

## Consistency Test on Mass Calibration of Set of Weights in Class E<sub>2</sub> and Lower

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### Abstract

On weight calibrations one by one there is not a way to check about the validity of the founded values on calibration due to systematic errors (p.e. a possible mass deviation of the mass standard used). The present work proposes a way to check the mass values of the calibration that would support calibration decisions, especially for the secondary labs.

### 1. Introduction

On weights calibration there is not a reference about the truthfulness of the resulting value in calibration.

Weights are delicate instruments to handle and it is common that weights could drift their mass, and there is not a way to note it before their calibration.

The consistency test proposed offers to the metrologist the possibility to perceive a attention signal if any of the values is not consistent in the calibration of mass sets, and from this way the metrologist would have the possibility to take decisions about it (correctives actions).

### 2. Development

The International Recommendation OIML R111 defines seven accuracy classes of

weights from E<sub>1</sub> to M<sub>3</sub>. The OIML R111 defines the nominal values of mass standards as  $1 \times 10^n$  kg,  $2 \times 10^n$  kg or  $5 \times 10^n$  where “n” could be a negative number, positive number or zero, and the sequence of the sets could be formed from the next options  $(1;1;2;5) \times 10^n$  kg;  $(1;1;1;2;5) \times 10^n$  kg;  $(1;2;2;5) \times 10^n$  kg or  $(1;1;2;2;5) \times 10^n$  kg [1].

On weights calibration of class E<sub>2</sub> and lowers mass standards of higher accuracy class are used, using for the calibration next model to calculate the conventional mass [2],

$$m_x^c = m_p^c - \left[ \rho_a - 1,2 \left( V_p - V_x \right) \right] \Delta m \quad (1)$$

Where,

$m_x^c$  Conventional mass of the unknown weight

$m_p^c$  Conventional mass of the standard weight

- $\rho_a$  Air density
- 1,2 Conventional air density
- $V_p$  Volume of the standard weight
- $V_x$  Volume of the unknown weight
- $\Delta m$  Mass difference between standard weight and the unknown weight read on the balance

The combined standard uncertainty of the conventional mass of the unknown weight is obtained from equation (2)

$$u_{m^c_x} = \sqrt{u_{m^c_p}^2 + \left( \frac{-V_x}{V_p} \right)^2 \cdot u_{\rho_a}^2 + \left( \frac{-1,2}{V_p} \right)^2 \cdot \left( u_{V_p}^2 + u_{V_x}^2 \right) + u_{\Delta m}^2} \quad (2)$$

Where,

- $u_i$  Standard uncertainty of the variable “i”

The expanded uncertainty of the conventional mass is obtained by multiplying the combined standard uncertainty by a confidence factor, usually  $k=2$  that amplify the confidence intervals of the uncertainty to approximately 95%, and this value must not be larger than 1/3 of the Maximum Tolerable Error (MTE) of the accuracy class of the calibrated weight.

$$U_{m^c_x} = k u_{m^c_x} \quad (3)$$

Where

- $U_{m^c_x}$  The expanded uncertainty
- $k$  Coverage factor associate to the confidence level

The consistency test consists in a calibration of the group of weights of a particular decade,

(all together as an unknown weight), using the corresponding mass standard equivalent. The sum of the individual values of the weights must to be equal to the found value on the calibration of the group of weights between the uncertainty values. For example in a weights set of nominal values from 100 g to 500 g, the weights 100 g, 200 g, 200 g(\*), and 500 g are calibrated one by one and finally all weights together as a group of 1 kg as nominal value vs. a 1 kg mass standard, see Table 1.

**Table 1.** Scheme of comparisons for 100 g to 500 g

Standard Weight		Unknown Weight
100 g	Vs	100 g
200 g	Vs	200 g
200 g	Vs	200 g(*)
500 g	Vs	500 g
1 kg	Vs	100 g + 200 g + 200 g(*) + 500 g

Difference between mass values of the group of weights and the sum of the individual mass values obtained must to satisfy next discernment,

$$e = \frac{\left| m^c_{\sum m^c_i} - \sum m^c_i \right|}{\sqrt{\left( U_{m^c_{\sum m}} \right)^2 + \left( U_{\sum m^c_i} \right)^2}} \quad (4)$$

$e \leq 1$  Found mass values of the calibration are consistent, the calibration was ok

$e > 1$  Found mass values of the calibration are not consistent between then, it is necessary take corrective actions.

Where,

$e$  Normalized error value

$m^c \sum m^c_i$  Conventional Mass of the group of the weights (obtained from the equation 1, taking as unknown weight the set of weights)

$\sum m^c_i$  Sum of the masses found on its individual calibration

$U_{m^c \sum m}$  Expanded uncertainty obtained on the group calibration accord with the equation 3

$U_{\sum m^c_i}$  Sum of the mass expanded uncertainties from the individual calibration of the weights, equation 5.

$$U_{\sum m^c_i} = \sum U_{m^c_i} = k \sum u_{m^c_i} \quad (5)$$

This calculus of uncertainty takes the uncertainties obtained with a correlation coefficient equal to 1 [3].

The use of the normalized error consists in comparing the difference between the mass of the group of weights and the sum of the mass individual values in the range of the combined uncertainty of both calculations. If the mass difference between both calibration is small than the combined uncertainty of both values, then the mass calibrations looks fine, but if the mass difference is large than the combined uncertainty shows that there is any problem in the calibrations and the uncertainty obtained do not cover it.

It is important to say that the uncertainty has a limit (1/3 MTE), and from this way there is not possibility to enlarge the uncertainty in order to cover this difference.

### 3. Numeric Example

On a set calibration are obtained next conventional mass values on weights from 100 g to 500 g

**Table 2.** Found values on a set of weights from 100 g to 500 g

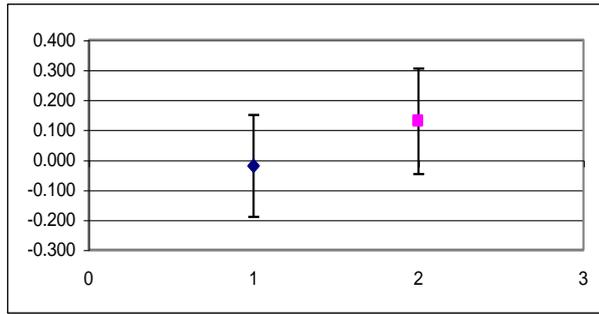
Nominal Value	Correction* mg	Uncertainty (k=2) mg
100 g	+ 0,153	±0,086
200 g	-0,006	±0,036
200 g	-0,003	±0,035
500 g	-0,016	±0,019
$\sum m^c_i$	<b>0,128</b>	<b>±0,176</b>
$m^c \sum m^c_i$ (1 kg)	<b>-0,021</b>	<b>±0,170</b>

\*Correction is equal to the conventional mass less to the nominal value of the weight

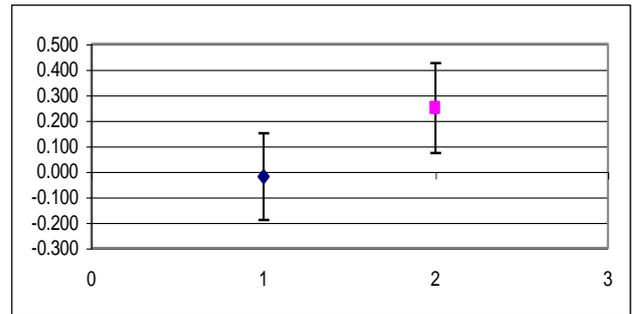
The normalized error is obtained from,

$$e = \frac{|m^c \sum m^c_i - \sum m^c_i|}{\sqrt{\left(U_{m^c \sum m}\right)^2 + \left(U_{\sum m^c_i}\right)^2}} = 0,61$$

The value of  $e$  is lower than 1, it means that the values of the calibration are consistent between then (Fig. 1). The value of  $e$  indicates that all the standards used on the calibration had not drift (the only possibility is that all they had drifted together), the calculation of the conventional mass and the uncertainty look ok.



**Figure 1.** Comparison between mass corrections of  $m^c \sum m_i^c$  and  $\sum m_i^c$  and its uncertainties respectively



**Figure 2.** Comparison between mass correction of  $m^c \sum m_i^c$  and  $\sum m_i^c$  and its uncertainties respectively

Now lets see an example where values obtained in the calibration are on table 3.

**Table 3.** Conventional Mass Values found in calibration in a set weight form 100 g to 500 g

Nominal Value	Correction mg	Uncertainty (k=2) mg
100 g	+ 0,153	±0,086
200 g	+ 0,080	±0,036
200 g	-0,003	±0,035
500 g	-0,021	±0,019
$\sum m_i^c$	<b>+ 0,248</b>	<b>±0,176</b>
$m^c \sum m_i^c$ (1 kg)	<b>- 0,021</b>	<b>±0,170</b>

The normalized error has next value,

$$e = \frac{|m^c \sum m_i^c - \sum m_i^c|}{\sqrt{\left( U_{m^c \sum m_i^c} \right)^2 + \left( U_{\sum m_i^c} \right)^2}} = 1,1$$

$e$  on this example is larger than 1, and it means that values found on the weights calibration are not consistent between them, although the uncertainty bars touch between them (Fig 2).

This situation give us an alert message about the results of the calibration offers us a chance to check the calibration process (data transferring, calculus, calibration of the standards, sensors, etc), before to submit a calibration certificate standards, sensors, etc), before to submit a calibration certificate.

If the mass laboratory has a history of  $e$  values for different decades it could make a statistical analysis of these values and establish the intervals of calibration based on it.

#### 4. Discussion

The consistency test could be applied to any accuracy class of weight, but it is recommended for class E<sub>2</sub> and lowers because for mass calibration of weights class E<sub>1</sub> are used models of subdivision where check standards are introduced in the models for the statistical control. For accuracy classes lowers than E<sub>1</sub> the consistency test could offer confidence to the mass laboratory on their certificates and for the customs for the service received.

By other hand it is necessary to say that this test is not sensitive to an equal drift in all standards involved in a calibration, because the results obtained would be consistent between them, but deviates from the real mass value, this possibility is avoided with the calibration program of the standard.

On mass calibration of weight classes E<sub>1</sub> and E<sub>2</sub> are required the volumes of the weights, and its volume are been determinate by

hydrostatic weighing usually, the consistency test could be adequate to volume measurements where the sum of volumes could be compared vs. the volume of the group of weights (normally a decade).

## 5. Conclusions

The  $e$  value represents how much of the uncertainty is taken for the difference between the sum of mass and the mass of the group of weights, then if  $e = 1$  means that the difference is on the limit of the uncertainty, and  $e = 0,5$  means that the difference takes a half of the interval of uncertainty and  $e = 0$  means that the difference is zero, perfectly consistent.

The consistency test could be a useful tool for the metrological assurance of the mass labs, because with the  $e$  values the lab could define calibration intervals and support all values declared on certificates or reports of calibration. The confidence of the customs could increase about the service required knowing that  $e$  values obtained on the calibration of their weights were less than 0,7 for example.

On evaluations of technical capability of Mass laboratories could be useful for the auditors checking the values of  $e$  obtained on routine calibration services.

## 6. References

[1]. OIML, *R111 International Recommendation N° 111 -Weights of classes  $E_1, E_2, F_1, F_2, M_1, M_2, M_3$* , 1994.

[2]. OIML, *R33 International Recommendation N° 33 Conventional Value of the result of weighing in air*", 1979.

[3]. Bich W., 1990, *Variances, Covariances and Restraints in mass metrology*, Metrologia 27, 111-116 (1990).

[4]. OIML, *Draft International Recommendation N° 111 - Weights of classes  $E_1, E_2, F_1, F_2, M_1, M_2, M_3$ , (including weights for testing of high capacity weighing machines) Part :1 Metrological and Technical Requirements-* February 2000.

[5]. BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, *Guide to the expression of uncertainty in measurement" Corrected and reprinted*, 1995.

[6]. Wolfgang Wöger -*Remarks on the  $E_n$  - Criterion Used in Measurement Comparison*, PTB-Mitteilungen 109 1/99, Internationale Zusammenarbeit.

[7]. European cooperation for Accreditation of Laboratories -*EAL Interlaboratory Comparisons-* (March 1996).

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