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The Generation and Measurement of Arbitrarily Directed Forces and Moments:

the Project of a Multicomponent Calibration Device Based on a Hexapod Structure

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Abstract

In the metrology of the mechanical quantities of force and torque, standard machines are used that allow these components to be generated in a given direction. For force in most cases this is the vertical, for torque both the horizontal or the vertical directions are in use. In contrast to this realization, for some applications it is necessary to generate forces or moments in arbitrary directions. Moreover, the standard machines generate only one main component and in addition small disturbing components. But it is often desired to have a superposition of forces and moments without a main component.

The present work describes a new method of generating and measuring arbitrary directed forces and moments using a set of six drives on one side of the machine and six force transducers on the other. Both the drives and the force transducers are arranged in a hexapod structure. A theoretical overview is followed by a description of the general design of the machine. Some optimization procedures show the dependence on the chosen geometry of the load decomposition into the six forces. Due to the limited stiffness of the transducers and the structural parts of the machine, the set of equations describing the system must be found by a process of iteration. Some considerations relating to the practical realization conclude the paper.

1. Introduction

The mechanical quantities force and torque can at this time in metrology be determined and measured quasistatically with a very small uncertainty of the order of 10^{-5}. To do this, in the past as well as today, standard measuring devices were developed and built - for force this has been the case for several decades, for torque increasingly over the last ten years. The basic principle of these devices is similar in each case: the given component acting in a certain direction, namely the main component, is determined and the other components, so called disturbing components, are minimized as far as possible. The transducers that must be calibrated with these devices are thus usually designed for a single force or torque component. In the case of multicomponent transducers, sophisticated auxiliary setups must be used for the calibration and frequently measurements must be done using different equipment, in order to at least be able to apply forces and moments consecutively.

The concept to be presented here should overcome these restrictions, in particular for the calibration of multicomponent transducers. The aim is to load a transducer with a combination of a force and a moment, both arbitrarily directed in space. For rotation of the force or torque vector in space, the output signal of the transducer will yield an array of characteristic values, so defining the sensor and its properties.

These special multicomponent transducers find application in robotics and automation technology, in machine and automobile construction, for example, and in measurements of complex mechanical structures to determine strains and their behaviour. Sensors for the monitoring of buildings, such as bridges, houses and industrial complexes, are currently being developed [1]. The multicomponent measuring technique can also be employed in the investigation of the sensitivity of force and torque transducers towards disturbing components.

2. The Basics of Multicomponent Measurements

As the applications mentioned already show, in general there are two objectives: on the one hand the determination of strain states in a continuum (materials testing, problems of strength, monitoring) and on the other hand the measurement of mechanical components (forces and/or moments) in individual construction elements. These two cases are closely coupled: forces/ moments acting on a system produce mechanical strains within it, and conversely, the cause for the strains in the system can be - among others - the acting components. A three-dimensional state of strain, referred to a point in the continuum, can therefore be generated technically by applying suitable forces and moments to the surface of the object. And so we can reduce the two applications named above to the representation of spatially arbitrarily directed forces and/or moments in a point.

The project presented here aims to demonstrate the application of a multicomponent measuring device in metrology to generate and measure the mechanical quantities force and moment, and to support this using mathematics and physics. The value of the force was fixed as 10 kN, for the moment 1 kN·m. The relative uncertainty shall take a value around 10^{-3}.

2.1. Comparison of Various Geometries

A force can generally be generated or measured in space if one applies a drive or force transducer, respectively, in the desired direction. The same applies to the moment. However, the problem of positioning and determining the direction arises. This can be solved by using three suitable arranged systems for the force (not parallel in pairs, and not in the same plane) and, analogous to the decomposition of a vector in a coordinate system, generating the required force components in the individual directions. For the combination of force and torque, therefore, six systems are required. In simple cases the individual systems are arranged parallel to the axes of a Cartesian coordinate system (Figure 1). But there are many more possible arrangements besides. Since in all these cases the drives or transducers are attached via joints to the test object on the one hand and to the device's base on the other, these setups are called parallel kinematics. Another name, which relates to the six individual links, is hexapod. For several years now these systems, especially in constructional engineering [2, 3, 4], have been of great interest.

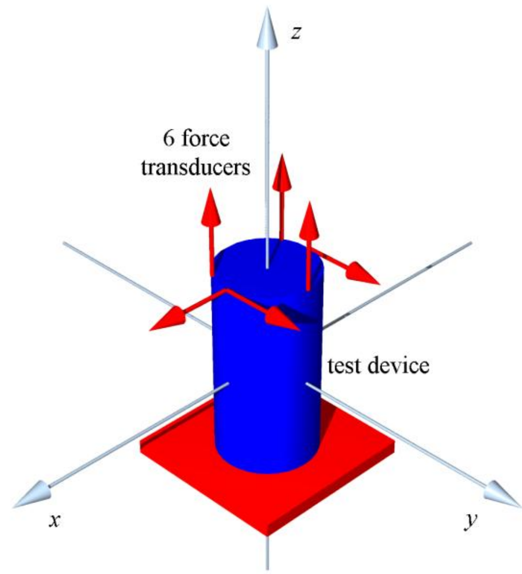


Figure 1. Design principle for a multicomponent measurement (example)

The aim for a measurement uncertainty in the region of 10^{-3} sets high standards for the construction and geometrical precision of the setup. For this reason, from the start, a symmetrical arrangement was favoured which was characterised by numerous similar parts. Despite relatively compact design, it was important to realise maximum working space for the test objects. The arrangement we preferred is shown in Figure 2: it satisfies the symmetry criterion and still leaves room for optimization (cf. 3.2).

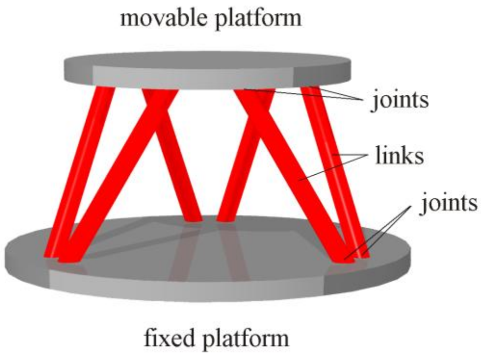


Figure 2. Hexapod design investigated in this project

2.2. Separation of Drive and Measurement

The measurement of forces and moments is inseparable from elastic deformation in all loaded construction elements. Here the geometry of the hexapod naturally changes and, in conjunction, the decomposition of force and moment into the individual forces in the single links of the hexapod. To deal with this, we separated force generation and force measurement in space. A combination of actuator (motor) and sensor (transducer) in each link would be easier to realise construction-wise, but besides strongly affecting the geometry could also have other undesirable results such as thermal and possible electromagnetic effects on the transducers. In addition, each link would have to be significantly longer if it were to encompass both the drive and the transducer elements. Moreover, the geometry of the all-important measuring side of the device can be better determined if it contains less single parts. Finally, it is possible, depending on the transducers employed, to optimize this region to maximum stiffness and so to minimize changes in geometry. A principle scheme of the setup with separate measurement and drive hexapod is shown in Figure 3.

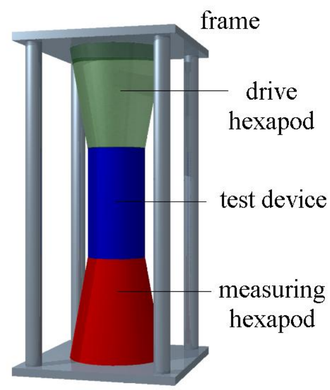


Figure 3. Design principle with separated force generation (drive) and force measurement

3. Measuring Hexapod

As already noted, the measuring hexapod is the component group of the machine which essentially determines the uncertainty that can be achieved. The drive hexapod is attached to the setup in mirror-image position with respect to the reference point (centre of the machine), with the same geometry. Thus optimization (see 3.1 and 3.2) was carried out for the measuring side only.

3.1. Force Decomposition

To optimize the hexapod it is important to know how the externally acting forces F (F_x, F_y, F_z) and moments M (M_x, M_y, M_z), referred to the reference point, are translated to individual link forces. The criterion is that one should be able to measure maximum and equal signals for the force and moment components in designated directions given by a virtual coordinate system attached to the machine. This conditions can only be approximately satisfied.

The hexapod arrangement studied can be theoretically described as follows: the articulated head and foot points of neighbouring links (see Figure 4) form alternate pairs whose centres lie on coaxial circles with radii r_u (foot) and r_o (head). The axial distance between these two circles is h and the tangential distance between the points in the pair is $2 \cdot f$ (foot) or $2 \cdot g$ (head). It is these quantities r_u, r_o, f, g and h that are regarded as the hexapod parameters that are subject to optimization. To calculate the force components F, G, H, I, J and K in the links, the system of equations (1) must be solved by the help of computer algebra software. We assume here that the reference point for the decomposition is the centre of the (upper) circle formed by the head points. Then l is the distance between the reference point of the measuring device and this reference point for the decomposition of the force.

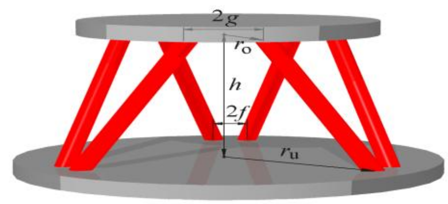


Figure 4. Hexapod with parameters

$$-F \cdot \cos(\beta) + G \cdot \sin\left(\frac{\pi}{6}+\beta\right) + H \cdot \sin\left(\frac{\pi}{6}-\beta\right) + I \cdot \sin\left(\frac{\pi}{6}-\beta\right) + J \cdot \sin\left(\frac{\pi}{6}+\beta\right) - K \cdot \sin(\beta) = \frac{F_x}{\cos\alpha}$$

$$F \cdot \sin(\beta) - G \cdot \sin\left(\frac{\pi}{6}+\beta\right) - H \cdot \sin\left(\frac{\pi}{6}-\beta\right) + I \cdot \sin\left(\frac{\pi}{6}-\beta\right) + J \cdot \sin\left(\frac{\pi}{6}+\beta\right) - K \cdot \sin(\beta) = \frac{F_y}{\cos\alpha}$$

$$F + G + H + I + J + K = \frac{F_z}{\sin\alpha}$$

$$(F - K) \cdot (b - g) + (G - J) \cdot b + (H - I) \cdot g = \frac{M_x - F_y \cdot l}{\sin\alpha} \qquad (1)$$

$$(-F - K) \cdot c - (G + J) \cdot a + (H + I) \cdot r_o = \frac{M_y + F_x \cdot l}{\sin\alpha}$$

$$\left.\begin{array}{l} (F - K) \cdot [(b - g) \cdot \cos\beta + c \cdot \sin\beta] + (-G + J) \cdot \left[b \cdot \sin\left(\frac{\pi}{6}+\beta\right) + a \cdot \cos\left(\frac{\pi}{6}+\beta\right)\right] + \cdots \\ \ldots + (H - I) \cdot \left[r_o \cdot \cos\left(\frac{\pi}{6}-\beta\right) - g \cdot \sin\left(\frac{\pi}{6}-\beta\right)\right] \end{array}\right\} = \frac{M_z}{\cos\alpha}$$

with

$$a = \frac{r_o - \sqrt{3}\cdot g}{2}, \quad b = \sqrt{r_o^2 + g^2 - a^2}, \quad c = a + \sqrt{3}\cdot g,$$
$$\tan\beta = \frac{\sqrt{3}\cdot\frac{r_o}{2} - \frac{g}{2} - f}{\frac{r_o}{2} + \sqrt{3}\cdot\frac{g}{2} - r_u} \quad \text{and} \quad \tan\alpha = \frac{h}{\sqrt{\left(\sqrt{3}\cdot\frac{r_o}{2} - \frac{g}{2} - f\right)^2 + \left(\frac{r_o}{2} + \sqrt{3}\cdot\frac{g}{2} - r_u\right)^2}} \quad . \tag{2}$$

3.2. Optimization

Table 1 collates the values for the geometrical parameters quoted above, found by optimization.

Table 1. Values of the hexapod parameters found by optimization

Para-meter	r_u	r_o	$2{\cdot}f$	$2{\cdot}g$	h
Value in m	0,50	0,23	0,18	0,18	0,3484

Further results are shown in Table 2: for each of the two vectors force and moment and each of the three coordinate directions, the links forces.

One can see that the maximum signals for the various directions differ by up to a factor of 5. Metrologically this should not pose too much of a problem, if one uses suitable high quality force transducers. In this context it is important that the values shown in Table 2 do not represent the maximum link forces for arbitrarily directed components. To determine these, the force and

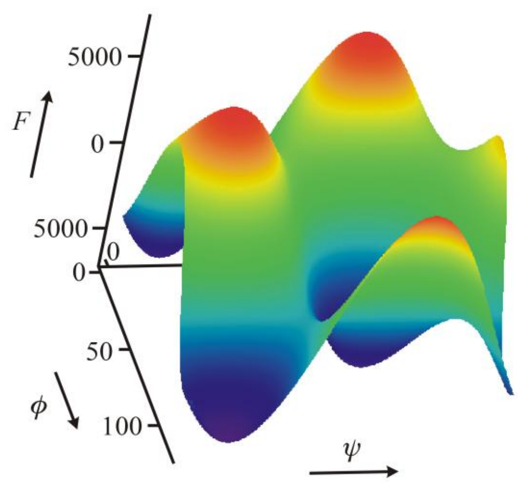


Figure 5. Value of the force F in link 1 for different directions of the external force, ϕ - azimuth and ψ - inclination (both with 120 steps of 3°)

Table 2. Additional results (see text)

Load	F_x = 10 kN	F_y = 10 kN	F_z = 10 kN	M_x = 1 kN·m	M_y = 1 kN·m	M_z = 1 kN·m
Link	**link force in N**					
1	-5721	-3023	2243	2306	-482	1308
2	243	-6466	2243	1570	-1756	1308
3	5478	-3443	2243	-736	2238	1308
4	5478	3443	2243	-736	2238	1308
5	243	6466	2243	-1570	-1756	1308
6	-5721	3023	2243	-2306	-482	1308
	the absolute values related to the maximum from above					
1	0,885	0,467	0,347	0,357	0,075	0,202
2	0,038	1	0,347	0,243	0,272	0,202
3	0,847	0,533	0,347	0,114	0,346	0,202
4	0,847	0,533	0,347	0,114	0,346	0,202
5	0,038	1	0,347	0,243	0,272	0,202
6	0,885	0,467	0,347	0,357	0,075	0,202
	the sum of the above relative link forces					
	3,539	4	2,081	1,426	1,384	1,214

3.3 Stiffness and Iteration

As indicated in 2.2 above, the chosen symmetrical arrangement has a disadvantage which leads to a complication of the theory and the technical realization.

Table 3. Maximum forces in each link for all possible directions of the external force and moment

Link	**1**	**2**	**3**	**4**	**5**	**6**
Load	maximum link force in N					
F	6910	6910	6910	6910	6910	6910
M	4603	4603	4603	4603	4603	4603

The problem is that the loads taken up by the links cause length changes in the latter, and this in turn alters the geometry of the hexapod in parts of a millimetre or a degree. This limited stiffness can be determined theoretically and experimentally, and accounted for in the force decomposition formulae. In the simplest case an iterative calculation is performed, where the hexapod in a first step is regarded to be infinitely stiff and the calculated link forces go into the geometry for the second step via the appropriate stiffness.

In a reverse process, the known unloaded system, and so here too the calculation of the outer components is possible. Moment vectors were rotated in given steps of arc in two mutually orthogonal directions in space, and for each step the components were calculated. Figure 5 shows how the force in link 1 changed during this process. The maximum link forces realized may be seen in Table 3.

4. Drive Hexapod

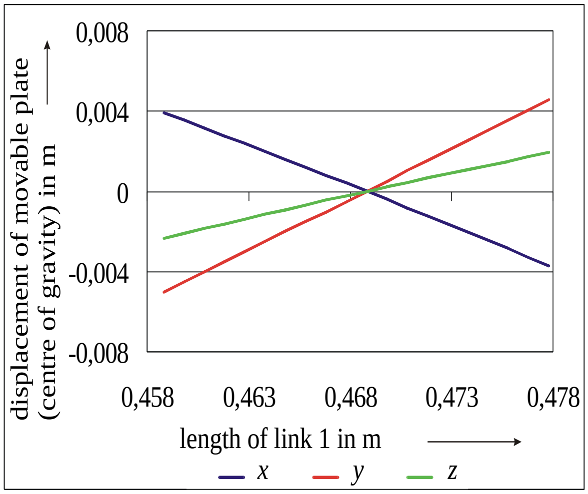
On the drive side it is less the force decomposition but rather the kinematics that are of great interest, because it is there that the total deformation of the system is compensated. One should take into account that, besides the very small elastic deformation of the measuring hexapod, larger deformations of the machine frame and of the test object are possible.

4.1. Kinematics

For this study of kinematics a system of equations was solved numerically for diverse combinations of various link lengths. The aim was to find the coordinates of the joint centres of the movable plate (head) under the condition that the distances between all foot points, but also between all head points are fixed. The result obtained was the dependence of the displacement of the centre of gravity and the rotation of the movable plate on the lengths of the individual links. Examples of the generally linear or quadratic relationships, may be seen in Figure 6.

4.2. Type of Drive

Two methods were considered for the drive, each having advantages and disadvantages. Using hydraulic aggregates allows the realization of direct force-pressure transformation, so that one has the advantage of direct information about the force acting in a drive link. A disadvantage is the more complicated installation



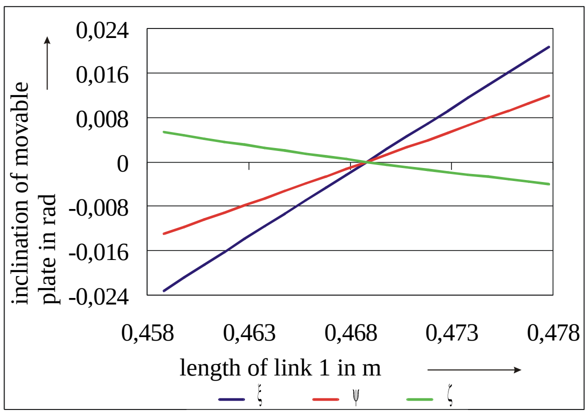


Figure 6. Results of the kinematics study (examples):

Displacement (above) and inclination (below) of the movable plate in dependence on the length of link 1

of the working medium, and the larger oil and dirt levels. The second method was to try electric drives, but there the drive control must be more sophisticated. There is no direct information on the force in a drive link, so a change of length must be induced in the link and the reaction observed on the measuring side. This way is less complicated from the point of view of the installation. Connection with the machine control can be realized more easily, and so this second method was chosen.

5. Technical Problems

The experimental realization of the theoretical results obtained so far poses some problems. This particularly applies to the realization of the force decomposition in the joining points. Here only minimum disturbance (bending moments, transverse forces, torque) should ensue, which may in some cases affect the whole system and alter the force decomposition. The outer components should almost ideally be transformed into pure axial forces in the links. The problem of drive and machine control has already been mentioned.

5.1. Joints

In a first stage elastic joints were considered (see Figure 7) and calculated using the finite element method for axial force and bending moment loads. A simpler theoretical description required these joints to be rotationally symmetric (Figure 7b).

However, this showed that, even for very small deformations of the hexapod, the bending moments induced in the joint are of the order of several N·m, considered too high. A second step therefore is the investigation of commercial ball and cardan joints for their suitability in the setup.

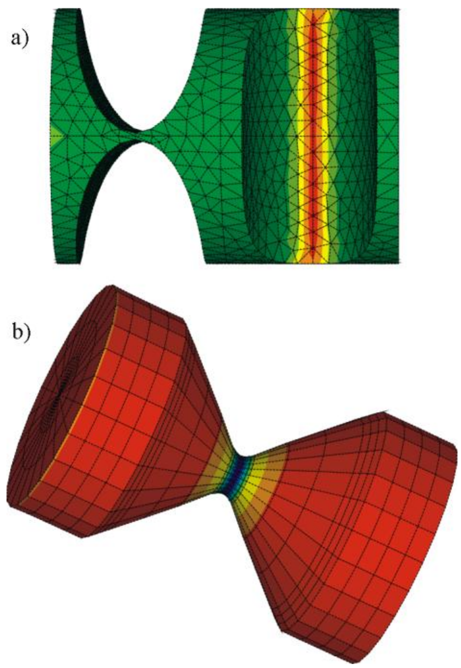


Figure 7. Finite element models of elastic joints:
a - with two separated bending regions,

b - version with rotational symmetry

These measurements are currently ongoing, the results will be reported.

5.2. Machine Control

As indicated above, with the chosen drive system it is not possible, without a good deal of extra control effort, to generate a given load situation with the small uncertainty of 10^{-3} and to keep it there in a step-by-step calibration. On the other hand, this is actually not required: a range of values for the force and the moment shall be continually scanned. The load states reached are recorded with the quoted uncertainty. To these the output signals of the test object are attributed, which allows description of the latter's properties. The concept and design of the control will be reported.

6. Conclusion

This project deals with the development of a measuring device in which a test object may be loaded with a combination of a force and a moment, whereby both quantities may be arbitrarily directed in space. As regards the coordinate description, this is a multicomponent measuring process. The theoretical background is described, numerical results are presented and some first ideas on the technical realization reported.

7. References

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