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# Investigation of the Dynamic Behaviour of PTB Deadweight Force Standard Machines

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## Abstract

This paper describes the behaviour of deadweight force standard machines of 100 kN and 1 MN at the Physikalisch-Technische Bundesanstalt(PTB). A build-up system was used to measure the dynamic behaviour of the deadweight force standard machines. The oscillating signals from the force-transducers in a build-up system are much larger than those from a single force-transducer. This is because the force-transducers in a build-up system are located several tens of millimeters from the centre of the force-standard machine. Hence, small movements of the deadweight result in changes in the reaction forces of the build-up system, which are amplified by the lever mechanism. The frequency and the trajectory of the pendulum's movements are investigated. The trajectories of the pendulum's movements are estimated by using force signals from the build-up system. In addition to that, the major and minor radii, direction of the elliptic motion and the rotational direction as functions of time are estimated to interpret the pendulum's movements more meaningfully. The two machines show somewhat different dynamic behaviour.

## 1. Introduction

In the international comparison of deadweight force machines in national metrology institutes, the maximum relative deviations were about $\pm 5 \times 10^{-5}$ in the worst case while the theoretical uncertainty of a deadweight force machine is less than $2 \times 10^{-5}$. This discrepancy has been ascribed to the interaction between deadweight machines and transfer standards, measurement procedure and parasitic force components of deadweight machines. A number of other factors which increase the measurement uncertainty in the realisation of force and the intercomparison of force standard machines are still not well defined. One profound example of such factors is the movement of the deadweight.

A build-up system provides a useful method to measure the deadweight movement. This is because small movements of the deadweight can be emphasised by the lever mechanism when the build-up system is placed on the deadweight force machine instead of a single force transducer. In previous papers[1-4], we investigated the movement of a deadweight force machine at the Korea Research Institute of Standards and Science (KRISS). The behaviour of the 540 kN deadweight force standard machine at the National Research Laboratory of Metrology (NRLM) was also investigated[5].

This paper describes the behaviour of the 100 kN and 1 MN deadweight force standard machines at the Physikalisch-Technische Bundesanstalt (PTB). The trajectory of the pendulum's movement is determined from the force signals of the build-up system. The relative uncertainty due to the pendulum movement is estimated. In addition to that, the major and minor radii and the rotational direction as functions of time are observed.

## 2. Deadweight Force Standard Machines

The 100 kN and 1 MN force standard machines of PTB are deadweight machines of totally different design and operation principles.

In the 1 MN machine (Figure 1) the first two force steps of 20 kN and 40 kN are generated by the lower loading frame and the scalepan, both of three-column type.. In the range from 50 kN to 1 MN the force can be changed in steps of 10 kN by selecting a combination of deadweights [6]. The deadweights hang from top to bottom in the following order: 9 x 100 kN, 1 x 50 kN, 2 x 20 kN, 1x 10 kN. The deadweights that participate in the loading are carried in the scalepan. Scalepan and deadweights can be lifted by the working cylinder. During a change in the deadweight combination, the force is controlled by an additional hydraulic loading system. To reduce the oscillating movements of the deadweights during the change of the force step, the frames are guided by clamping devices with roller guides. The deadweights which do not participate in the loading are deposited in the weight basket. The stack of deadweights is placed in a pit of 9 m depth.

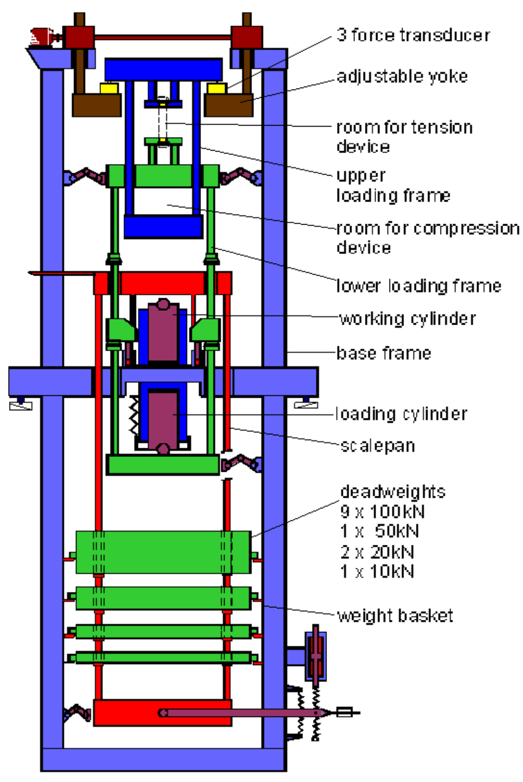


**Figure 1.** Schematic diagram of the PTBs 1 MN deadweight force standard machine.

In the 100 kN force standard machine (Figure 2) the first force step is generated by the loading frame of 2 kN which is also of the three-column type. The other steps are realised by a stack of deadweights which are coupled by special coupling elements and connected to the loading frame. The same principle is used in PTBs 20 kN deadweight force standard machine [7]. The stack of about 2 m height is placed on a lift table, which can be moved down, or up to increase or decrease the force. To carry out calibrations of 20 kN, 50 kN and 100 kN in steps of 10% the following force steps are possible in kN: 2, 4, 5, 6, 8, 10, ...,20, 25, ...,50, 60, ..., 100. Oscillation movements of the deadweights are reduced by the design of the special coupling elements.

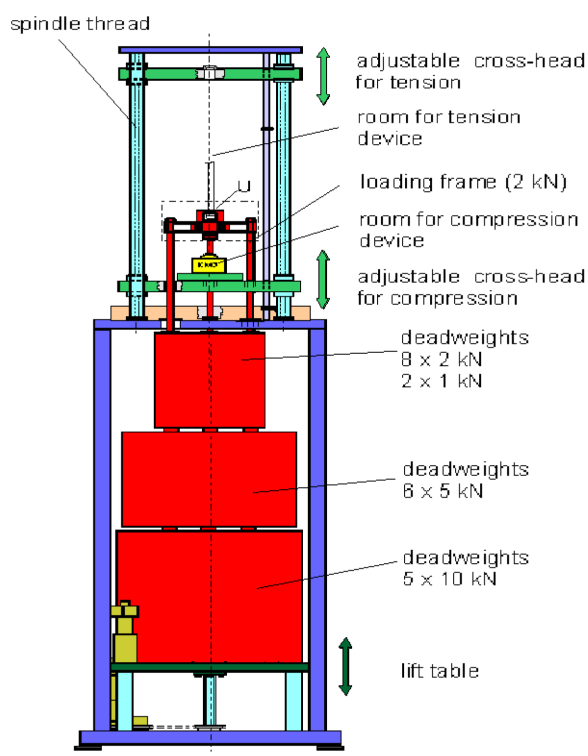


**Figure 2.** Schematic diagram of the PTBs 100 kN deadweight force standard machine.

## 3. Description of the Experiment

The experimental set-up to measure the dynamic behaviour of the force machine comprises a build-up system and a signal processing system. The signals from the force transducers in the build-up system were amplified and sampled by a signal conditioning amplifier of type HBM MGCplus. All signals were sampled simultaneously at a sampling rate

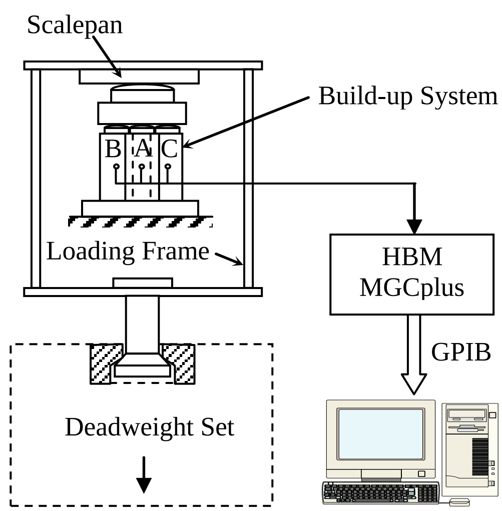


**Figure 3**. Experimental set-up

of 50 Hz, and transmitted to a personal computer through a GPIB interface. The number of data points for each signal is 16384. This implies that the total data length is 327.68 s, and the FFT frequency resolution is 0.003 Hz. Figure 3 illustrates the build-up system and signal processing system.

## 4. Pendulum Movement

In the build-up system on a deadweight force machine as shown in Figure 3, the scalepan of the machine delivers the load to the force transducers through the spherical cap of the load button. Assuming that the force transducers are identical and are located at the exact position and if the normal to the plane surface of the scalepan coincides with the vertical axis, the output signals measured from the force transducers must be the same. However, the scalepan is generally tilted. This gives an offset of the loading point to the build-up system.

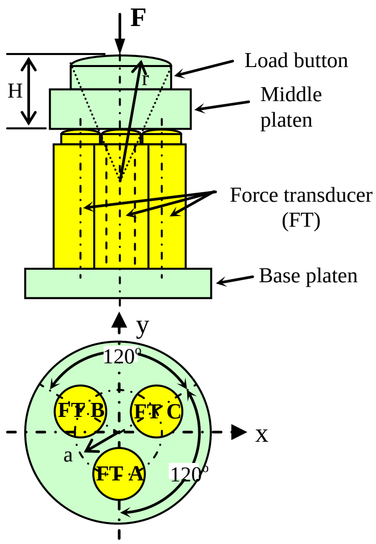


**Figure 4**. Schematic diagram of the build up system

By considering the equilibrium of force and moment of the build-up system, the tilt angle of the deadweight force machine can be represented as

$$\alpha = tan^{-1}\left(\frac{\sqrt{3}}{2}\frac{a}{r-H}\frac{R_C - R_B}{R_A + R_B + R_C}\right),$$
$$\beta = tan^{-1}\left(\frac{1}{2}\frac{a}{r-H}\frac{R_B + R_C - 2R_A}{R_A + R_B + R_C}\right), \qquad (1)$$

where $R_A$, $R_B$, $R_C$ are signals from the force transducers *A*, *B*, *C* respectively and $\alpha$, $\beta$ are the tilt angles along the x- and y- axes respectively. *H*, *a* and *r* are geometric quantities of the build-up system as shown in Figure 4. For details of the derivation of Eq. (1), see [4]. From Eq. (1) one can see that the tilt angle of the force machine can be expressed by using the 3 reactions of the force transducers of the build-up system.

The pendulum movement may be modelled as an elliptic motion, which implies that

$$\alpha(t) = r_l \cos\theta \cos\omega t - r_s \sin\theta \sin\omega t \;,$$
$$\beta(t) = r_l \sin\theta \cos\omega t + r_s \cos\theta \sin\omega t \;, \qquad (2)$$

where $r_l$, $r_s$ are the major and minor radii of ellipse respectively, $\theta$ is the angle between the major axis of ellipse and the x axis and $\omega$ is the radial frequency of the motion. $r_l$, $r_s$ and $\theta$ may be functions of time.

The major and minor radii $r_l$, $r_s$ and the direction $\theta$ of the elliptic motion can be estimated as functions of time by using the Hilbert transform [8]. For the details, see [4].

Because of the pendulum movement of the force machine, some losses of force occur due to the presence of a side force. The relative uncertainty due to the pendulum movement can be represented as

$$\frac{\Delta F}{F} = \frac{Max(\alpha^2 + \beta^2)}{2}\;, \qquad (3)$$

where $Max(\alpha^2 + \beta^2)$ is the maximum tilt angle of the pendulum movement.

## 5. Experimental Results

The frequencies of the pendulum movement for several loading conditions are estimated. Table 1 shows the frequencies for the 100 kN deadweight force machine and Table 2 for the 1 MN deadweight force machine. From the tables one can see that the frequency gets lower as the loads are increased for the 100 kN force machine and higher for the 1 MN machine.

In the case of the 100 kN machine the centre of mass moves downwards with each increase in force step and therefore the pendulum length increases, which reduces the oscillation frequency.

**Table 1.** Frequency of the pendulum movement ( 100 kN deadweight force machine )

| Load (kN) | Frequency (Hz) |
|---|---|
| 20 | 0.525 |
| 40 | 0.444 |
| 60 | 0.408 |
| 80 | 0.384 |
| 100 | 0.363 |

In the 1 MN machine the small masses which are used for the 100 kN step are located at the bottom. The force is increase from 100 kN to 500 kN in steps of 100 kN by the selection of additional masses in the sequence from top to bottom. Therefore from 100 kN to 200 kN the centre of mass moves up which results in an increase in the frequency. A more quantitative analysis is necessary for the interpretation of the oscillation frequency for the steps from 300 kN to 500 kN.

**Table 2.** Frequency of the pendulum movement ( 1 MN deadweight force machine )

| Load (kN) | Frequency (Hz) |
|---|---|
| 100 | 0.186 |
| 200 | 0.204 |
| 300 | 0.213 |
| 400 | 0.213 |
| 500 | 0.210 |

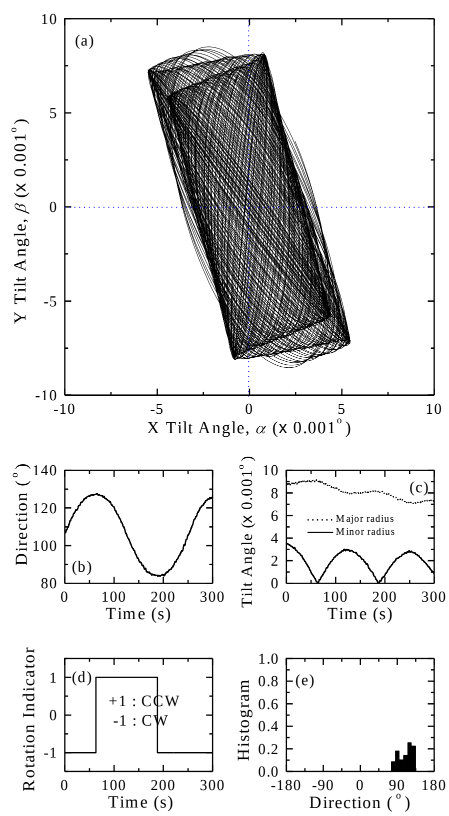


**Figure 5.** Pendulum movement of the 100 kN deadweight force machine when 60 kN is loaded: (a) trajectory; (b) direction of the ellipse; (c) major and minor radii; (d) rotational direction; (e) histogram of the direction (b)

Figure 5 describes the elliptic motion of deadweights of the 100 kN machine when 60 kN is loaded. Figure 5 (a) is the trajectory of the tilt angles along the x- and y-axes over a period of about 300 s. Because of the complexity of the figure, it cannot give sufficient information to resolve the perfect behaviour of the deadweights. Figure 5 (b) shows the direction of the ellipse, and indicates the angle between its major axis and the x-axis. From the figure, one can observe that the direction shows oscillation with a period of about 250 s in the range of about 82$^o$ to 127$^o$. This means that the direction of the pendulum movements rotates periodically.

Figure 5 (c) shows the tilt angles of the major and minor radii as functions of time. The ellipse is narrow when its direction turns to the opposite. Figure 5 (d) is the rotational direction of the elliptic motion, where +1 indicates counter-clockwise and –1 clockwise. From the figure, one can see that the rotational direction changes at about 70 s and 180 s, when the minor radius of the elliptic motion becomes zero; the deadweights therefore move in a straight line. Figure 5 (e) is a histogram of the direction of the elliptical motion. This figure shows that the direction of the elliptical motion is spread over a range of about 60 degrees.

The oscillating phenomena of the direction of the elliptic motion are also observed for other loading conditions, except when 100 kN is loaded. When 100 kN is loaded, the direction of

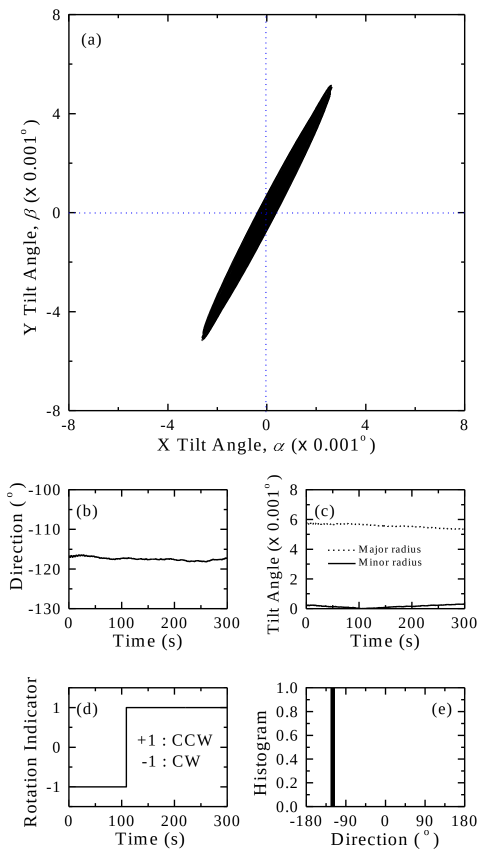


**Figure 6.** Pendulum movement of the 1 MN deadweight force machine when 300 kN is loaded: (a) trajectory; (b) direction of the ellipse; (c) major and minor radii; (d) rotational direction; (e) histogram of the direction (b)

the pendulum movement is concentrated in a narrow range of angles.

Figure 6 shows the elliptical motion of the deadweights of the 1 MN machine when 300 kN is loaded. The structure of the figure is the same as for Figure 5. From Figure 6 (b), one can see that the direction of the elliptical motion is conserved at about -117$^o$. Figure 6 (c) indicates that the major radius decreases with time. The minor radius decreases during about 100 s, and then increases with time. Figure 6 (d) illustrates that the rotational direction changes from clockwise to counter-clockwise at about 100 s when the minor radius of the elliptical motion is zero. Figure 6 (e) shows that the direction of the elliptical motion is confined to a narrow range of angles.

**Table 3.** Relative uncertainty ( 100 kN deadweight force machine )

| Load (kN) | Relative uncertainty |
|---|---|
| 20 | $1.89\times10^{-9}$ |
| 40 | $2.39\times10^{-8}$ |
| 60 | $1.27\times10^{-8}$ |
| 80 | $9.23\times10^{-9}$ |
| 100 | $3.03\times10^{-9}$ |

**Table 4** Relative uncertainty ( 1 MN deadweight force machine )

| Load (kN) | Frequency (Hz) |
|---|---|
| 100 | $1.26\times10^{-8}$ |
| 200 | $8.43\times10^{-9}$ |
| 300 | $5.13\times10^{-9}$ |
| 400 | $2.61\times10^{-9}$ |
| 500 | $6.24\times10^{-9}$ |

For all loading conditions, the pendulum movement of the deadweights of the 1 MN machine shows similar results. The direction of the elliptical motion is conserved over a narrow range of angles for all loading conditions.

The relative uncertainty due to pendulum movement of the force machine is estimated by using Eq. (3), and is represented in Table 3 for the 100 kN machine and Table 4 for the 1 MN machine. Overall, the relative uncertainty decreases as the load increases for both force standard machines.

## 6. Conclusions

In this study, we analyzed the dynamic behaviour of the deadweights in the 100 kN and 1 MN deadweight force standard machines.

We investigated the trajectory of the pendulum movement by using the force signals. In order to extract some useful information from the trajectory, we estimated the direction of the elliptic motion, major and minor radii of the ellipse and the rotational direction as functions of time. The estimated quantities provided useful methods to investigate the movement of deadweight force machines.

A force standard machine is in interaction with the force transducer. The dimensions of the build-up system are totally different to single force transducers. Therefore, the oscillation motion investigated in this paper can be different for other types of transducer. Nevertheless the measurement results demonstrate that the investigated PTB force standard machines show very low oscillation motion which is of the order of thousandths of the relative uncertainty of the deadweight force standard machines, 2 x $10^{-5}$.

## 7. References

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