Dynamic Properties and Investigations of Piezoelectric Force Measuring Devices

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Abstract

Piezoelectric measurement chains are widely used in many applications of dynamic force measurements, since these sensors offer a wide useable frequency range due to their high stiffness. Typical applications of piezoelectric force sensors are described in this paper. In general, the transducers are calibrated by quasistatic or continuous calibration methods. This contribution describes the dynamic properties of piezoelectric force measuring devices and methods to determine the dynamic response of the piezoelectric force measuring chain.

1. Introduction

In a great variety of applications, piezoelctric force measuring devices are used for the measurement of dynamic forces. A reduction of the measurement uncertainties requires that calibrated force transducers with well-known dynamic properties are used. In the past, dynamic calibration procedures were therefore developed at PTB to determine the dynamic response of force transducers [1, 2]. Nevertheless, in special applications the arrangement of the force transducer, the mounting conditions and the whole mechanical structure of the measuring arrangement may significantly influence the uncertainty of dynamic force measurement in these particular cases. It is the aim of this paper to discuss possible influences and describe some methods according the basic vibration theory in order to reduce dynamic errors.

2. Typical applications

Piezoelectric force sensors are used in many different applications like demonstrated in Fig. 2a. Continous or stepwise calibration methods are used for the determination of the senitivity of the piezoelectric force sensor. In dynamic applications like shown in Fig. 2a the dynamic properties of the sensors should be well known.

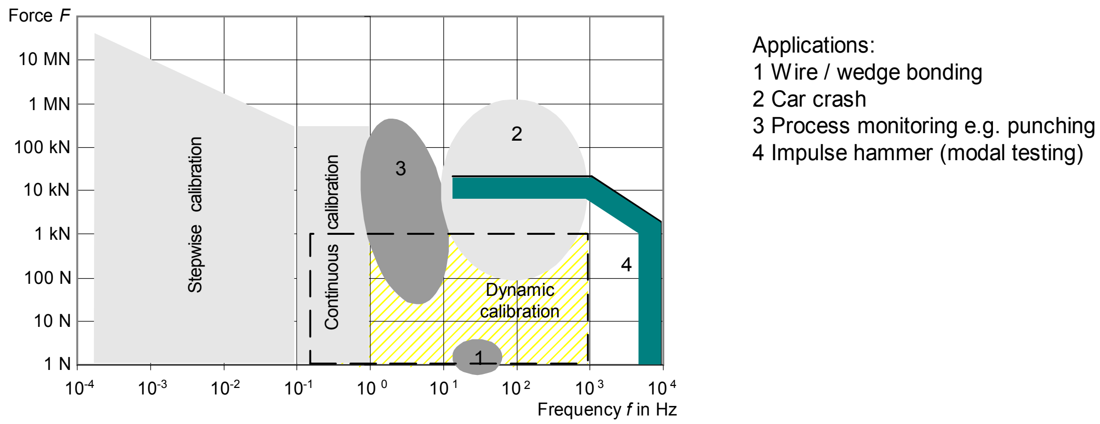

Fig. 2a: Force-frequency plot of dynamic calibrations and applications of piezoelectric force transducers

For a clear understanding of the dynamic behavior of force sensors, the following three cases are to be distinguished:

a) Sensor without additional external mass
A typical application for rigidly mounted force sensors with no additional mass is measuring the forces during a car crash. Fig 2b shows the situation after a crash against a car crash barrier. The crash was executed at an impact velocity of 70 km/h.

Fig. 2b: Situation after car crash at v=70 km/h against a crash barrier with piezoelectric force sensors

On the rigid block (right) several force measuring platforms are rigidly mounted, each of them consisting of normally four force measuring sensors and one cover plate. The dynamic behavior of such a system can easily be determined by means of a frequency analysis, resulting in the frequency response function (magnitude and phase). The excitation of the system may be performed by means of an instrumented impulse force hammer.

b) Sensor with additional external mass
The rigidly mounted sensor is affected by some additional mass, upon which the dynamic behavior is depending. Example: cutting force measurements, where the rigidly mounted mass of the workpiece changes the dynamic behavior (i. e. natural frequency) of the sensor without additional mass. In addition, under acceleration this extra mass causes an unwanted inertial force, which is superimposed to the process force under investigation and may cause an error, hence. Knowing the additional mass, the effects of the inertial force can be significantly reduced by means of measuring the acting acceleration on the force sensor and an appropriate signal processing. Fig. 6a shows a typical setup for such an inertia compensated force measuring device.

c) Sensor mounted into a structure
When a force sensor is mounted under preload into a structure, the dynamic behavior of the system, consisting of the sensor and the structure, has to be taken into account. Compared to the case of the sensor without additional mass, the modal properties of the whole structure dominate the dynamic behaviour.

3. Dynamic description of piezoelectric force transducers
The dynamic behaviour of force transducers can often be described by the simple spring mass model represented in Fig. 3a, i.e. by the motion of two masses with a spring of zero mass and stiffness k_f, and with a zero mass damper of damping coefficient b_f connected in parallel (Voigt model). The masses of the transducer are discrete in this model and divided into an internal upper mass m_{ti} and an internal lower mass m_{bi}. Force introduction often does

not take place directly but through additional external masses denoted here by m_{ta} and m_{ba}. Moreover, if it is assumed that the external masses are rigidly connected to the force transducer, the oscillation behaviour of the force transducer can be described by the movements of the upper and lower masses, m_t and m_b. The displacements of the upper and lower masses from their rest positions are denoted by x_t and x_b. F_t and F_b are the external forces acting on the upper and lower mass respectively.On the assumption that the sensor can be described as a non-delayed proportional element, the transducer output signal U_f is directly proportional to the spring force $k_f \cdot r_f = k_f \cdot (x_t - x_b)$.

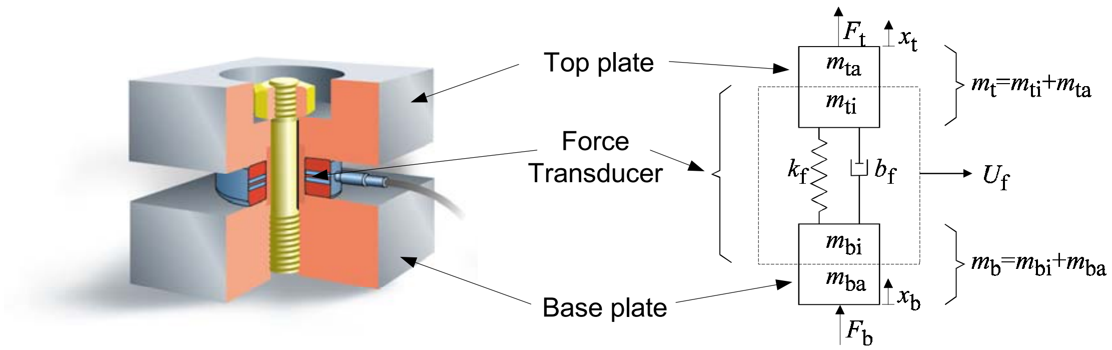

Fig. 3a: Model of a force transducer with additional external masses and external forces.

The system of differential equations can, therefore, be rewritten as follows:

$$m_t \cdot \ddot{r}_f + b_f \cdot \dot{r}_f + k_f \cdot r_f = F_t - m_t \cdot \ddot{x}_b \quad (1a)$$

$$m_b \cdot \ddot{r}_f + b_f \cdot \dot{r}_f + k_f \cdot r_f = -F_b + m_b \cdot \ddot{x}_t \ . \quad (1b)$$

The static sensitivity of a force measuring device S_{f0} is defined as the quotient of the change in output signal and the change in active force. Assuming that the static sensitivity is constant and that there is a linear relationship between the spring force $k_f \cdot r_f$ and output signal U_f of the force transducer, it follows that: $k_f \cdot r_f = S_{f0}^{-1} \cdot U_f$ (2)

During the measurement of dynamic forces the transducer is subjected to external force excitation. The resonance behaviour of the force transducer can be derived from Eq. 1a for the excitation with a external sinusoidal force F_t in the case of a rigid base (unaccelerated lower mass).

The resonance behaviour is described by the amplification function

$$V\left(\frac{\omega}{\omega_0}, D\right) = \frac{k_f \cdot \hat{r}}{\hat{F}} = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + 4D^2\left(\frac{\omega}{\omega_0}\right)^2}} \quad (3a)$$

and the matching phase curve

$$\varphi_{rF} = \varphi_r - \varphi_F = -\arctan\left(\frac{2D\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}\right) \quad (3b)$$

plotted in Fig. 3b for different damping factors D. Piezoelctric force transducers usually have a very small damping factor of $0<D<0{,}01$. The amplification function shows that the resonance behaviour of a transducer can lead to large systematic deviations.

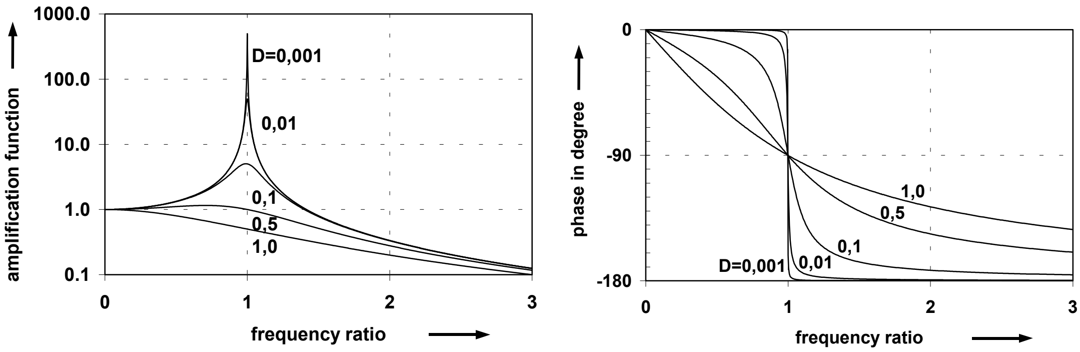

Fig. 3b: Amplification function (amplitude response) V(ω/ω_0,D) according to Eq. 3a for various damping factors D as a function of the frequency ratio ω/ω_0. Phase shift φ_{rF} according to Eq. 3b for various damping factors D as a function of the frequency ratio ω/ω_0.

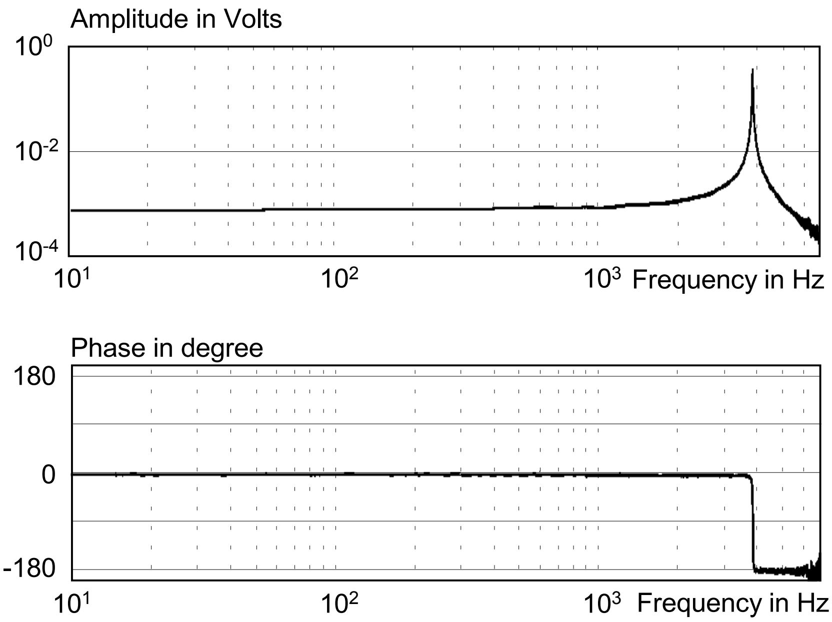

Fig. 3c: Frequency response of a 5 kN piezoelectric sensor with a additional external mass of 0,626 kg. The resonant frequency was measured to be 3,88 kHz. The damping ratio and the spring constant derived are: D = 0,002, k_f= 3,71·10^8 N/m.

For the unloaded piezoelectric force transducer the resonance frequency f_r lies in the kHz range but when coupled with external masses the resonance frequency can be considerable reduced. The measurement results shown in Fig. 3c are obtained with the arrangement according Fig. 3a with a 5 kN piezoelctric force sensor and an additional external mass of 0,626 kg. The spring mass system was mounted on a very rigid base and exposed to a fast stepwise force signal. In this setup, the resonant frequency was measured to be 3,88 kHz.

The resonance measurements are also performed with a similar 5 kN piezoelectric force transducer on a shaker system with two different large masses of 22,5 kg and 18,5 kg [1]. From the resonance frequencies of 432 Hz and 492 Hz the stiffness was calculated to be k_f= 1,74·10^8 N/m and k_f = 1,78·10^8 N/m, respectively. This value differs from the value determined with the step response setup. The resonance behaviour is strongly related to the experimental setup and can show considerable differences depending on the stiffness of the whole mechanical structure which is in interaction with the piezoelcetric force transducer.

Furtheron it follows that the resonance frequency of the force transducer is considerably reduced by the coupling of large load masses, compared with the resonance frequency with no additional masses, given in the manufacturer's data sheet.

The relationship between the resonance frequency f_r and characteristic frequency f_0 of a force transducer is given by

$$f_r = f_0\sqrt{1-2D^2} \ . \qquad (4a)$$

When a transducer has negligible damping, as is true for piezoelectric sensors (see Fig. 3c), its resonance frequency f_r and eigenfrequency f_d are equal to the characteristic frequency f_0 and change with the coupled mass:

$$f_r = f_d = f_0 = \frac{1}{2\pi}\cdot\sqrt{\frac{k_f}{m_{ti}+m_{ta}}} \ . \qquad (4b)$$

Here m_{ti} is the inner (co-oscillating) mass of the transducer and m_{ta} the external additional mass. The fundamental eigenfrequency f_{0g} of a force transducer is defined as the eigenfrequency without external additional mass:

$$f_{0g} = \frac{1}{2\pi}\cdot\sqrt{\frac{k_f}{m_{ti}}} \ . \qquad (4c)$$

It follows that the resonance frequency of the force transducer is considerably reduced by the coupling of large load masses, compared with the resonance frequency with no additional masses. Considerable measurement errors may therefore occur if the change in resonance behaviour due to the coupling of additional masses is not taken into account.

4. Dynamic calibration of piezoelectric force measuring devices

4.1 Principle of dynamic calibration

Dynamic investigations are being carried out at PTB with piezoelectric force measuring devices with the facility shown in Fig. 4a. The piezoelectric force transducer is mounted on a shaker and a load mass m_l is coupled to the force transducer. Excitation by the shaker results in a dynamic force *F* acting on the force transducer:

$$F = (m_l + m_e)\cdot\ddot{x}_l \qquad (5)$$

where $\ddot{x}_l$ is the acceleration of the load mass m_l and m_e is the end mass of the force transducer. The acceleration on the load mass is measured by acceleration transducers which are calibrated by interferometric procedures [3]. The simple equation (5) does not take into account the effects of the relative motion of the load mass and the influences of side forces which must be considered because force is a vector quantity. If large masses are used the influence of side forces is reduced by an air bearing system [1]. If small masses are used like in this investigation the influence of side forces can be reduced by measurements in different mounting positions and averaging of the measurement data [1]. To allow for the effect of relative motion, the dynamic force must be determined from the acceleration distribution $a(x,t)$ and the mass distribution with density ρ according to

$$F = \int_V \rho\cdot a(x,t)\cdot dV \ . \qquad (6)$$

For the determination of the acceleration distribution, multicomponent acceleration measurements must be carried out as shown in Fig. 4a, and the theory presented in [1,4] must be used to calculate the dynamic force. According to e.q. (5), (6) the dynamic force is traceable to the definition of force according to Newton's law. This calibration procedure allows the dynamic sensitivity of the force measuring device to be determined, comprising the piezo-electric force transducer and the charge amplifier.

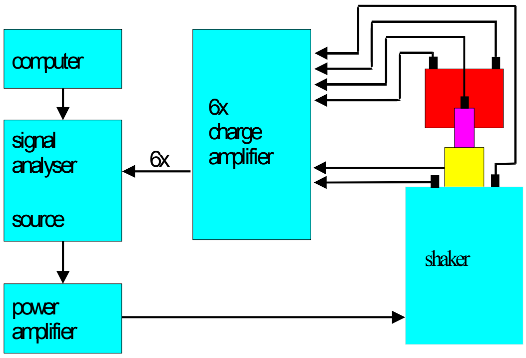

Fig. 4a: Calibration facility for dynamic force measurement.

4.2 Dynamic response of charge amplifier

The structure and signal flow of piezoelectrical force measuring devices are usually represented as a block diagram according to Fig. 4b. The static and dynamic properties of such a force measuring device are influenced by all components shown in the block diagram.

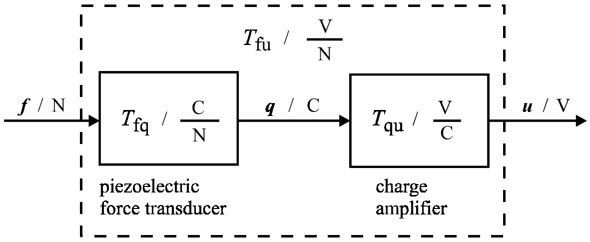

Fig. 4b: Signal flow of a piezoelectric force measuring device and model of a force transducer with additional external masses and external forces.

According to the signal flow in Fig. 4b the frequency response of a piezoelectric force measuring device T_{fu} is the product of the frequency response of the force transducer T_{fq} and of the frequency response of the charge amplifier T_{qu}, provided that transducer and amplifier are non-interacting:

$$T_{fu} = T_{fq} \cdot T_{qu} \tag{7}$$

If the frequency response of the force measuring device and charge amplifier is known, the frequency response of the piezoelectric force transducer can be calculated by division. The charge amplifiers are therefore calibrated with a well-known capacitance. Figure 4c shows a typical result of the measured frequency response T_{qu} of a charge amplifier for different time constants. The dynamic measurements are carried out with the short time constant of 1 sec. But in principle the measurements are also possible with the long time constant which is used in quasistatic measurements, if care is taken to the overload of the charge amplifier.

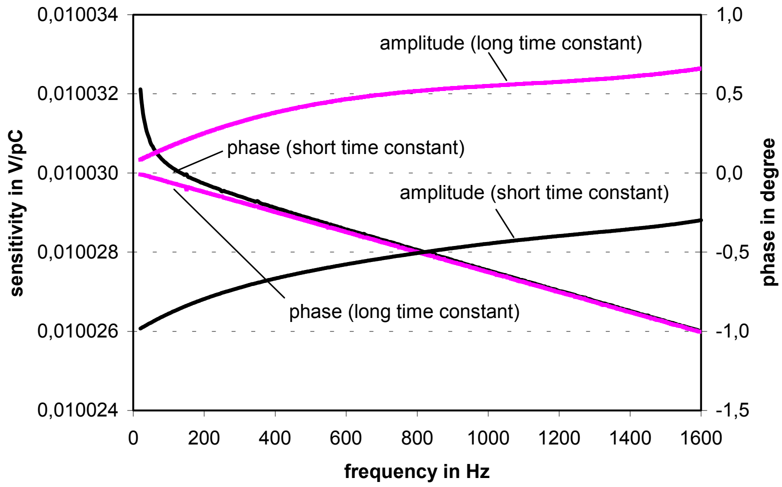

Fig. 4c: Typical amplitude and phase response of a charge amplifier for different time constants (long and short). Note: The amplitude (long time constant) varies as less as 0,02 % in the frequency range of 20 ... 1600 Hz.

4.3 Dynamic response of piezoelectric force transducers

To determine the dynamic characteristics of piezoelectric force measuring devices, measurements with a set of load masses m_l were carried out. Because of the smaller dimensions and the smaller endmass of piezoelectric force transducers, calibration is also possible with smaller masses [5]. Therefore the frequency response was measured with steel load masses of mass 0,44433 kg, 0,84447 kg and 1,2449 kg. From the measurement results obtained with these masses, the dynamic sensitivity was determined according to the theory presented in [1,4]. The end mass was determined by a least square fit to 96,5g. For the cylindrical steel masses which are screwed directly by a small adapter screw on the top of the force transducer the following equation was used to evaluate the influence of the relative motions:

$$F = (m_\mathrm{l} + m_\mathrm{e}) \cdot \ddot{x}_{\mathrm{lo}} \cdot \mathrm{K}_{\mathrm{mo}} \qquad (8)$$

with the correction factor $\mathrm{K}_{\mathrm{mo}} = \sin\left(\sqrt{\frac{\rho}{E} \cdot \omega_0^2} \cdot l\right) \cdot \frac{1}{\sqrt{\frac{\rho}{E} \cdot \omega_0^2} \cdot l}$,

the acceleration measured on top of the load mass $\ddot{x}_{\mathrm{lo}}$, the excitation frequency $f = 2\pi \cdot \omega_o$, the material density ρ , the elasticity modulus E and the length of the load mass l.

The results are shown in Fig. 4d. The previously determined frequency response of the charge amplifier (Fig. 4c) had already been taken into account. The measurements of the dynamic sensitivity in the frequency range up to 800 Hz showed no significant deviations. There is only a relative small decrease of sensitivity of about 0,5% up to 1600 Hz. Some irregularities in the frequency range are caused by the transverse motion of the shaker which results in an increase of the relative standard deviation of the measurements. The measurements show a relatively constant dynamic sensitivity and good correspondence with the quasistatic sensitivity of 3,91 pC/N which was determined by contionus and stepwise calibration described below. The piezoelectric force transducer is, therefore, well-suited for dynamic measurements.

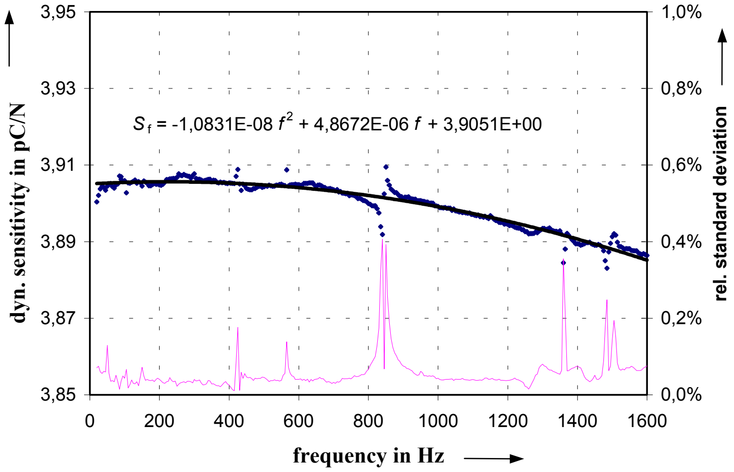

Fig. 4d: Dynamic sensitivity and relative standard deviation calculated from the measurements carried out with masses of 0,44433 kg, 0,84447 kg and 1,2449 kg calculated with a end mass of 96,5 g.

5. Comparison with quasistatic and continous calibration

In practical application piezoelectric force transducers are characterized by a linear sensitivity coefficient, which is well defined in a fixed load range. Typical load ranges are the nominal load range of the force transducer and a partial load range up to 10 % of the nominal load range. To minimize the influence of drift, manufacturers of piezoelectric force transducers determine the sensitivity by continuous calibration methods. The results are evaluated by means of the method of the best straight line (BSL), and are specified as sensitivity, linearity error and hysteresis. The BSL is defined as a straight line through zero, which minimizes the maximum deviations from the response curve (Tchebycheff approximation). A continuous calibration of the 5 kN force sensor yielded a sensitivity of 3,91 pC/N in the partial load range up to 2 kN [6].

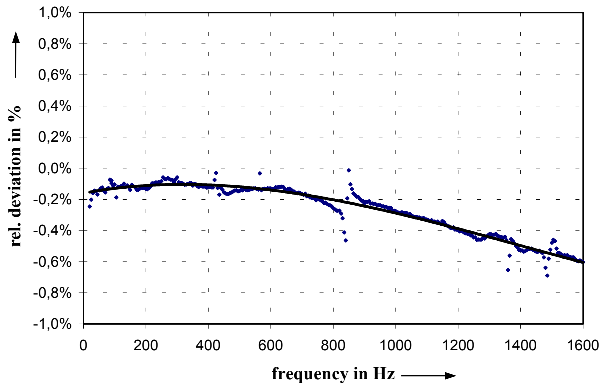

Fig. 5a: Relative deviation between the dynamic sensitivity and the quasistatic sensitivity of a piezoelectric force transducer in the partial load range up to 2 kN obtained with continous calibration method.

As shown in Fig. 5a the relative deviation between the dynamic sensitivity and the quasistatic sensitivity is in the whole frequency range small compared to the uncertainty of the dynamic calibration method which is less 1%. Therefore the dynamic calibration is in good accordance with the sensitivity determined by the continuous calibration method. The small differences can be caused by different influences. The quasistatic sensitivity is determined only in the compression range. In the dynamic use on the shaker compression and tension forces are acting on the transducer. Therefore the different sensitivities in the tension and compression range have to be considered for a detailed analysis [1]. Furthermore the dynamic load is small compared to the partial load range of 2 kN used for the continous calibration. Nonlinearities can influence the sensitivity [7]. The decrease of the dynamic sensitivity with increasing frequency can often be explained by additional spring effects between the load mass and the sensing element of the force transducer which results in a decrease which is proportional to the square of the frequency [1].

At PTB piezoelectric force transducers are calibrated with static forces, which are generated with deadweight force standard machines in different force steps. To ensure comparability with continuous calibrating methods the loads are applied stepwise in increasing and decreasing load steps. The transducer is mainly specified by the repeatability in the same mounting position, the repoducibility in different mounting positions and the hysteresis. The sensitivity is determined by means of a least square fit with forced zero. In opposite to the continous calibration methods the sensitivity is strongly influenced by drift effects. This is illustrated by twelve independent measurements m_1 to m_{12} with the force transducer in the partial load range up to 2 kN (see figure 5b).

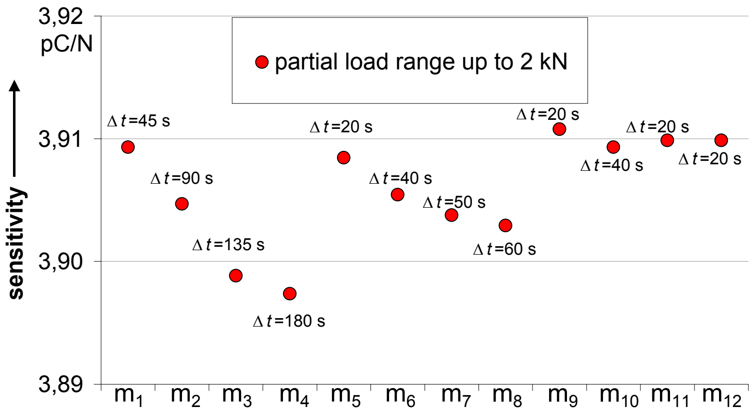

Fig. 5b: Value of sensitivity of the piezoelectric force transducer in the partial load range up to 2 kN

The measurement values are recorded at equidistant time intervals between Δt = 20 s and Δt = 180 s. Due to the linear drift of the charge amplifier and different time intervalls Δt the value of the sensitivity shows a relative variation of $4 \cdot 10^{-3}$ between 3,8974 pC/N and 3,9108 pC/N. Nevertheless stepwise and continous calibration methods are directly comparable for short time intervals Δt. In this case the sensitivity determined stepwise with static forces is in good accordance with the continous sensitivity of 3,91 pC/N. Long time intervals Δt, for example due to slow load changes of the force calibration machine require unique calibration methods [6,7].

6. Consequences for dynamic applications

To avoid systematic errors, inertial forces and damping forces must be taken into consideration in dynamic applications. According to the equations in chapter 3, this requires that the masses acting between the measuring spring and the points of force application be known, as well as the stiffness and damping of the force transducer. For small uncertainties in the measurement of dynamic forces it is indispensable that the force transducer can actually be described by the model on chapter 3. From the differential equation follows the interpretation

that the force transducer is a 2nd order measuring component (PT_2-term) which has become important in numerous applications. If the base acceleration is negligible (e.g. for a rigid support), the results may also be used to compensate systematic deviations. Anyhow, it must be pointed out that for larger base accelerations these must be measured, too, and considered in the compensation as well [8].
If the relationship between deformation and force transducer signal is given by Eq. 2, the equation may be rewritten as:

$$F_t = S_{f0}^{-1} \cdot U_f + m_t \cdot \ddot{x}_t + \frac{b_f}{k_f} \cdot S_{f0}^{-1} \cdot \dot{U}_f \quad (9)$$

According to this equation the dynamic force F_t to be measured can be determined from the transducer signal U_f and from the measurement of the acceleration $\ddot{x}_t$ of the effective mass m_t. For piezoelcetric force transducers the damping is small which simplifies the equation. Such a compensation according to Fig. 6a is commercially available and described in [9].

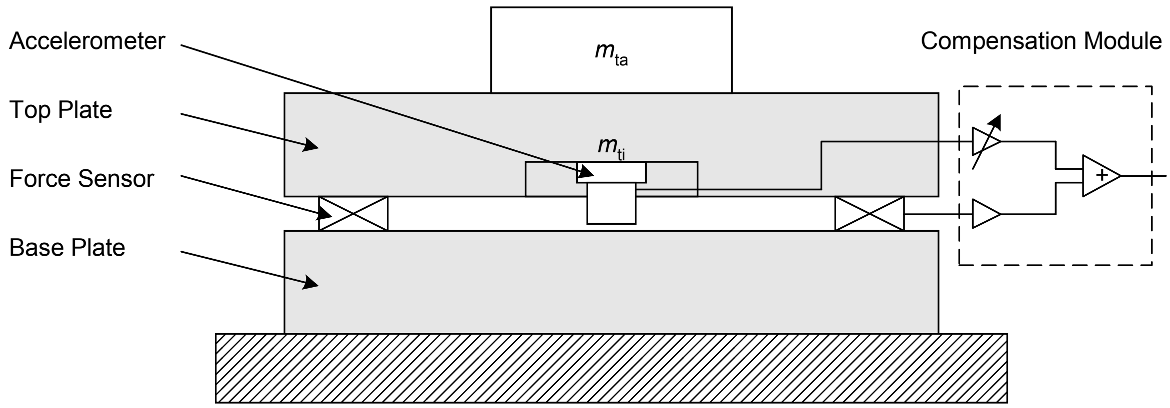

Fig. 6a: Schematic of commercially available force measuring sensor with compensation of inertial forces.

7 Conclusions

This paper describes the dynamic response of piezoelectric force transducers which has to be taken into account in dynamic applications. Different influences are described and methods are discussed to reduce the measurement uncertainty with respect to applications in the field of dynamic force measurement.

It is shown that in many cases the force indicated by the force transducer can noticeably differ from the force which must be determined in dynamic application. The deviation increases with increasing frequency and mainly depends on the stiffness and the mass distribution in this particular case. A reduction of these systematic influences can be achieved by application of a theoretical model which describes the vibration behaviour of the system. Methods are discussed which allow the systematic influences to be reduced. If the forces are measured with force transducers, only, the resonance behaviour must be well known, and it must be reproducible so that the systematic influences can be taken into account. Moreover, it must be considered that, in addition to the resonance behaviour of the force transducer, the resonance behaviour of the surrounding mechanical structure can significantly influence the measurement results. Another method takes into account the acceleration of the acting masses. This demonstrates that dynamic force measurement is strongly related to acceleration measurement.
The particularly high rigidity and small dimensions of these piezoelectric force sensors are unique features for dynamic applications. It is pointed out that the frequency response is determined by the properties of the charge amplifier and the piezoelectric transducer. The sen

sitivity as a function of frequency of the piezoelectric force transducer is determined with the dynamic calibration methods. The dynamic sensitivity of piezoelectric force measuring devices is affected by every component of the measuring chain. Main influences come from the charge amplifier and from the force introduction. If the influences of charge amplifiers are taken into account, the dynamic sensitivity of the piezoelectric force transducers exhibits a small change in the frequency range from 20 Hz to 1600 Hz. A good agreement could be achieved between dynamic sensitivity and quasistatic sensitivity. Therefore, it is possible to perform dynamic force measurements with piezoelectric force sensors, provided the quasistatic sensitivity of the sensor as well as the dynamic behaviour of the whole structure is known. In principle quasistatic calibration of piezoelectric force measuring devices is possible with force standard machines, but the charge drift of the charge amplifier must be taken into account.

The resonance behaviour of piezoelectric force measuring devices must be observed in additional dynamic measurements. Piezoelectric force transducers may have high resonance frequencies in the kHz range. The resonance can, however, be considerably be reduced to less than 1 kHz if large masses are coupled to the transducers.

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