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# STUDY ON A SIMPLIFIED IMPLEMENTATION OF HOMODYNE TIA METHOD FOR LOW FREQUENCY PRIMARY VIBRATION CALIBRATION

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**Abstract** – In this paper, the shortcomings and their causes of the conventional homodyne time interval analysis (TIA) method is described with respect to its software algorithm and hardware implementation, based on which a simplified TIA method is proposed with the help of virtual instrument technology. Equipped with an ordinary Michelson interferometer and dual channel synchronous data acquisition card, the primary vibration calibration system using the simplified method can perform measurements of complex sensitivity of accelerometers accurately, meeting the uncertainty requirements laid down in pertaining ISO standard. The validity and accuracy of the simplified TIA method is verified by simulation and comparison experiments with its performance analyzed. This simplified method is recommended to apply in national metrology institute of developing countries and industrial primary vibration calibration labs for its simplified algorithm and low requirements on hardware.

**Keywords**: primary vibration calibration, time interval analysis method, homodyne interferometry.

## 1. INTRODUCTION

The sine-approximation method (SAM), a commonly applied primary vibration calibration method in many national metrology institutes (NMIs), has been investigated and well implemented for calibration of complex sensitivity of accelerometers in various versions. Based on phase demodulation of laser interferometer signals, its implementation normally requires a quadrature homodyne or heterodyne interferometer, typically a modified Michelson or Mach Zehnder type as described and recommended in [1]. However, there are some NMIs and a large number of industrial laboratories where fringe-counting method (FCM) in conjunction with ordinary Michelson interferometer still dominates the frequency range from 1 Hz to 800 Hz in primary vibration calibration and provides the only information about sensitivity amplitude of accelerometers.

Previous study has successfully solved this problem by providing both sensitivity and phase shift information of accelerometers from 0.1 Hz to 1 kHz with high requirement on signal acquisition and processing hardware. The implementation of so-called homodyne reconstruction and time interval analysis (TIA) methods are based on high performance data acquisition card (50 MS/s) and time interval analyzer (50 ps) respectively. Obviously, not every laboratory can afford the setup of such a VXI-bus calibration system though it is a good basis for accurate measurement.

Aiming at similar measurement accuracy as VXI system but with a simplified algorithm, our theoretical and experimental work fully investigates the possibility and validity of frequency demodulation of ordinary Michelson interferometer signal digitized at a relatively slow sampling speed (2 MS/s). With a high speed data acquisition card (max.5 MS/s) added to computer, the conventional national medium frequency vibration standard (FCM) in National Institute of Metrology (NIM), China is successfully transformed to a calibration system whose measurement uncertainty from 20 Hz to 800 Hz is 0.5% and 1° (confidence level 95%) respectively for sensitivity and phase shift measurement of accelerometers, with the exception of 0.5% and 0.5° at reference conditions.

## 2. CONVENTIONAL MEASURING METHOD AND PRACTICAL CONSIDERATIONS

Homodyne TIA is based on the measurement of time intervals between successive zero-crossings of the interferometer signal. This method has the advantage of suppressing the disturbing motion quantities because the time history of velocity is recovered by frequency demodulation algorithm before the calculation of acceleration amplitude and initial phase values are performed. Traditionally, the time intervals are either measured directly by a high-resolution time interval analyzer or obtained by identification of time locations of all the zero-crossings of the interferometer signal that is digitized by a high speed DAC card. In the latter case, third order polynomial fitting is applied for identification of a zero-crossing to either sample data within each half period around this crossing [2] or two samples before and after it [3]. This procedure can be time consuming with a large number of sampling data.

It is well known that homodyne interferometer uses laser light operating at a single frequency and measure changes in light intensity and therefore, is prone to any effect of voltage disturbance such as random noise. This effect can be clearly seen in figure 1, where interferometer output signal modulated by a vibration motion of 1 kHz is sampled at 2MS/s. In figure 1(a), the interferometer signal is approaching the turning point while instant vibration velocity decreases to about zero. Within a time interval of 1.5 us, three successive zero-crossings occur due to the influence of random noise, which is clearly shown in figure 1(b). In such a case, the noise obscures the actual time location of the zero-crossing and thus, nullifies the high resolution of 50 ps ensured by time interval analyzer or 20 ns provided by high speed DAC card with maximum sampling rate of 50MS/s.

On the other hand, the smallest number of samples is acquired when instant vibration velocity increases to the maximum value. This is presented in figure 2. With fringe frequency increasing, the signal appears quite smooth in figure 2(a) without any jitter around zero-crossing as shown in figure 2(b). In this case, neighboring samples in each side of zero-crossing are almost linear with each other, representing the maximum slope of the sine waveform. Therefore, third order polynomial fitting seems not to be the best efficient algorithm to achieve accurate time location of zero-crossing.

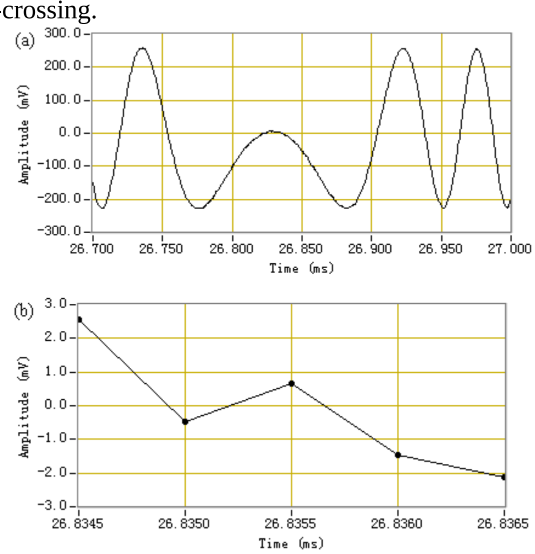



Figure 1. Noisy homodyne interferometer signal modulated by vibration motion at 1 kHz while approaching the turning point (a) and crossing zero repeatedly (b).

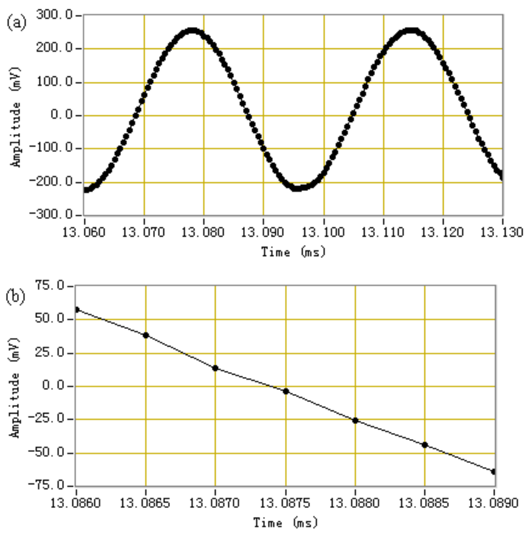



Figure 2. Smooth homodyne interferometer signal modulated by vibration motion at 1 kHz while approaching the maximum fringe frequency (a) and crossing zero linearly (b).

Based on the knowledge of homodyne interferometer signal from real situations, two important conclusions can be drawn which lay a solid basis for the feasibility and measurement accuracy of simplified implementation of TIA: (1) though time interval analyzer can measure time intervals of interferometer signal with a resolution of 50 ps, this measurement accuracy ensured by hardware can not assure accurate identification of zero-crossings because random noise can easily obscure the actual time location of zero-crossings. The same is true with high speed DAC card; (2) third order polynomial is not the most suitable algorithm for detection of time location of zero-crossings in either case described above.

## 3. ANALYSIS OF SIMPLIFIED HOMODYNE TIA ALGORITHM

The measurement of the time interval between successive zero-crossings basically provides time locations at which velocity values of a vibration motion are determined.

The homodyne TIA starts with the single photoelectric signal from ordinary Michelson interferometer, which is theoretically expressed as:

$$u(t) = \hat{u}\cos[\varphi_0 + \hat{\varphi}_M \cos(\omega t + \varphi_s)] \qquad (1)$$

where $\hat{\varphi}_M$ is the amplitude of a modulation term, proportional to the displacement, $\hat{\varphi}_M = \frac{4\pi\hat{s}}{\lambda}$. $\varphi_0$ is the initial phase angle of the photoelectric signal, $\varphi_s$ displacement initial phase angle and $\omega$ vibration angular frequency.

The interferometer output signal is equidistantly sampled during a measurement period $t_0 < t < t_0 + t_{Meas}$. The series of measurement values $\{u(t_i)\}, i = 0,1,\dots,N-1$ sampled within $t_0 < t < t_0 + t_{Meas}$ have a constant sampling interval $\Delta t = t_{i+1} - t_i$. After the offset of this series is eliminated by a mean value calculation procedure, the occurrence of a zero-crossing can be easily detected by a conditional judgment in algorithm:

$$u(t_i) \cdot u(t_{i+1}) \le 0 \qquad (2)$$

Ideally, a positive and a negative zero-crossing occur when the argument of the cosine function (1) changes by a displacement step of half the wavelength. In real situation, however, some of those occurrences of zero-crossing are very close in time sequence. This phenomenon, as shown in figure 1(b), is caused by the influence of voltage disturbances, especially random photoelectric noise. In this situation, only one zero-crossing should be selected or identified as the result of denoise processing. Considering the fact that time interval between two ideal neighboring zero-crossings becomes longer as the fringe frequency decreases in this situation, any of the three zero-crossings will not cause significant error in following frequency demodulation and almost contribute nothing to the calculation of acceleration, based on sine approximation of velocity values. So, the first zero-crossing is always selected whenever this phenomenon occurs, which ensures the accuracy and efficiency of the TIA algorithm.

Instead of using third order polynomial fitting, the identification of the time location of a zero-crossing between two neighboring samples can be performed simply by arithmetic operations:

$$t'_i = t_i + \frac{|u(t_i)|}{|u(t_i)| + |u(t_{i+1})|} \cdot \Delta t, i = 0,1,\ldots, M-1 \quad (3)$$

Based on the time locations $t'_i$ of all the zero-crossings of the modulated interferometer signal, a series of time intervals between successive positive or negative zero-crossings can be calculated by $\Delta t'_i = t'_{i+1} - t'_i, i = 0,1,\ldots, M-2$. A non-equidistant time series of instant frequencies can be obtained by the relationship:

$$f(t^*_i) = \frac{1}{\Delta t'(t^*_i)} \quad (4)$$

where $t^*_i = \frac{t'_i + t'_{i+1}}{2}, i = 0,1,\ldots, M-2$. The velocity series is given by:

$$v(t^*_i) = \frac{\lambda}{2} f(t^*_i) \quad (5)$$

Because the single-channel Michelson interferometer does not provide any information on the direction of velocity, the time history of velocity recovered from this procedure appears as only half the sinusoidal motion, shown as dotted plots in figure 3. Two possible waveforms are presented as solid plots in figure 3(a) and (b), respectively. Though the actual motion waveform can be determined with the helpful directional information of accelerometer under calibration corresponding to the direction of acceleration, this is not necessary since only the amplitude and initial phase values of velocity are of interest, not the exact velocity time history. The amplitude and initial phase of velocity can be obtained then by the calculation of sine-approximation, which finally leads to those of acceleration using the known relationships valid for sinusoidal time dependencies. The same amplitude value of acceleration is obtained from either case. As for initial phase angle, the 180 degrees phase difference between two possibilities does not have any influence on the final calculation of phase shift of accelerometers where two parameters have to be converted into the same angular range [-90°, 90°]. The detailed procedure for sine approximation can be referred to [1].

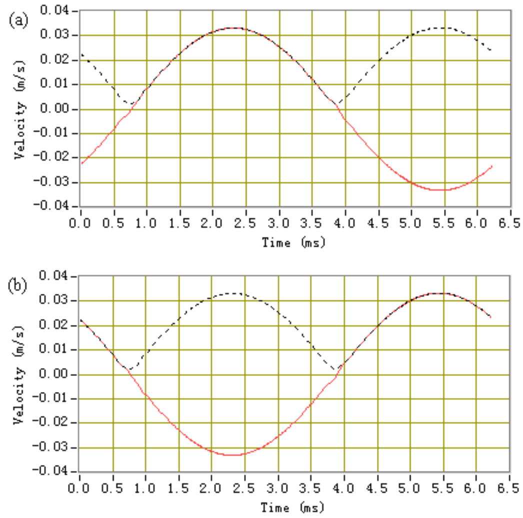



Figure 3. Velocity time history recovered by frequency demodulation at vibration frequency of 160 Hz with two possible motion directions shown as solid plots in (a) and (b).

## 4. SIMULATIONS

The theoretical model for Michelson interferometer signal is expressed in equation 1. However, the real measurement condition is more complicated, affected by random noise, phase disturbance, etc. The interferometer signal then can be written as:

$$u(t) = \hat{u}_o + (\hat{u}_r + 1)\hat{u}\cos[\varphi_0 + \varphi_p + \hat{\varphi}_M \cos(\omega t + \varphi_s)] \quad (6)$$

where $\hat{u}_o$ is offset of the signal; $\hat{u}_r$ is random noise level. They are all presented as percent of ideal signal voltage amplitude $\hat{u}$. $\varphi_p$ denotes the random phase disturbance in degree.

In our simulation tests, the simulated interferometer output signal has an offset of 5% in relation to its voltage amplitude. The random noise level of the signal is 5%, and the maximum phase disturbance is 5°. Other conditions concerning the simulated signal generated are: ADC effective resolution 8 bits, sampling rate 2 MS/s, and number of samples 200K.

Since the initial phase angle of vibration motion has certain influence on measurement accuracy, it is randomly generated within the angle range from 0° to 180° for each simulation test. On account of this influence, one hundred simulation tests are performed under all the same test conditions with the only exception of different random-generated displacement initial phase angle. Based on simulated digital interferometer signal with limited precision described above, the arithmetic mean and experimental standard deviation of the calculated deviations between nominal acceleration amplitude and initial phase values and those calculated by TIA can be estimated.

In our simulations, the influences from hum, distortion and relative motion are ignored because they are minor uncertainty resources if relevant requirements laid down in ISO 16063-11 are met.

In table 1, some of the results from the numerical simulation tests over frequency range from 20 Hz to 800 Hz are presented to illustrate the effectiveness of TIA for good measurement results of acceleration amplitude and initial phase, except for initial phase measurement at frequencies higher than 500 Hz. The mean values of initial phases above 500 Hz are from -0.23° to -0.67°. So, a further test is performed at 800 Hz with ideal 8-bit simulated interferometer signal affected by neither noise nor phase disturbance, the results of which is listed in Table 2.

Table 1. Simulation test results for TIA from 20 to 800 Hz.

| Frequency (Hz) | Acceleration ($m/s^2$) | Amplitude | | Initial phase | |
|---|---|---|---|---|---|
| | | Mean (%) | Standard deviation (%) | Mean (°) | Standard deviation (°) |
| 20 | 10 | 0.00 | 0.00 | 0.00 | 0.00 |
| 25 | 10 | 0.00 | 0.00 | 0.00 | 0.00 |
| 31.5 | 10 | 0.00 | 0.00 | 0.00 | 0.00 |
| 40 | 10 | 0.00 | 0.00 | 0.01 | 0.00 |
| 50 | 10 | 0.00 | 0.00 | 0.01 | 0.01 |
| 63 | 10 | 0.00 | 0.00 | 0.02 | 0.01 |
| 80 | 10 | 0.00 | 0.00 | 0.05 | 0.02 |
| 100 | 10 | 0.00 | 0.00 | 0.10 | 0.04 |
| 125 | 50 | 0.00 | 0.00 | 0.01 | 0.00 |
| 160 | 50 | 0.00 | 0.00 | 0.01 | 0.00 |
| 200 | 50 | 0.00 | 0.00 | 0.04 | 0.00 |
| 250 | 50 | 0.00 | 0.00 | 0.08 | 0.01 |
| 315 | 100 | 0.00 | 0.00 | 0.05 | 0.00 |
| 500 | 100 | 0.01 | 0.00 | -0.23 | 0.02 |
| 630 | 100 | 0.02 | 0.00 | -0.42 | 0.03 |
| 800 | 100 | 0.05 | 0.01 | -0.67 | 0.03 |

Table 2. Simulation test results for TIA at 800 Hz.

| Acceleration ($m/s^2$) | Number of fringes | Times of fringe frequency | Amplitude | | Initial phase | |
|---|---|---|---|---|---|---|
| | | | Mean (%) | Standard deviation (%) | Mean (°) | Standard deviation (°) |
| 50 | 25.0 | 63.6 | 0.30 | 0.03 | -2.47 | 0.05 |
| 100 | 50.0 | 31.8 | 0.07 | 0.02 | -0.67 | 0.03 |
| 200 | 100.1 | 15.9 | 0.02 | 0.00 | -0.34 | 0.01 |
| 300 | 150.1 | 10.6 | 0.01 | 0.00 | -0.19 | 0.01 |
| 400 | 200.1 | 8.0 | 3.17 | 0.53 | -0.42 | 0.13 |

It is clear that the mean value of initial phase decreases significantly with the increase in acceleration amplitude, with the exception of 400 $m/s^2$. This is because more fringes, and therefore more zero-crossings, per vibration period are available, which leads to more velocity data produced by frequency demodulation can be used in sine approximation calculation. Normally, the size of velocity data should be larger than 100 for accurate results from sine approximation algorithm. Insufficient velocity data has more unfavorable influence on initial phase measurement, e.g. at acceleration level of 50 $m/s^2$ shown in Table 2. On the other hand, the fringe frequency rises up dramatically with the increase in acceleration. With constant sampling rate of 2 MS/s applied, fewer samples are acquired from the shortest period of photoelectric signal. This can reduce the accuracy in identifying the time location of zero-crossings. Therefore, the sampling rate should be at least 10 times higher than fringe frequency. Inadequate sampling rate has more unfavorable influence on acceleration amplitude measurement, e.g. at acceleration level of 400 $m/s^2$ shown in Table 2.

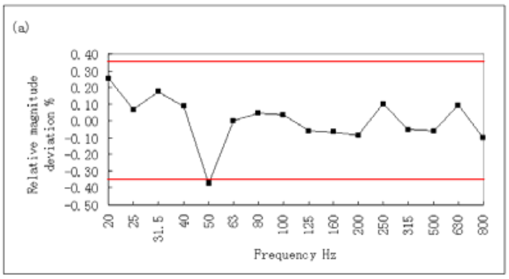
In summary, the simulation tests have proven the high accuracy of simplified TIA algorithm for acceleration amplitude and initial phase measurement, even when the interferometer signal is noisy. Because of the limited number of fringes at higher frequency, this method is normally applied to 800 Hz, preferably with high acceleration level at frequencies higher than 500 Hz.

## 5. COMPARISON EXPERIMENT

The comparison experiment has been conducted with a standard calibration set (B&K 8305 and 2650) from 20 Hz to 800 Hz between national medium and high frequency vibration standards, based on homodyne and heterodyne interferometry respectively, at NIM, China. By adding a PCI dual channel synchronous data acquisition card (max.5 MS/s), the simplified TIA has been successfully implemented with original medium frequency vibration standard (FCM).

Evaluation of the results follows the protocol used for CCAUV.V-K1. For the sensitivity and phase shift measurement, the weighted mean, with its associated uncertainty of measurement, is calculated for each frequency point and used as the comparison reference value. The degree of equivalence is then determined for the sensitivity as well as the phase shift measurements [4].

Sixteen frequencies and reference acceleration levels in the comparison are the same as those in simulations, shown in table 1. Figure 4 shows the results obtained for the difference in the sensitivity and phase shift values obtained between the TIA and the reference values. For the frequency range from 20 Hz to 800 Hz, the expanded measurement uncertainty for high frequency vibration standard is 0.5% and 0.5° (k=2); for medium frequency vibration standard 0.5% and 1° (k =2). So, the uncertainty associated with the degree of equivalence between the TIA and the reference values is 0.35% and 0.9° (k =2) for sensitivity and phase shift measurements, shown as upper and lower limits in figure 4 (a) and (b).



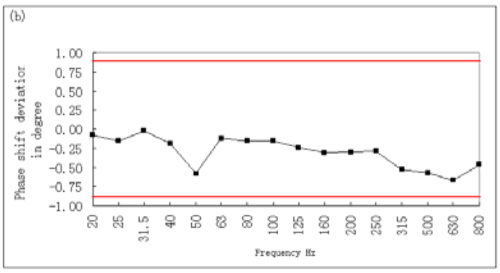




Figure 4 Frequency response of the difference between TIA and the reference values in sensitivity (a) and phase shift (b) measurements for B&K calibration set in comparison.

It is evident in figure 4 that all the TIA measurement results are within the limits, except for sensitivity measurement at 50Hz. The deviation at this frequency is -0.37% (beyond the lower limit) and -0.58° because the voltage output signal from the accelerometer is easily affected by various voltage disturbances under this condition.

For the phase shift deviation plot in figure 4(b), the trend agrees with the expectation from simulation test quite well, except at 50Hz. This further verifies that the TIA is better applied within 1 kHz and preferably with high acceleration level at high frequency to ensure more zero-crossings per vibration period available for more accurate measurement of acceleration initial phase.

## 6. CONCLUSION

A simplified implementation of the time interval analysis method has been described. Based on the configuration of primary vibration calibration system for fringe counting method, the new method can ensure high accurate measurement of complex sensitivity of accelerometers, with a dual channel synchronous data acquisition card added and calculation software programmed. Since the real signal from Michelson interferometer is noisy, common data acquisition card is preferred rather than high resolution time interval analyzer or high speed DAC card, which lowers the hardware requirement. For identification of time location of zero-crossings, simple algorithm is used instead of third order polynomial fitting, which improves the software efficiency.

Various simulation tests and real comparison experiment are conducted to investigate performance and characteristics of TIA. Certain sampling rate (10 times of fringe frequency) and acceleration level should be ensured for accurate measurement, which also leads to the normal frequency limitation of 1 kHz for this method. The comparison experiment from 20 Hz to 800 Hz between NIM's medium and high vibration standards verifies the stated uncertainty for TIA: 0.5% and 1° (k=2).

Because of its high accuracy for complex sensitivity measurement of accelerometer at low and medium frequency range and simplified algorithm and low requirements on hardware, the simplified TIA method is ideal to apply in national metrology institute of developing countries and industrial primary vibration calibration labs where the primary vibration calibration system based on fringe counting method already exits.

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