IMEKO International Conference on
Metrology for The Sea
Naples, Italy, October 11-13, 2017

Enhanced pseudorange weighting scheme using local redundancy

Antonio Angrisano[1], Silvio Del Pizzo[2], Salvatore Gaglione[2], Salvatore Troisi[2], Mario Vultaggio[1]

[1] G. Fortunato University, via R. Delcogliano 12, Benevento, Italy
[2] Department of Science and Technology, Parthenope University of Naples, Centro Direzionale di Napoli, Is. C4 80143, Napoli, Italy.

Abstract **- The absolute positioning is the most common GNSS operational mode, being used, for instance, by ships satellite receivers; it is based on pseudorange measurements and its functional model, relating measurements to unknowns, is well defined. On the other hand the stochastic model, describing the measurement error behaviour, is currently under investigation and could be used to reduce the position errors, by defining a suitable weighting scheme. In signal-degraded scenario, as high density traffic harbours or canals, a weighted approach is especially suited, because the pseudorange accuracies are significantly different, so equally weighting all the measurements would bring to large errors.**

The redundancy numbers are the diagonal elements of the redundancy matrix and they represent the degree of controllability of the measurements; a large redundancy number corresponds to a well-controlled measurement, while a small one corresponds to a leverage observation, with a high potential to influence the solution.

In this work, the redundancy number is proposed as indicator, in addition to the largely adopted signal-to-noise ratio and satellite elevation angle, for defining a weighting scheme, based on the concept of limiting the effect of leverage measurements.

I. INTRODUCTION

Global Navigation Satellite System (GNSS) navigation is critical in scenarios, where natural or artificial obstacles reduce the signal quality, producing gross errors on the measurements. Examples of such critical scenarios are narrow canals or crowded harbour, where the GNSS signals are typically affected by multipath interference and non-line-of-sight (NLOS) reception. The multipath interference happens when a signal is received through multiple paths; the multiple signals combination causes a distortion of the correlation function (between received and locally generated signals) and yields a range error up to several tens of meters [1]. The NLOS reception happens when the direct signal is blocked and only reflected signals are received; this phenomenon could yield range errors of km order [2].

Absolute or Single Point Positioning (SPP) is a widespread GNSS operational mode, being used by mass-market receivers, mounted for instance in car navigation devices and smartphones [3-4] and often integrated with other sensors (maps, inertial sensors, vision systems).

In critical scenario, GNSS devices are typically affected by blunders related to the aforementioned phenomena. The problem is commonly tackled in two ways:

- using Receiver Autonomous Integrity Monitoring (RAIM) techniques, able to detect and reject blunders [5-6-7];
- using robust estimation techniques, able to absorb blunders effect [8].

In this work, alternatively, suitable weighting schemes, able to de-weight potentially dangerous measurements, are taken into account; some widespread measurement weighting schemes are reviewed and tested and an additional quality criterion is proposed in order to limit the effect of potential blunders.

In literature, the GNSS measurements are differently weighted according to two quality indicators: satellite elevation angle and carrier-to-noise ratio.

Satellite elevation ($\boldsymbol{El}$) is considered a measurement quality indicator because satellites at low elevation are usually noisier, due to typical behaviour of multipath and tropospheric errors [1].

The Signal-to-Noise ratio (S/N), or better the Carrier-to-Noise ratio ($\boldsymbol{C/N_0}$), is the ratio between the carrier power and the noise power per unit bandwidth and is usually expressed in decibel-Hertz; $\boldsymbol{C/N_0}$ is a measure of signal strength, so it is a suitable indicator of measurement quality [6].

The $\boldsymbol{El}$ and $\boldsymbol{C/N_0}$ indicators are adopted individually or synergistically to define a model of measurement error variance; the measurements are usually weighted inversely to the modelled error variance.

The measurements of a dataset do not equally influence the solution; indeed, some of them (called leverage observations) have high potential to influence the solution, while others are less influent. Therefore, leverage observations are potentially critical, because if affected by a blunder they can yield large errors in the solution. It can be demonstrated that the concept of leverage observation is related to local redundancy one [9], which is well represented by the redundancy numbers, that are the diagonal elements of redundancy matrix.

In critical scenario, the presence of blunders is very common and the geometry is often weak. In this context, if a blunder is present on a leverage observation, the solution could be strongly degraded.

In this work, a weighting approach, taking into account the local redundancy, besides indicators $\boldsymbol{El}$ and $\boldsymbol{C/N_0}$, is proposed and its effectiveness is verified in harsh environment. The strategy is applied to GPS only, demonstrating evident usefulness, but it can be simply generalized to multi-constellation case.

In the following sections, the existing weighting schemes are described, then the weighting strategy based on local redundancy concept is defined, at the end the considered weightings are applied to real data and the results are discussed. The used data are collected in urban environment, which properly represent a critical scenario.

II. WEIGHTING SCHEMES REVIEW

Most of weighting schemes in literature are based on $\boldsymbol{C/N_0}$, satellite elevation $\boldsymbol{El}$ or both; the most commonly used are reviewed in this section.

The most widespread variance models for GPS carrier phase, based on satellite elevation, depend on squared sin of El [10-11]; despite designed for carrier phase, they are simply adapted for pseudorange observation [12-13] as shown below:

$$\boldsymbol{\sigma_{PR}^2 = \frac{\sigma_0^2}{\sin^2(El)}} \quad (1)$$

where $\boldsymbol{\sigma_0^2}$ is the pseudorange error variance.

In [10], a variance model for carrier phase observation is proposed; it is re-used and adapted in [2] for pseudorange measurement. The model is exclusively based on $\boldsymbol{C/N_0}$ and has the following form:

$$\boldsymbol{\sigma_{PR}^2 = c * 10^{-\frac{C/N_0}{10}}} \quad (2)$$

where $\boldsymbol{c = 10^4}$ m^2 is a constant, defined empirically.

In literature, few weighting schemes, based on both satellite elevation and $\boldsymbol{C/N_0}$, are available. In [14] a model, based on both satellite elevation and $\boldsymbol{C/N_0}$, designed for land navigation, is defined:

$$\boldsymbol{\sigma_{PR}^2 = k * \frac{10^{-\frac{C/N_0}{10}}}{\sin^2(El)}} \quad (3)$$

The model (3) is clearly a fusion between the elevation-based model (1) and the $\boldsymbol{C/N_0}$-based model (2). The constant k is 1 if the signal is received in line-of-sight (LOS) and 2 or +∞ if the signal is received after reflection by obstacles surrounding the antenna (NLOS); in [14], a fish-eye camera is used to distinguish LOS and NLOS signals.

In this work, the constants included into the considered variance models are modified to obtain, in a particular condition, i.e. $\boldsymbol{El = 90°}$ and $\boldsymbol{C/N_0 = 55\ db - Hz}$, $\boldsymbol{\sigma_{PR} = 3\ m}$.

III. REDUNDANCY MATRIX

In absolute positioning mode, the measurement model is

$$\boldsymbol{z = H\Delta x + \varepsilon} \quad (4)$$

where

$\boldsymbol{z}$ is the vector of measurements, defined as the difference between measured and computed (with a priori information) pseudorange,

$\boldsymbol{H}$ is the design matrix,

$\boldsymbol{\Delta x}$ is the state vector, containing the corrections to update the receiver coordinates and clock offset,

$\boldsymbol{\varepsilon}$ is the measurement error vector.

The number of measurements is indicated as *m*, the number of unknowns as *n*; *m* is usually larger than *n* and the equation (4) is solved using least squares (LS) method.

LS solution is:

$$\boldsymbol{\widehat{\Delta x} = (H^T H)^{-1} H^T z} \quad (5)$$

The residuals $\boldsymbol{v}$, defined as the difference between the actual measurements and their estimated values ($\boldsymbol{\hat{z}}$), are an indicator of the measurement mutual agreement [9]:

$$\boldsymbol{v = z - \hat{z} = z - H\widehat{\Delta x}} \quad (6)$$

It can be simply demonstrated that the relationship between the measurement errors $\boldsymbol{\varepsilon}$ and the LS residuals is:

$$\boldsymbol{v = R\varepsilon} \quad (7)$$

The matrix $\boldsymbol{R}$ is called "Redundancy Matrix" and can be obtained by the following expression:

$$\boldsymbol{R = I - H(H^T H)^{-1} H^T} \quad (8)$$

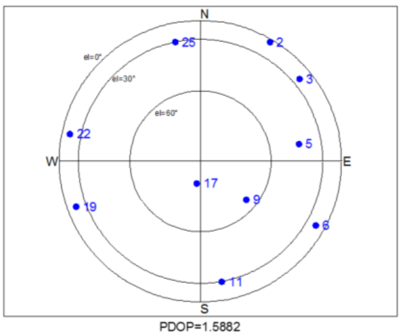
The trace of the matrix $\boldsymbol{R}$ is the total redundancy (or the degree of freedom) of the equation, that is (*m-n*); the i-th diagonal element $\boldsymbol{r_i}$ of $\boldsymbol{R}$ is called "redundancy number" and is the contribution of the i-th measurement to the total redundancy [15]. The redundancy number assumes values between 0 and 1. Small values of $\boldsymbol{r_i}$ (near 0) correspond to measurements providing little contribution to total redundancy and so hardly controlled; on the other hand, approximately equal values of $\boldsymbol{r_i}$ are desirable, being every measurement controllable.

Measurements with small $\boldsymbol{r_i}$ values are leverage observations and have high potential to influence the solution; if a blunder or a large bias is present on a leverage observation, harmful effects can be evident on the positioning. In this context, de-weighting leverage observations could be a successful strategy, above all in difficult environments characterized by frequent gross errors.

In table 1, an example of GPS geometry, in terms of satellite elevations (El) and azimuths (Az), is shown; from El and Az, the design matrix H and the redundancy matrix $\boldsymbol{R}$ can be computed. The diagonal elements of $\boldsymbol{R}$ are the redundancy numbers.

Tab. 1 – Example of GPS geometry, in terms of satellite elevation and azimuth

PRN	*El* [deg]	*Az* [deg]
2	12	30
3	24	50
5	45	80
6	21	119
9	65	130
11	30	170
17	81	190
19	20	250
22	19	282
25	30	348

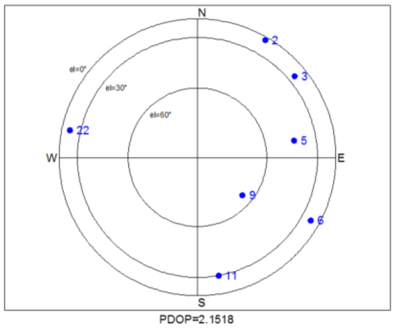


PRN	Redundancy number
02	0.56
03	0.71
05	0.76
06	0.51
09	0.66
11	0.57
17	0.52
19	0.53
22	0.56
25	0.63

Fig. 1 – Sky-plot of GPS satellite configuration; the total redundancy is 6

In figure 1 the sky-plot of the satellite geometry, detailed in table 1, is shown; all the listed satellites are considered, so the number of visible GPS satellites is 10 and the PDOP value is 1.59. The total redundancy of the measurement model is $(\boldsymbol{m}-\boldsymbol{n}) = \boldsymbol{6}$, therefore the average redundancy number is 0.6. The sky-plot is accompanied by the values of the redundancy numbers and it is evident that all the values are near the average.

In figure 2 the sky-plot of the satellite geometry, detailed in table 1, is shown, but satellites with PRNs 17, 19 and 25 are excluded; consequently, the number of visible GPS satellites is 7 and the PDOP value raises to 2.15. The total redundancy is 3, so the average redundancy number is about 0.43. The redundancy numbers are all around or above the average value, except the one corresponding to satellite 22, which is a leverage observation. From figure 2, it is evident a geometric interpretation of leverage observation, for GPS absolute positioning, as a measurement corresponding to a satellite "isolated" with respect to the others.



PRN	Redundancy number
02	0.45
03	0.65
05	0.63
06	0.41
09	0.33
11	0.43
22	0.10

Fig. 2 – Sky-plot of GPS satellite configuration; the total redundancy is 3

In case of multi-constellation approach, a redundancy number is null when a single satellite from a system is present. For instance in a GPS/Glonass combination, if there is only one GPS measurement and several Glonass, the redundancy number corresponding to the unique GPS observation is 0 and the observation is uncontrollable (because it is the only one able to estimate the receiver clock offset to GPS time).

IV. CONTRIBUTION OF REDUNDANCY MATRIX TO WEIGHTING SCHEMES

The diagonal elements of the redundancy matrix, the redundancy numbers $\boldsymbol{r_i}$, represent the local reliability of the measurement model and so the controllability of the measurements [15].

Measurements with low $\boldsymbol{r_i}$ are critical for the solution, because they can strongly influence it. For this reason, an effective weighting strategy should take into account for the redundancy number, above all in scenarios with high probability of blunders.

In [16] the redundancy number is used, together with several other parameters, as an indicator of the measurement quality for robust estimation. In [6] the extra-diagonal elements of the redundancy matrix are used to evaluate the correlation among the measurements, in order to avoid erroneous blunder rejections. Recently, Falco et al. (2016) [17] adopted a purely geometric strategy to identify measurements which could strongly influence the solution, in order to de-weight them.

In this paper, the information contained in the redundancy matrix are used to correct the original weighting matrix, which could be defined according to one of the methods

previously described. In general, the weight w_i, associated to the i-th measurement, is obtained inverting the corresponding pseudorange error variance σ^2_{PRi}.
The contribution of redundancy number to the weighting strategy could be introduced making w_i is proportional to r_i as shown below

$$\begin{cases} w_i = \dfrac{r_i}{\sigma^2_{PRi}}, & \textit{redundant measuremens} \\ w_i = \dfrac{1}{\sigma^2_{PRi}}, & \textit{lack of redundancy} \end{cases}$$

In case of lack of redundancy $r_i = 0$, so the weights must be computed simply inverting the measurement error variances.
The strategy is tested on a multi-constellation static data collections, carried out in urban scenario, and demonstrate its efficiency in GPS only case, exhibiting significant improvements on positioning accuracy.

V. TEST

To demonstrate the benefits of including the redundancy number into a weighting scheme in difficult environment, a static data collection is carried out in urban scenario; specifically the test location is the "Centro Direzionale di Napoli" (CDN), a district of Naples (Italy) characterized by several skyscrapers. A specific point in CDN is surveyed with topographical methods, in order to determine its coordinates with mm order [18]. The used equipment consists in a NVS receiver, a single frequency, double constellation (GPS/GLONASS) and high-sensitivity device, connected to a patch antenna; the device is placed on the defined point to collect pseudorange measurements for some hours and the collected data are processed in single point to test several weighting schemes.
In figure 3 the considered point (indicated as P) is shown and the urban canyon scenario is evident.

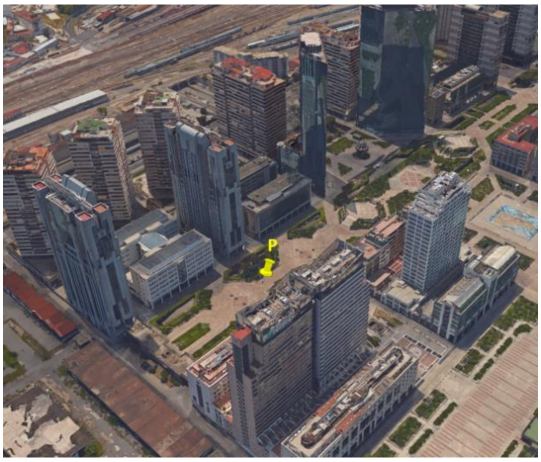


Fig. 3 – Test location

The data collection was carried out on 1-st July 2016; the NVS receiver was placed on P point for about 2 hours. In figure 4 the number of GPS visible satellites and the corresponding PDOP values during the session are shown.

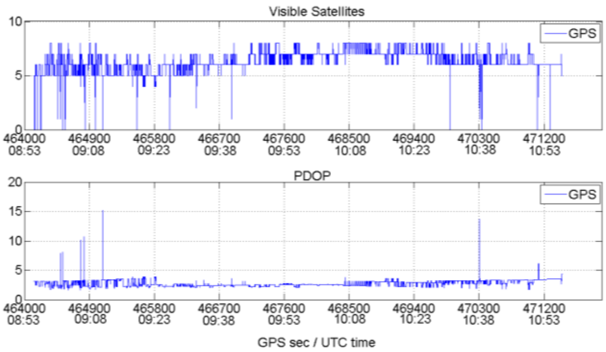


Fig. 4 – Number of visible GPS satellites and corresponding PDOP during session

The number of GPS visible satellites ranges between 0 and 8, with an average of 6.4. The PDOP average value is about 2.7, with maximum and minimum respectively about 15 and 1.6.
The GPS solution availability, defined as the time percentage where the solution can be computed (at least 4 satellites should be visible), is about 99.3%.
Despite the urban canyon scenario, the solution availability is high, owing to the use of a high-sensitivity receiver, able to acquire very weak signals. The rapid oscillations of the number of visible satellites, is due to frequent signal loss, typical of hostile environment.

VI. RESULTS

The weighting strategies described in section 2 are adopted to process data collected in urban scenario, in order to compare their performance.
The considered configurations, implementing GNSS SPP with a distinct weighting scheme, are:

- The baseline configuration with all measurements equally weighted, shortly indicated as EQW;
- weights depending merely on El, as expressed by formula (1), shortly indicated as ELV;
- weights depending merely on C/N_0, as expressed by formula (2), shortly indicated as CN0;
- weights depending on both El and C/N_0, as expressed by formula (3), shortly indicated as ELC.

The figure of merits adopted for the comparison are the mean, RMS and maximum errors of both horizontal and vertical components of position.

In Table 2 the configuration performance are summarized.

Tab. 2 – Figure of merits obtained with GPS only, using classic weighting methods

		EQW	ELV	CN0	ELC
Mean [m]	Horiz.	16.0	10.4	15.4	5.9
	Vert.	18.2	12.9	17.6	5.8
RMS [m]	Horiz.	44.9	28.2	43.4	14.9
	Vert.	45.9	33.3	45.1	19.5
Max [m]	Horiz.	351.6	187.7	341.0	130.1
	Vert.	316.6	224.0	305.8	173.4

The results obtained with the EQW configuration is clearly inaccurate; in fact, the errors, on both components and for each figure of merit, are very large owing to the frequent blunders in the dataset. The considered weighting schemes reduce drastically the errors; the model based uniquely on satellite elevation (ELV) show better performance with respect to the model based uniquely on carrier-to-noise ratio (CN0). The ELC scheme, considering both quality indicators, demonstrates significant improvements relatively to the other configurations. In particular, ELC RMS and mean errors are reduced, with respect to EQW ones, between 58 and 68%, and the maximum horizontal and vertical errors are reduced respectively of 63 and 45%. The error reduction of ELC with respect to ELV configuration is 43% and 55% on horizontal and vertical mean error, 47% and 41% on horizontal and vertical RMS error, 31% and 23% on horizontal and vertical maximum error. The error reduction of ELC with respect to CN0 configuration is 62% and 67% on horizontal and vertical mean error, 66% and 57% on horizontal and vertical RMS error, 62% and 43% on horizontal and vertical maximum error.

The considered weighting schemes are augmented, using information embedded in the redundancy matrix as described in previous sections; the configurations using this information are shortly indicated as RDM. For instance, a configuration using the elevation-based model (1), augmented with redundancy matrix-based approach, is shortly indicated as ELV+RDM. In table 3 the performance of the configurations, obtained combining the considered classical weighting schemes with redundancy matrix information, are resumed. Moreover, in the table, alongside the errors, the percentage improvements, with respect to corresponding configurations without the use of redundancy numbers, are reported. It is evident that the introduction into the weighting schemes of information from redundancy matrix brings to significant improvements. The best configuration is ELC+RDM, characterized by horizontal mean, RMS and maximum errors respectively about 4.5, 8.8 and 87.2 meters; the vertical mean, RMS and maximum errors are respectively 3.3, 11.4 and 115.9. The obtained performance of ELC+RDM configuration are very good, considering the strongly degraded scenario.

Tab. 3 – Figure of merits obtained with GPS only, using classic weighting methods, augmented with information from redundancy matrix

		EQW RDN	ELV RDN	CN0 RDN	ELC RDN
Mean [m]	Horiz.	14.0 (12.2%)	8.0 (22.9%)	13.3 (13.7%)	4,5 (23.6%)
	Vert.	15.5 (14.9%)	9.0 (29.8%)	14.7 (16.6%)	3.3 (43.8%)
RMS [m]	Horiz.	36.4 (19.0%)	18.1 (35.7%)	34.4 (20.9%)	8.8 (41.1%)
	Vert.	37.2 (18.9%)	22.8 (31.6%)	35.8 (20.6%)	11.4 (41.7%)
Max [m]	Horiz.	276.5 (21.4%)	121.1 (35.5%)	261.7 (23.2%)	87.2 (33.0%)
	Vert.	241.5 (23.7%)	154.7 (30.9%)	230.3 (24.7%)	115.9 (33.2%)

VII. CONCLUSIONS

The main objectives of this paper are: to analyse the existing weighting strategies for GNSS absolute positioning and to assess the possible benefits of including the information embedded into the redundancy matrix to improve the existing weighting schemes.

GNSS navigation is critical in environments characterized by probable presence of blunders among the observations, which could cause very large position errors. In this context, a measurement weighting scheme is essential, because it could limit the blunder effects. The existing schemes are based on measurement quality indicators as satellite elevation and/or carrier-to-noise ratio; so some schemes, described in literature, are tested in urban scenario (a typical critical environment), in order to compare their performance. The best results are surely obtained with the schemes based on both the abovementioned quality indicators.

The redundancy matrix contains, on its diagonal, information related to the local redundancy, which indicates the measurement degree of controllability; a measurement with low local redundancy is difficultly controlled and, if affected by gross errors, it strongly influences the solution. In this work, a strategy is proposed in order to include the local redundancy into the existing weighting schemes; the approach consists in setting the measurement weights proportionally to the corresponding local redundancy numbers. In this way, the most influencing measurements (i.e. the leverage measurements) are de-weighted, limiting their effects on the solution.

The proposed approach is tested on a 2-hour dataset, collected in urban scenario; GPS only configurations are analysed, but the analysis could be simply extended to multi-constellation case. The obtained results demonstrate that the proposed weighting strategy effectiveness; indeed, for all the considered configurations the position errors are significantly reduced, applying the weighting scheme based on local redundancy numbers. The best results are obtained with a weighting scheme, including satellite elevation, carrier-to-noise ratio and redundancy number, which allows a positioning with horizontal accuracy below 5 meters in a severely degraded environment.

REFERENCES

[1] Kaplan, E.D. and Hegarty, J. (2006). Understanding GPS: Principles and Applications. Artech House Mobile Communications Series.

[2] Groves, P. and Jiang, Z. (2013). Height aiding cn0 weighting and consistency checking for gnss nlos and multipath mitigation in urban areas. Journal of Navigation, 66(5), 653-669.

[3] Van Sickle, J. (2015). GPS For Land Surveyors, CRC Press, New York, NY, USA.

[4] Yoon, D., Kee, C., Seo, J. and Park, B. (2016). Position Accuracy Improvement by Implementing the DGNSS-CP Algorithm in Smartphones. Sensors, 16(6), 16 pages

[5] Brown, R. G. and Chin, G. Y. (1997). GPS RAIM: calculation of threshold and protection radius using chi-square methods-a geometric approach. Global Positioning System: Institute of Navigation, 5, 155–179.

[6] Kuusniemi, H. (2005). User-level reliability and quality monitoring in satellite based personal navigation. Ph.D. dissertation, Tampere University of Technology, Tampere, Finland.

[7] Castaldo, G., Angrisano, A., Gaglione, S. and Troisi, S. (2014). P-RANSAC: An Integrity Monitoring Approach for GNSS Signal Degraded Scenario. International Journal of Navigation and Observation, Volume 2014, Article ID 173818, 11 pages.

[8] Knight, N. and Wang, J. (2009). A Comparison of Outlier Detection Procedures and Robust Estimation Methods in GPS Positioning. Journal of Navigation, 62(4), pp. 699-709.

[9] Leick, A. (2004). GPS Satellite Surveying. JohnWiley and Sons, Inc.

[10] Hartinger, H. and Brunner, F. (1999). Variances of GPS phase observations: The sigma-ε model. GPS Solutions, 2, 35-43.

[11] Collins, J.P. and Langley, R.B. (1999). Possible Weighting Schemes for GPS Carrier Phase Observations in the Presence of Multipath. Final contract report for the U.S. Army Corps of Engineers Topographic Engineering Center, No. DAAH04-96-C-0086 / TCN 98151, March, 33 pp.

[12] Petovello, M. (2003). Real-time Integration of a Tactical-Grade IMU and GPS for High-Accuracy Positioning and Navigation. PhD Thesis, Department of Geomatics Engineering, University of Calgary, Canada.

[13] Wieser, A. (2007). How important is GNSS observation weighting? GNSS Solutions Column, Inside GNSS, January-February Issue, pp. 26-28, 2007.

[14] Tay, S. and Marais, J. (2013). Weighting models for GPS Pseudorange observations for land transportation in urban canyons. Proceedings of the 6th European Workshop on GNSS Signals and Signal Processing, Munich, Germany.

[15] Schaffrin, B. (1997). Reliability measures for correlated observations. Journal of Surveying Engineering, 123(3), 126–137.

[16] Wieser, A. (2001). A Fuzzy System for Robust Estimation and Quality Assessment of GPS Data for Real-Time Applications. Proceedings of the 14th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 2001), Salt Lake City, UT, US.

[17] Falco, G., Nicola, M. and Falletti, E. (2016). Constellation-Aware Method for Computing the Covariance Matrix of GNSS Measurements. Proceedings of the European Navigation Conference 2016, Helsinki, Finland.

[18] Ackermann, S., Angrisano, A., Del Pizzo, S., Gaglione, S., Gioia, C., Troisi, S. Digital surface models for GNSS mission planning in critical environments(2014) Journal of Surveying Engineering, 140 (2), art. no. 0000119.