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VIBRATION-BASED METHODS FOR EVALUATING LINEARITY OF DC-COUPLED VIBRATION ACCELEROMETERS

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Abstract: This paper describes a shaker technique for measuring nonlinearity of accelerometers. The method has application to acceptance testing of DC-coupled accelerometers where the use of a centrifuge is cost prohibitive. To demonstrate the utility of shaker-based methods, nonlinearity tested by both centrifuge and shaker is compared.

Keywords: accelerometer, nonlinearity

1. INTRODUCTION

An accelerometer's nonlinearity is an important product characteristic. In vibration applications, sensor nonlinearity can cause rectification of high-level, high-frequency acceleration leading to erroneous low frequency response. In calibrating DC accelerometers on a shaker, influence of sensor nonlinearity can manifest itself as a reported sensitivity that depends on sensor orientation with respect to earth's gravity.

For AC-coupled piezoelectric accelerometers, RMS vibration methods are universally recognized to specify and qualify accelerometer nonlinearity [1]. Excited at a single frequency the accelerometer is stepped through its acceleration range. The RMS output is plotted against peak acceleration level. A best fit line is forced through the data points. Nonlinearity is specified as the largest deviation from a best fit of the data points. A major shortcoming of the RMS method is that it does not fully characterize sensor nonlinearity. The RMS method cannot detect even order nonlinearity; and as it will be shown in this paper, it is the even order terms that cause problems of vibration rectification.

Centrifuge testing can fully characterize sensor nonlinearity and is universally recognized for testing inertial grade accelerometers. However, precision centrifuges are expensive and the time required to calibrate an accelerometer can be cost prohibitive for routine product acceptance. Therefore for DC accelerometers used in vibration applications, most manufacturers and corporate quality assurance laboratories do not use centrifuges for product acceptance. Instead they use vibration methods using shaker excitation. With proper signal processing, it is possible to obtain nonlinearity similar to centrifuge testing.

The problem is that in the absence of recognized standards, each company has developed its own methods for processing and evaluating shaker data. In a small private survey of accelerometer manufacturers and laboratories that calibrate DC-coupled vibration accelerometers, the authors observed a surprisingly diverse number of methods for measuring nonlinearity. The issue is that each method results in a different value for nonlinearity, making statements of conformance to product specification problematic.

This paper will demonstrate that a shaker based vibration method can provide reasonable measurement of accelerometer nonlinearity that is comparable to a centrifuge. It is hoped that this paper becomes a starting point for a discussion on standardizing vibration-based methods for linearity testing. One caveat is that unlike a centrifuge, a shaker cannot measure hysteresis. But this is not usually a limitation, as hysteresis is of interest for inertial grade accelerometers and is of less interest for vibration applications. Another caveat is that accelerometer nonlinearity is frequency dependent. Nonlinearity measured at very low frequency will approach the centrifuge nonlinearity. There is an underlying assumption that shaker testing is performed at a frequency that is low compared to any high frequency sensor dynamics.

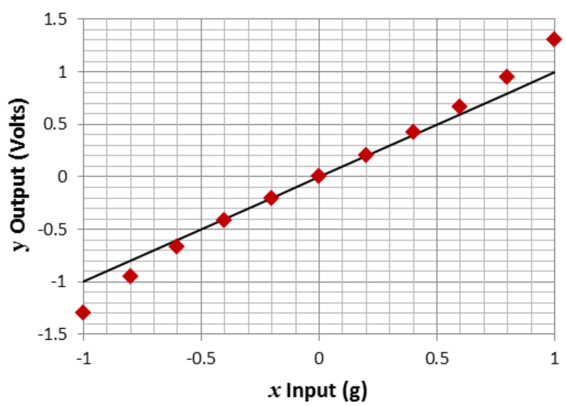
2. EXPRESSION OF NONLINEARITY

Nonlinearity of DC-coupled accelerometers is determined on a centrifuge by stepping the accelerometer through its acceleration range and at each step measuring the voltage output. Figure 1a displays accelerometer output at eleven equally spaced acceleration levels ranging from -1g to +1g (g = 9.80665 m/s^2). Also shown in the figure is a best fit line through the data. The difference between the data and the best fit line is displayed in Figure 1b. Nonlinearity is usually stated as the largest deviation from the best fit line expressed as percent of the full scale output.

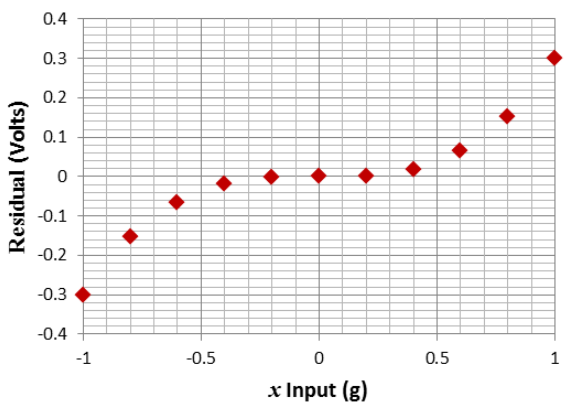
Given sensor input *x* and output *y* (as in Figure 1a), the nonlinear response can be modelled by a polynomial with constant coefficients b_n:

$$y = b_0 + b_1 x^1 + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + b_6 x^6 + \ldots \quad (1)$$

The offset bias at zero acceleration is represented the coefficient b_0. The linear sensitivity is represented by b_1 and nonlinearity is captured by the higher order terms of b_n.



(a)



(b)

Figure 1. *Nonlinear accelerometer input-output relationship (a) and residual from best fit line (b).*

In the paragraphs that follow, the nonlinear response of Equation (1) to sinusoidal input is analysed in some detail. It is assumed that the excitation is sinusoidal and the excitation frequency is low compared to the accelerometer resonance and any low-pass filter influence. Thus the influence of nonlinear sensor dynamics (e.g nonlinear damping, and nonlinear resonance) is small. Assume the input x is sinusoidal with acceleration amplitude equal to A:

$$x = A\sin\omega t \qquad (2)$$

Inserting Equation (2) into (1) results in the harmonic response:

$$y = a_0 + a_1 \sin\omega t + a_2 \sin^2\omega t + a_3 \sin^3\omega t + \ldots \qquad (3)$$

Where:

$$\begin{aligned} a_0 &= b_0 \\ a_1 &= Ab_1 \\ a_2 &= A^2 b_2 \\ a_3 &= A^3 b_3 \\ &etc\ldots \end{aligned} \qquad (4)$$

Applying trigonometric identities and power reduction formulas, it can be shown that the even and odd order terms in Equation (3) will respectively produce even and odd harmonics as follows:

Even order terms

$$\begin{aligned} \sin^2\omega t &= (1-\cos 2\omega t)/2 \\ \sin^4\omega t &= (3-4\cos 2\omega t + \cos 4\omega t)/8 \\ \sin^6\omega t &= (10-15\cos 2\omega t + 6\cos 4\omega t - \cos 6\omega t)/32 \\ &\text{etc}\ldots \end{aligned} \qquad (5)$$

Odd order terms

$$\begin{aligned} \sin^3\omega t &= (3\sin\omega t - \sin 3\omega t)/4 \\ \sin^5\omega t &= (10\sin\omega t - 5\sin 3\omega t + \sin 5\omega t)/16 \\ &\text{etc}\ldots \end{aligned} \qquad (6)$$

Note that the odd order terms of the polynomial produce components at the fundamental and the even order terms do not. This means that if one were to measure linearity using FFT methods looking at changes in amplitude at the fundamental frequency, even order terms will have *no* effect on measured linearity. Note also that the even order terms produce a DC component and odd order terms do not. This means that for DC-coupled sensors, the even order distortion components will produce a change in bias level as the sensor is stepped through its range. Monitoring of the DC bias can be a good way of detecting even order distortion components.

For sinusoidal excitation, a direct approach for evaluating nonlinearity would be to measure the positive and negative peak output as the accelerometer is stepped through its amplitude range resulting in a plot similar to Figure 1. The obvious approach would be to directly identify the peak values in the time domain. In this paper this approach will be called “peak pick” method.

Historically, the “peak method” did not gain widespread application as implementation of the method requires modern data acquisition and computer processing. Instead, methods were developed that utilized available electronic meters. Peak output was estimated by summing the DC bias and RMS signal output. In this paper this method is called “RMS+2*bias”. A variation of this approach is to sum the fundamental amplitude, obtained in the frequency domain, with the DC bias. In this paper this method is called “FFT+2*bias”. The math justifying these approaches is not as straightforward as the “peak pick” method. Therefore in the paragraphs that follow, the relationship between the time domain peak and the frequency domain spectrum is examined in more detail.

It can be shown that the output y in Equation (3) has positive and negative peak values at respectively $\omega t = \frac{\pi}{2}$ and $\omega t = -\frac{\pi}{2}$. Inserting these values of ωt into Equation (3) we find that the positive and negative peak y values are equal to:

$$y_{\max} = a_0 + a_1 + a_2 + a_3 + a_4 + a_5 ... \quad (7)$$

$$y_{\min} = a_0 - a_1 + a_2 - a_3 + a_4 - a_5 ... \quad (8)$$

Thus once the polynomial coefficients a_n have been identified, the positive and negative peak output can calculated from Equations (7) and (8). A process for identifying the coefficients a_n is illustrated by a simple example where we ignore terms greater than the 6th order. Ignoring higher order terms Equations (3), (5), and (6) can be combined and then spending a bit of time rearranging terms we get the harmonic response:

$$\begin{aligned} y &= c_0 + c_1 \sin \omega t + c_2 \cos 2\omega t + c_3 \sin 3\omega t + c_4 \cos 4\omega t \\ &+ c_5 \sin 5\omega t + c_6 \cos 6\omega t \end{aligned} \quad (9)$$

where:

$$c_0 = a_0 + \frac{1}{2} a_2 + \frac{3}{8} a_4 + \frac{10}{32} a_6 \quad (10)$$

$$c_1 = a_1 + \frac{3}{4} a_3 + \frac{10}{16} a_5 \quad (11)$$

$$c_2 = -\frac{1}{2} a_2 - \frac{1}{2} a_4 - \frac{15}{32} a_6 \quad (12)$$

$$c_3 = -\frac{1}{4} a_3 - \frac{5}{16} a_5 \quad (13)$$

$$c_5 = \frac{1}{16} a_5 \quad (14)$$

$$c_6 = -\frac{1}{32} a_6 \quad (15)$$

The terms c_n can be identified in the frequency domain from the spectral components and are thus known. The coefficient a_0 is known from the measurement of bias taken at the zero acceleration level. Equations (10) through (15) are an over determined set of seven equations from which the six unknown coefficients a_n can be calculated from the seven known values of c_n. Interestingly, even if the bias term c_0 is not known (such as in an AC coupled piezoelectric accelerometer) it is still possible to calculate the theoretical peak output that would have been produced if the sensor were DC coupled. Such an approach might have applicability to identify the nonlinearity of piezoelectric AC coupled accelerometers.

Oftentimes the accelerometer nonlinearity is dominated by only the quadratic term and higher order terms can be ignored. In such cases it can be shown that Equation (9) will reduce to:

$$y = a_0 + \frac{1}{2} a_2 + a_1 \sin \omega t + \frac{1}{2} a_2 \cos 2\omega t \quad (16)$$

Summation of the first two terms in the above equation is the bias level. Thus a_2 can be calculated directly from the change in bias as follows:

$$a_0 + \frac{1}{2} a_2 = bias \quad (17)$$

$$a_2 = 2 * (bias - a_0) = 2 * \Delta bias \quad (18)$$

The third term in Equation (16) is the fundamental. The coefficient a_1 can be identified from spectral component at the fundamental frequency:

$$a_1 = \sqrt{2} * FFT(\,fundamental) \quad (19)$$

For a 2nd order polynomial, Equations (7) and (8) reduce to:

$$y_{\max} = a_1 + a_2 \quad (20)$$

$$y_{\min} = -a_1 + a_2 \quad (21)$$

Combining Equations (17), (19), (20), and (21) we get expressions for the maximum and minimum peak outputs:

$$y_{\max} = 2 * \Delta bias + \sqrt{2} * FFT(\,fundamental) \quad (22)$$

$$y_{\min} = 2 * \Delta bias - \sqrt{2} * FFT(\,fundamental) \quad (23)$$

The above is exact for quadratic nonlinearity. If higher order nonlinear terms exist then the above becomes approximate.

An RMS measurement can approximate the amplitude of the fundamental. It can be shown that for quadratic nonlinearity the RMS reading will be equal to:

$$y_{rms} = \frac{1}{\sqrt{2}} \sqrt{a_1^{\,2} + a_2^{\,2}} \approx \frac{1}{\sqrt{2}} a_1 \quad (24)$$

$$y_{rms} \approx \frac{1}{\sqrt{2}} a_1 \text{ (for small } a_2 \text{)} \quad (25)$$

Many laboratories find it convenient to use a meter to measure RMS and DC bias voltage from which the peak value can be approximated. The RMS voltage approximates the amplitude at the first harmonic and the bias is equal to the DC voltage. In such cases the peak output is estimated by:

$$y_{\max} = 2 * \Delta bias + \sqrt{2} * y_{rms} \quad (26)$$

$$y_{\min} = 2 * \Delta bias - \sqrt{2} * y_{rms} \quad (27)$$

The above closely approximates the peak value for quadratic nonlinearity. For higher order nonlinearity the approximation will may not be as good.

3. MEASUREMENT OF NONLINEARITY

To demonstrate feasibility of vibration methods, the nonlinearity of a PCB Piezotronics Model 3741M06A was evaluated using a DC centrifuge and using a shaker. The 3741M06A is a 20 mV/g capacitive accelerometer with a nonlinearity specification of <1% full scale output. Sensor resonant frequency is 15 kHz. Centrifuge testing was performed at an outside laboratory. With the shaker, three methods were applied to the data processing:

1) "Peak Pick" Positive and negative peak values are identified directly in the time domain.

2) "FFT+2*Bias" Fundamental component summed with bias according to Equations (22) and (23).

3) "RMS+2*Bias" RMS output summed with the bias according to Equations (26) and (27).

For the shaker testing a quartz piezoelectric ICP® accelerometer is used as the reference. The shaker is oriented horizontally so that the SUT is not influenced by earth's gravity. A resonant ring fixture provides amplification to achieve peak levels of +/-100g at a frequency of 500 Hz. Positive and negative peak amplitudes of the sensor under test (SUT) and reference are recorded as the SUT is stepped through its range. Note that at each acceleration step both positive and negative acceleration peaks are acquired.

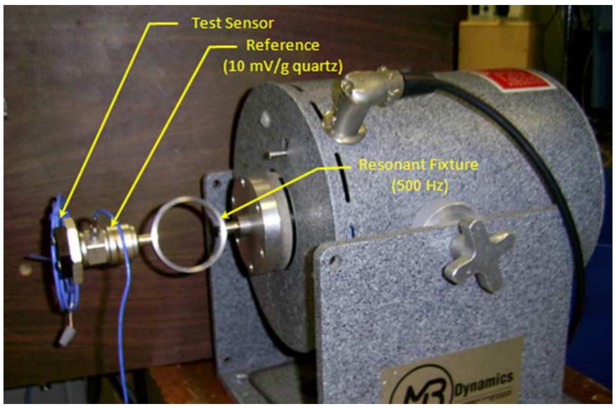


Figure 2. *Test sensor on shaker with quartz reference and resonant fixture.*

Validity of the shaker method depends on the following:

1) The reference is linear. Given that a 10 mV/g (500g range) reference is used over only a 100g range, this is a reasonable assumption.
2) An AC coupled reference can be used to identify peak acceleration value. For arbitrary motion the zero level of an AC coupled reference is not known and thus the absolute peak acceleration values cannot be known. However with shaker excitation motion is constrained and the DC acceleration level must be zero. Therefore in the special case of shaker excitation with a linear reference, the peak acceleration values can be obtained even though the reference accelerometer is AC coupled.
3) The SUT must be DC coupled.
4) Test frequency is low compared to SUT resonant frequency. Nonlinearity is frequency dependent. At higher frequencies the nonlinearity will be influenced by nonlinear sensor dynamics such as nonlinear damping and nonlinear sensor resonance. Also, the fundamental frequency and harmonics considered must low frequency relative to any sensor low-pass filtering that may influence the measurement.

A National Instruments 16-bit sigma-delta acquisition board with Labview® software was used to acquire the data. Acquisition settings were as follows: 10000 samples at 5000 samples/sec. The Labview® user interface is shown in Figure 3. The shaker excitation frequency was 500 Hz and this translates to 1000 cycles acquired at each acceleration level. At each acceleration level the data set was processed by the three methods: "Peak Pick", "FFT+2*Bias", and "RMS+2*Bias".

Data was acquired at 12 equally spaced intervals over the acceleration range from -100g to +100g resulting in a plot similar to Figure 1. A linear least square best fit straight line was forced through the data. Sensitivity is determined from the slope of the best fit line. Residual is calculated from the difference between the data and best fit line. The residual is scaled to g (g = 9.80665) based on identified accelerometer sensitivity. Accelerometer nonlinearity specification is usually taken as the maximum residual value, often expressed as a percentage of the full scale output.

Figure 4 compares residual by centrifuge with the "Peak Pick" vibration method. At this point in time a detailed analysis of uncertainty has not been performed. A rigorous comparison of data should be based on uncertainty and normalized error (E_n value). However, qualitatively the data looks reasonable. Magnitudes of the residuals by the two methods are similar and plots have similar shape. Figure 2 compares the centrifuge with "FFT+2*bias" and "RMS+2*bias" vibration methods.

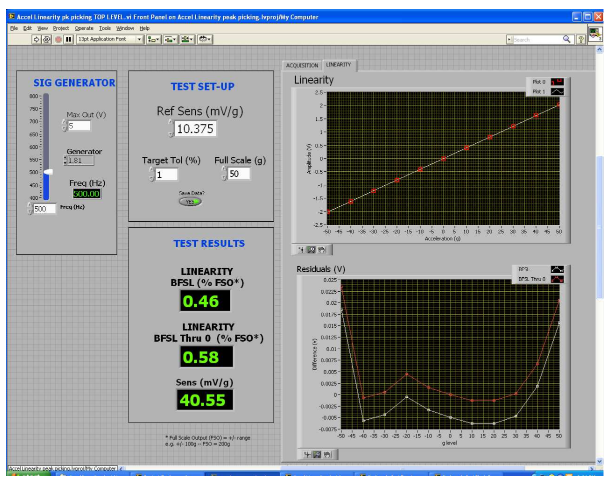


Figure 3. *Labview user interface.*

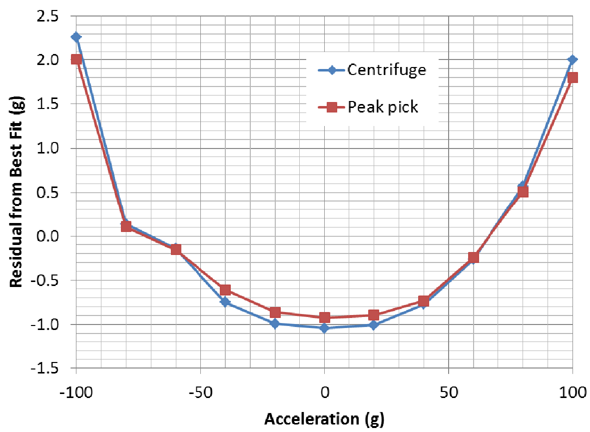


Figure 4. *Centrifuge nonlinearity compared with vibration "peak pick" method. Model 3741M06A SN 7934.*

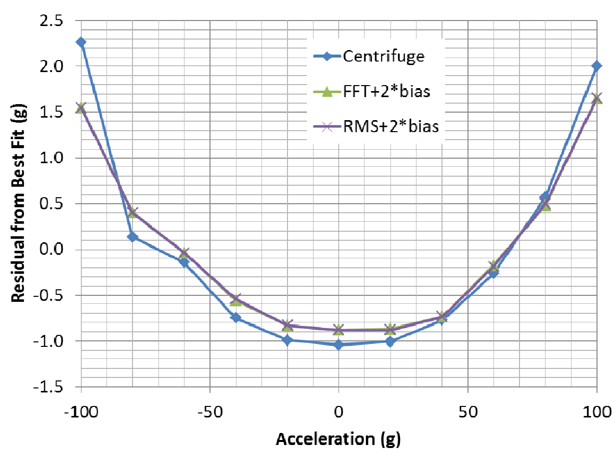


Figure 5. *Residual from best fit line. Comparison of centrifuge nonlinearity with 2 vibration methods. Model 3741M06A SN 7934.*

4. CONCLUSION

The centrifuge is the "gold standard" for measuring accelerometer nonlinearity. But because of the cost and complexity of centrifuge testing, many laboratories are making statements of product acceptance based on shaker tests. The problem is that there are no standards for shaker-based linearity testing leading to ambiguity when making statements of product conformance. This paper has shown that for vibration applications, shaker methods can provide reasonable assessment of accelerometer nonlinearity. Shaker based methods can be comparable to centrifuge methods as long as the test frequency is low compared to the potential influence of any high frequency dynamics.

This paper only examines the reasonableness of vibration based methods. A formal uncertainty budget needs to be determined before for any formal inter-laboratory comparisons are performed.

5. REFERENCES

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