Proceedings of the 17th International Conference on Force, Mass, Torque and Pressure Measurements,
IMEKO TC3, 17-21 Sept. 2001, Istanbul, Turkey

Preliminary Uncertainty Estimation of the Pressure Distortion Coefficient of a Pressure Balance by FEM Calculations

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Abstract

The great development of the Finite Element, the Volume Element and the Boundary Element Methods allows the solution of metrological problems with estimated uncertainties that are improving as far as experience is reached and methods are validated by experimental results. Consequently the uncertainty estimation in the numerical Finite Element Method is still an open object of debate. In the present work an iterative method based on Finite Element Method, developed by the IMGC-CNR in Turin and the University of Cassino, is applied to characterize the national primary standard for liquids from 10 MPa to 100 MPa (IMGC-100-NN_{tc}). Particular attention is paid to the evaluation of the different contributions to the uncertainties in the calculation of the pressure distortion coefficient of the piston-cylinder unit.

1. Introduction

Over the last few years the development of innovative high pressure production technologies and the scientific interest in determining the thermophysical properties of materials has increased the attention paid to high pressure metrological traceability. The pressure distortion coefficient, λ, and the piston fall-rate, v, are very important parameters for the characterization of pressure balances especially when operating in high pressure conditions [1, 2]. These parameters depend on piston-cylinder design and on other operating parameters (e.g. applied pressure on sealing).

The methods used to evaluate λ can be classified as experimental ("*cross-floating method"* [2]), "*similarity method*" [3], *"piston fall rate method"* [1] or the *"flow method*" [3]) or theoretical ones, based on the evaluation of the pressure distortion coefficient by the analytical or numerical solution of a system of differential equations. Even if different analytical methods were developed in the last years (based on the *"Lamè equation"* or on the combined analysis of the laminar flux in the clearance and the elastic distortions of the unit [4]), generally the complexity of the geometry and of the boundary conditions make *"Finite Element Methods"* (FEM) more reliable and useful to solve mechanical problems. In the last few years, many authors have used the FEM to characterize pressure balances and, in particular, to evaluate the pressure distortion coefficient [5-9]. This method can be also successfully applied to design complex units extending the pressure range up to few GPa [10]. To design and optimize the use of pressure balances the Finite Element Method developed by the University of Cassino and the IMGC-CNR of Turin [5] was optimized and applied to different kinds of pressure balances [11]. In [12] the proposed method was applied to the pressure balance IMGC-100-NN_{tc} operating in liquid media from 10 MPa to 100 MPa to evaluate the influence of size, geometrical uncertainties and surface roughness of piston and cylinder on the calculated pressure distortion coefficient. Even if the Finite Element Method represents the best numerical method to analyse pressure balances, there is not a wide international acceptance on how to develop an uncertainty budget on the FEM models of distortion. In fact, the uncertainty estimation related to the numerical evaluation of the parameters that characterise the pressure balances is often omitted or evaluated empirically by comparing the numerical results with the experimental ones. An estimation methodology to evaluate the uncertainty of the pressure distortion coefficient determined by means of mechanical and dimensional characteristics and the subsequent numerical estimation will allow to use FEM not only to design but also to be another method used to metrologically characterize the pressure balances. In the present paper a preliminary methodology to evaluate the uncertainty budget for the pressure distortion coefficient based on Finite Element Analysis is carried out for the IMGC-100-NN_{tc} pressure balance. It is based on the evaluation of the standard uncertainties and of the sensitivity coefficients of the most important mechanical, dimensional and operating parameters such as the elastic properties of the cylinder and of the piston (Young modulus, Poisson ratio), the piston and cylinder radius and the thermo physical properties of the working fluid. Furthermore, the numerical model uncertainty will be evaluated as a function of the nodes number and of the rounding (off) errors.

2. The Finite Element Method to Evaluate the Pressure Distortion Coefficient of a Simple Pressure Balance (IMGC-100-NN_{tc})

The numerical-iterative method [5] is based on the combined analysis of the laminar flow in the clearance and of elastic distortions of the piston-cylinder unit. In particular, under the hypothesis of axial-symmetrical unit, the elastic equilibrium mechanical theory allows the evaluation of the pressure distortions of the unit by knowing the clearance pressure distribution whereas the fluid and dynamic theory allows the determination of clearance axial pressure distribution by knowing the radial elastic distortions. The simultaneous determination of the elastic distortions and of the pressure distribution in the clearance involves the application of iterative methods to evaluate the effective area that can be determined by the following equation:

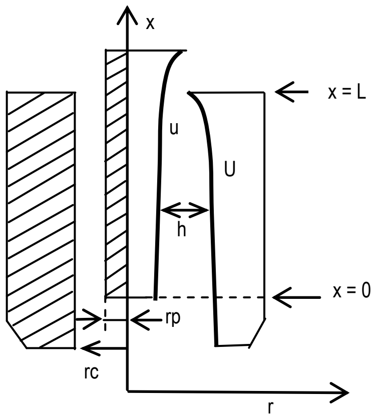
$$A_e = \pi r_p^2 \left(1 + 2 \frac{u(0)}{r_p} \right) + 2 \frac{r_p \cdot \pi}{p} \cdot \int_0^L \left(p \cdot \frac{du}{dx} - \frac{h}{2} \cdot \frac{dp}{dx} \right) dx \quad (1)$$

where r_p is the piston radius, u(x) are the radial distortions of the piston, $h(x)$ is the radial clearance, L is the engagement length and p(x) is the axial pressure distribution in the clearance. The pressure distortion coefficient, λ, can subsequently be evaluated from the following simplified equation:

$$A_e = A_0 \cdot (1 + \lambda \cdot p) \quad (2)$$

where A_o is the effective area at atmospheric pressure and at reference temperature, p is the measurement pressure.

A detailed analysis of the IMGC-100-NN$_{tc}$ (whose characteristics are shown in Fig. 1) using the FEM is given in references [5, 12].



Unit	IMGC-100-NN$_{tc}$
Geometry	simple
Range/MPa	from 5 to 100
Piston radius/mm	2.5231
Cylinder radius/mm	2.5237
Engagement length/mm	52
Material	Tungsten carbide
Young Modulus/MPa	$6.47 \cdot 10^5$
Poisson ratio	0.2178

Figure 1. Characteristics of the IMGC-100-NN$_{tc}$ unit

The comparison of the results obtained by using the proposed procedure with those obtained by the IMGC analytical method and the FEM-City University of London and determined experimentally by cross-floating against another standard furnishes valid indications on the reliability of this numerical model. In fact, the full accordance found between the results obtained and those published in literature (with a maximum relative difference in comparison to the experimental method of -3.6 10^{-2}) represents a valid test to confirm the reliability of the iterative method adopted (Table.1). The estimated uncertainty of about 6% on the pressure distortion coefficient [4] was obtained by experimental methods by measuring distortions only on cylinder .

Table 1. A comparison of the pressure distortion coefficients of the IMGC-100-NN_{tc} obtained using different methods

Method used	λ (ppm MPa^{-1})	Relative difference
FEM (University of Cassino)	0.720	-3.6 10^{-2}
FEM (City University of London)	0.722	-3.3 10^{-2}
Analytical model (IMGC)	0.741	-0.8 10^{-2}
Simplified elastic theory	0.718	-3.9 10^{-2}
Experimental value	0.747±0.044	-

As regards the solution of the equilibrium mechanical problem, by adopting the FEM, about 30000 iso-parametric and axial-symmetric quadrangular and triangular elements were used in the case of IMGC 100 NN_{tc} unit, and the energetic criterion of Zienkiewicz-Zhu was used to optimise the element dimensions [13]. It is evident that the number and the distribution of elements can affect the accuracy of the calculation of the elastic distortions and the pressure profile (determined at every iteration) and, therefore, the pressure distortion coefficient. From the mechanical point of view it is generally necessary to use a great number of elements in the areas of maximum concentration of stresses or deformations. Therefore, in the present case, the importance of evaluating the displacements of U and u makes it necessary to accumulate the elements near to the clearance. Fig. 2 shows the variations of λ as a function of the number of nodes present along the clearance for the IMGC-100-NNtc unit.

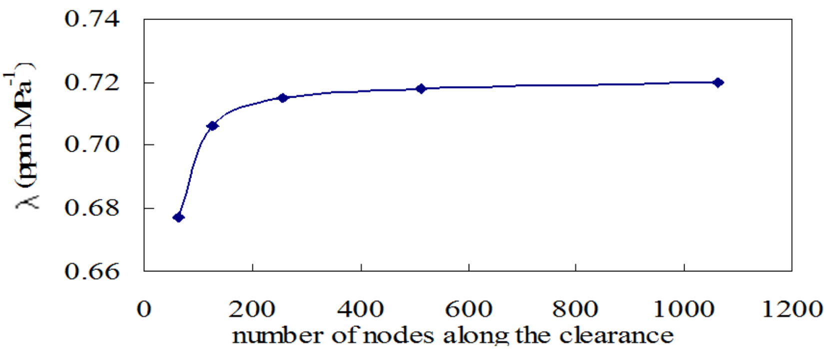


Figure. 2 Values of λ obtained for different numbers of nodes along the clearance for the IMGC-100-NN$_{t.c.}$ unit

From this figure it can be deduced that an insufficient number of elements gives rise to a virtual stiffening of the system with consequent underestimate of pressure distortion coefficient. On the contrary, from the fluid-dynamic point of view, the optimisation proves to be less problematic since the flow is considered monodimensional with a very regular pressure profile.

3. Uncertainty Estimation of the Pressure Distortion Coefficient of the IMGC-100-NN$_{t.c.}$ Simple Pressure Balance

The estimation of the pressure distortion coefficient using the FEM depends on, as previously shown, different parameters or physical quantities (dimensional, mechanical, fluid dynamics and thermodynamics ones). In the case of simple (free deformation) pressure balances, the functional link is:

$$\lambda = f(D, L, E_p, \upsilon_p, E_c, \upsilon_c, \rho, \mu, h, r) \qquad (3)$$

where D is the external cylinder diameter, L is the axial engagement length, E and ν are the Young modulus and the Poisson ratio respectively of the piston-cylinder unit, ρ and μ are the density and viscosity of the working fluid respectively which depend on pressure, h is the undistorted clearance and r represents the possible geometric shape of the piston and cylinder. The standard uncertainty, $u(\lambda)$, of the pressure distortion coefficient is evaluated by considering the standard uncertainties of the input quantities, $(\partial\lambda/\partial x_i)u(x_i)$, where $(\partial\lambda/\partial x_i)$ are the sensitivity coefficients of each parameter or physical quantity. Even if it is always possible to evaluate these sensitivity coefficients with a good reliability (with exception of clearance shape and dimension irregularities that need a carefully dimensional measurements of both piston and cylinder), the complexity and the variability of the functional link (as the characteristics of the pressure balances vary) do not allow to obtain an unique equation for every kind of pressure balance. Therefore, the most suitable approach in the determination of the sensitivity coefficients seems to be the numerical approach based on the differences [14]. In Table 2. the standard uncertainties of the input variables are reported in the case of the evaluation of the pressure distortion coefficient for the simple pressure balance analysed. It has to be mentioned that the present table is optimistic view, and valid only when E_p, E_c, ν_p and ν_c are directly experimentally measured, as it was for the considered IMGC-CNR piston-cylinder assembly. The estimated uncertainties of the elastic constants of materials are valid only under such conditions, otherwise it has been demonstrated that possible variations for tungsten carbide elastic constants can be extremely high (up to possible variation of 35 % due to binder content). Even when the method of the tungsten carbide production is well known and traceable, an uncertainty of E to about 2-3 % and of ν to about 5-10 %, can be assigned with direct consequences on the values of standard uncertainties of Table 2. The most important contributions are those related to the elastic constants of piston and cylinder materials and to the clearance geometry. The last one is also in some extent associated to the evaluation of the pressure distribution along the piston-cylinder clearance. This term was not directly considered as input quantity in Table 2 because its influence is directly related to the effect of radial distortions of piston and cylinder as a function of the applied pressure.

Table 2. Uncertainty analysis of the pressure distortion coefficient of the IMGC-100-NN_{tc} unit at fixed temperature and pressure (t_{ref}=20ºC, p=97.5 MPa)

Quantity	Standard uncertainty		Sensitivity coefficient		Contribution to the combined standard uncertainty	
x	$u(x_i)$	$u(x_i)/x_i$	$\frac{\partial\lambda}{\partial x_i}$		$u_i(\lambda)/10^{-6}$ MPa^{-1}	$u_i(\lambda)/\lambda$
E_c = 6.47 10^5 MPa	6470 MPa	1.0 10^{-2}	-1.55 10^{-12}	Mpa^{-2}	-0.010	-1.4 10^{-2}
E_p= 6.47 10^5 MPa	6470 MPa	1.0 10^{-2}	3.92 10^{-13}	Mpa^{-2}	0.003	0.35 10^{-2}
ν_c = 0.2178	6.5 10^{-4}	0.3 10^{-2}	9.58 10^{-7}	Mpa^{-1}	0.001	0.09 10^{-2}
ν_p= 0.2178	6.5 10^{-4}	0.3 10^{-2}	2.46 10^{-6}	Mpa^{-1}	0.002	0.22 10^{-2}
ρ = 1003 kg m^{-3}	10.0 kg m^{-3}	1.0 10^{-2}	Negligible		negl.	negl.
μ = 9.63 10^{-2} Pa s	9.6 10^{-4} Pa s	1.0 10^{-2}	negl.		negl.	negl.
h = 0.6 μm	0.2 μm	33 10^{-2}	1.9 10^{-8} MPa^{-1} μm^{-1}		0.004	0.5 10^{-2}
L = 52 mm	0.1 mm	0.2 10^{-2}	negl.		negl.	negl.
D = 30 mm	0.1 mm	0.3 10^{-2}	negl.		negl.	negl.
Clearance geometric taper shape r or barrel and hourglass shape r	0.20 μm 0.20 μm		4.6 10^{-8} MPa^{-1} μm^{-1} 9.3 10^{-8} MPa^{-1} μm^{-1}		0.01 0.02	1.3 10^{-2} 2.6 10^{-2}

The choice of clearance irregularities can be made either on the base of clearance measurements or by considering the shape which gives the maximum uncertainty contribution. Typical irregularity clearance shape such as barrel, hourglass and taper have been considered [12]; in the example of table 2 the barrel irregularity was used according to experimental measurements.

From the input quantities shown in table 2 the combined standard uncertainty has been evaluated considering the propagation of uncertainty law and the variance covariance matrix for the input quantities (where only Ep and Ec, νp and νc were considered fully correlated). As a result of the above calculations the uncertainty contribution due to mathematical model was evaluated considering a number of iterations and nodes to have differences less than 0.1 % on the λ value as demonstrated by repeated iterative calculations [11]. The standard combined uncertainty related to the pressure distortion coefficient λ of the IMGC-100-NN_{tc} pressure balance is 3%.

The comparison between the λ value and its uncertainty obtained from experimental measurements [4] (λ = 0.747 MPa^{-1} u = 0.044 MPa^{-1} as shown in table 1) and those found by using FEM method gives a standard normalized error of 0.55. The normalized error is defined by the ratio between the difference of experimental and calculated values and the square root of the sum of the squared standard uncertainties. The normalized standard error shows the agreement of the two methods.

4.Conclusions

The present work is really just a preliminary approach in order to evaluate a possible well established method for the estimation of the contribution of different parameters or physical quantities on the standard uncertainty of the pressure distortion coefficient of a piston-cylinder pressure balance.

The example here reported is based on experimental data and FEM calculations for the IMGC-CNR pressure balance named IMGC-100-$NN_{t.c.}$ operating up to 100 MPa in liquid media.

The proposed method has still to be internationally discussed and improved in order to include other parameters not taken into account in the present study. The method will be analysed based on experiences in other metrological high pressure laboratories both to derive a better way for λ calculations and also for the validation of results on the base of experimental data.

What we are interested to say is that, in itself and having evaluated their intrinsic limits, the FEM methods here used are extremely powerful to quickly obtain the results of the parametric analysis based on complex structures, as the one here presented, where different parameters (range of distortions, stress concentrations, boundary conditions, pressure sealing effects, fluid properties, elastic constants of materials, geometrical shape of piston and cylinder and their variation with applied pressure, establishment of a stable pressure distribution along the clearance, …) are playing a strong and interactive role between them.

The proposed method has advantages of giving quick replies to the input influence quantity effects; more difficult is its full validation that can be reliably assessed only having gained experience on comparing different calculation methods that can be referred as well to precisely defined experimental tests designed for the purpose of verification of the calculation results.

The above is the full and important purpose of the agreed EUROMET Project n. 463 (having PTB, Germany, as coordinator) for the analysis of different distortion calculations for a piston-cylinder unit in liquid media extending up to 1 GPa.

All this is even more important now as all international metrological laboratories, as a consequence of the CIPM-MRA, not only will need to compare their best realisations but also to improve their knowledge of some parameters, frequently under or over estimated as it is the case of the pressure distortion coefficient of high pressure balances, affecting the measurement uncertainty as a function of the pressure range and consequently having effects on uncertainty of calibration laboratories and users.

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