigid ($\theta_1 = \theta_2 = 0$), then from the (10) and (11) follows $l'_{jky} = l_{jky}$ $l'_{jk\alpha} = l_{jk\alpha}$. It means, that l'_{jky} and $l'_{jk\alpha}$ represent the "old" aerodynamic coupling coefficients, which can be determined by the old technique of aerodynamic loads measurement disregarding the airfoils deformation [3]. Thus, (10) and (11) enable to raise precision of the “old" aerodynamic coupling coefficients.

In fig. 5a there are presented "the old" aerodynamic coupling coefficients l'_{jky} (thin solid lines - variant of the cantilever supporting of airfoils, thin dashed lines - variant of the beam supporting of airfoils), and more accurate aerodynamic coupling coefficients l_{jky} (bold lines) in dependance on a relative dynamic pressure. The quantities l'_{22y} and l_{22y} are not shown in the figures because in our example $l'_{11y} = l'_{22y}$. Similar relations, valid for aerodynamic coupling coefficients $l'_{jk\alpha}$ and updated $l_{jk\alpha}$ of torsion vibrations, are depicted in fig. 5b, again in absolute values. The corresponding arguments are shifts of phases between an aerodynamic force on an airfoil and oscillation displacement of one from the airfoils inducing this force.

The airfoils in experiment were rather thin (the airfoil thickness was 0,033 of the chord). They were made of a composite material on the basis of carbon filaments. The relation of the lowest natural frequency of airfoils to operational frequency at cantilever supporting kf1 = kf2 = 3,29. For the airfoil supporting with turning joints on both ends k_{f1} = k_{f2} = 9,24. As it is visible from fig. 5a and fig. 5b, with the relative dynamic pressure increasing, i.e. with raising relation between aerodynamic airfoil loads and inertial ones, the error of measurements grows and achieves 18 % in absolute value and 2° in argument at cantilever supporting. At the beam airfoil supporting, the influence of the airfoil deformation does not exceed 4 % in absolute value and 0,5° in argument.



a) b)

Figure 5: Dependences of absolute value of aerodynamic coupling coefficients on relative dynamic pressure at flexural oscillations (bold line - more accurate aerodynamic coupling coefficients)

4. CONCLUSIONS

The mathematical model of a system for the non-stationary aerodynamic loads measurement at the flexural-torsion oscillations of measuring systems with airfoils in a cascade is developed. The mathematical model takes into account the finite flexural rigidity of airfoils and of other measuring systems elements.

It was found a method giving higher precision to the linear non-stationary aerodynamic forces obtained by the old technique at flexurel-torsion oscillations. The application of airfoils fabricated from a composite material on the basis of carbon filaments, and the airfoil beam supporting, increase the loads accuracy determination in the experimental rig.

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