Analysis of vibrations in a wind turbine excited by earthquakes

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***Abstract*— In this paper, the authors aimed to summarize the results of an analytical study that evaluated the vibration analysis control of an on-shore wind turbine subjected to earthquake forces. The on-shore wind turbine (model G80-2.0 MW, manufactured by Gamesa Company) was modeled mathematically as a simplified mechanical system. The primary purpose of this work is related to the analysis of the lateral displacement of the wind turbine and to undertake a mechanical vibration study to reduce the effects of such displacements induced by earthquake ground motions across the entire system. Simulated records corresponding to the maximum credible earthquake risk associated with the design spectra for the specific study site (Istmo de Tehuantepec in the state of Oaxaca, Mexico) were used to carry out the analysis. The mechanical system is modeled via the Euler–Lagrange formalism and the system is described as an inverted pendulum with a concentrated mass at the free end. Finally, some results are presented through numerical simulations, considering the real parameters of the on-shore wind turbine.**

***Index Terms*— Mechanical vibrations, earthquakes; inverted pendulum; on-shore wind turbine; exogenous perturbation.**

I. INTRODUCTION

Recently, in many countries, innovation, design, and research into new technologies relating to the generation of electricity using clean energy sources as solar and wind per are topics which have received an increasing amount of interest from industry, academics, and researchers. During the last few years in Mexico, several wind farms have been built using different technologies manufactured by, for example, Vestas, Gamesa, etc. These technologies are designed for use in both offshore and onshore applications. Specifically, these wind farms in this study were located in a geographic region known as the Istmo de Tehuantepec in the Oaxaca State in Mexico. This zone has interesting wind conditions with average speeds of ~10 m/s and the power potential is estimated to be 33,200 MW, making this one of the more important wind zones around the world [1].

Geographically, the Istmo de Tehuantepec is located in a subduction zone at the convergence of the Cocos and North America tectonic plates. The study site is located on the coast, 80 km from the town of Istmo de Tehuantepec. As can be observed in Fig. 1, according to the *Manual of Civil Structures* (MOC-15) the Istmo de Tehuantepec is located in an earthquake-prone area with a high level of seismic risk (indicated using red) [2,3]. This region also has the characteristic of having swarms of seismic epicenters, in which seismic events are common.

In general, a wind turbine is a machine that combines a wind rotor and an electrical generator. The wind rotor transforms the kinetic energy of the wind into rotational mechanical energy, which is transformed into electrical energy via a generator. Wind turbines can be classified into two basic types according to the way the rotational axis spins: horizontal or vertical. Wind turbines that rotate around a horizontal axis (parallel to the ground) are the most commonly used because of their efficiency and versatility. This kind of wind turbine has its main rotational axis on the upper side of a tower, and it needs an orientation mechanism to adjust to variations in the wind direction [4,5].

The innovation, manufacturing, and maintenance of wind turbines are complicated, thus becoming an important problem to solve in the engineering and the research in renewable energies. The wind turbines, regardless of the application or installation place (in-shore or off-shore), are designed such that the tower structure supports the entire load of the turbine (wind rotor with blades and nacelle). Thus, the towers have a strong and slender section, with heights up to 80 m. This structural and mechanical design significantly exposes the wind turbines to excessive vibrations and movements, caused by the same turbine operation and other external factors such as wind forces and earthquakes [6,7]. Because the system is not redundant (only one tower as support), it will be particularly vulnerable to earthquakes. The significantly amplitudes of the vibrations and displacements in wind turbine systems may compromise the safety of the tower and its performance. To mitigate these effects, in structural engineering and vibration control fields, three different control methodologies are used: passive, semi-active and active control [8-11]. These schemes of control can be used to reduce the undesirable negative effects of the vibrations and indirectly the displacement of the overall structure.

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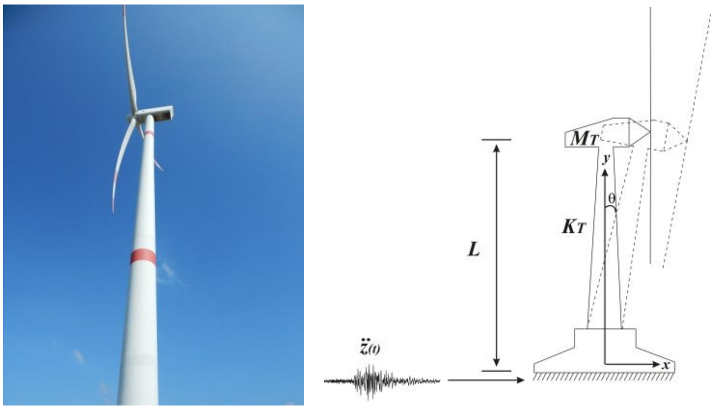
II. DESCRIPTION OF THE SYSTEM

In this work, a vibration analysis of an on-shore wind turbine (model G80-2.0 MW, Gamesa Company) was carried out [12-14]. This turbine model is currently installed on several wind farms located in the Istmo de Tehuantepec, Oaxaca, Mexico. The system was modeled with a nominal stiffness $K = 3EI/L$, where (E) is the Young´s modulus, (I) is the moment of inertia and (L) is the total height of the tower. Fig. 1b shows a schematic diagram of the turbine, which has blades 39 m long. It must be noticed that in this study, we only consider the lateral vibrations induced in the structure by earthquakes. To describe the performance of the structure, the tower, all three blades and the corresponding mass of the wind rotor and nacelle are considered to be structural members. The tower is 78 m high and is considered to be mounted in the foundation (fully fixed). Its structural form is a tapered tubular tower whose cross-section is a circular corona. The base diameter is 3.5 m with a wall thickness of 0.045 m. A linear variation between the base and the upper end of the tower with respect to the diameter and the thickness is considered. The construction material is steel with a Young's modulus of $2.1 \times 10^{11} N/m^2$, a shear modulus (G) of $8.4 \times 10^{10} N/m^2$ and the effective density of the steel (ρ_v) being 7900 kg/m^3. The total mass of the tower is 201,000 kg and the tower's mass per unit of length is 3908.926 kg/m. The weight of each blade is 6500 kg, the mass of the rotor is 38,000 kg and the top head mass is 108,000 kg (including the wind rotor, nacelle, blades and hub).



(a) The La Venta wind farm (b) Schematic diagram of the tower

Fig. 1 The G80-2.0 MW wind turbine

III. ANALYSIS OF THE TOWER

Megastructures like wind turbines are submitted to mechanical stresses at both the base and the entire surface of the tower. In this section, we make an estimation of the natural frequency of the tower which is described [4,7,15]. The parameters of the tower and their respective values obtained from the technical specifications of the wind turbine are summarized in Table I [12-14].

Table I. Parameters of the model G80-2.0 MW wind tower manufactured by Gamesa Company

Young's Modulus of Elasticity E (N/m^2)	Moment of Inertia I (kg/m^4)	Height of Tower L (m)	Mass per unit of length m (kg)	Volumetric density ρ_v (kg/m^3)	External diameter D (m)
2.1×10^{11}	0.728937	100	3908.926	7900	3.5

The natural frequency of a wind turbine is proposed using the following criteria to avoid any coincidence with other operating frequencies such as the wind rotor that causes resonance within the system. Fundamentally, the first mode of vibration in a wind turbine tower is the most conflicting because it is the smallest and is the closest to the natural frequency of the rotor. The first natural frequency of the tower is calculated by assuming the principle of free vibration of a beam. However, it has an infinite number of normal modes, and one natural frequency can be associated with each normal mode [16]. The natural frequency of the beam can be expressed as follows:

$$\omega_T = \frac{3.516}{2\pi L^2}\sqrt{\frac{EI}{m}} = 0.3501 \quad (1)$$

where:

$$I = \frac{\pi}{64}[D^4 - (D - 2(t))^4] \quad (2)$$

and:

$$m = \pi D t \rho_v \quad (3)$$

The moment of inertia in wind turbines is related to the moment of inertia of a hollow circle, considering the external diameter D and the wall thickness t of the tower.

The stiffness of the tower is obtained by considering the tower to be prismatic with a flexural rigidity of EI, where E is Young's Modulus and I is the second moment of area of the section. The stiffness (K_T) is then considered to be [17,18]:

$$K_T = \frac{3EI}{L} = 4.59 \times 10^9 \ N/m \quad (5)$$

IV. A SIMPLIFIED MATHEMATICAL MODEL O THE WIND TURBINE

The on-shore wind turbine was modeled by considering the system to be an equivalent inverted pendulum [5,8,19]. In this case, the nacelle and the wind rotor, including the hub and blades, are concentrated in the mass (M_T) of the upper end of the beam representing the tower. The beam's bending stiffness is assumed to be large enough to be modeled as a rigid rod, which has a length L and a uniform mass per unit of length m. The viscous damping associated with the pendulum is considered to be proportional damping and is calculated via Rayleigh's method. The equivalent lateral stiffness of the tower is considered as a constant linear spring K_T, which is associated with the pendulum (see Fig. 2).

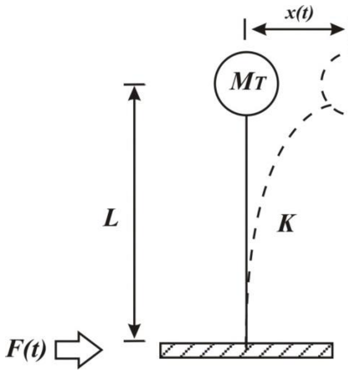


Fig. 2 Cantilever beam considering the lateral stiffness (*k*) of the beam

The cantilever structure with a single degree of freedom (see Fig. 2), can be considered as an inverted pendulum with a small beam displacement in order to parameterize the stiffness (K_T) of the system. For this purpose, it is necessary to establish an affinity with the schematic diagram in Fig. 1b. We assume a linear correspondence between the angular displacement of the tower θ and the lateral displacement of the beam (x), which are both time-dependent variables. If we consider the coordinates system shown in Fig. 1b, the position of the structure (x, y) is obtained by using the Euler–Lagrange formalism. As mentioned above, the angular displacement of the tower θ is small and hence the system can be assumed to display linear behavior. The representation of the overall system in terms of the kinetic (U_E) potential (T_E) and dissipative (D_E) energies is as follows:

$$x = L\sin\theta = L\theta;\; y = L\cos\theta = L \tag{6}$$

$$x_b = \frac{L}{2}\sin\theta \approx \frac{L}{2}\theta;\; y_b = \frac{L}{2}\cos\theta \approx \frac{L}{2} \tag{7}$$

$$U_E = \frac{1}{2}\left(M_T L^2\dot{\theta}^2 + \frac{1}{3}\rho L^3\dot{\theta}^2 + 2M_T L\dot{\theta}\right) \tag{8}$$

$$T_E = \frac{1}{2}\left[K_T\,\theta^2 + \left(2M_T\,l + \rho_L L^2\right) g\cos\theta\right] \tag{9}$$

$$D_E = \frac{1}{2}C_s\dot{\theta}^2 \tag{10}$$

where g is gravitational acceleration, ρ_L is the linear density of the tower, (x) and (y) are the M_T coordinates, and (x_b) and (y_b) are the coordinates of the tower's center of gravity. The system can be represented in linear matrix form by considering them as a typical translational second-order mechanical system using an ordinary differential equation with constant coefficients in which $x(t)$ denotes the displacement of the structure:

$$M\frac{d^2x}{dt^2} + C\frac{dx}{dt} + Kx(t) = 0 \tag{11}$$

Eq. (11) is the equation of motion for a linear system with a single degree of freedom, with M, C and K being the matrices of mass, damping and stiffness of the system, respectively.

V. Free Vibration of the Wind Turbine

In a system where no external forces are acting upon the structure, the motion induced by any initial disturbance will be a free vibration. The free response of the wind turbine, modeled in Fig. 1b, is obtained by considering the simplified inverted pendulum model in Fig. 2. In that case, we establish a set of equations for a system with one degree of freedom as described in the following expressions:

$$\left[\frac{\rho_L L^3}{3} + M_T L^2\right][\ddot{\theta}] + C[\dot{\theta}]\left[K_T - M_T g L - \frac{\rho_L g L^2}{2}\right][\theta] = 0, \tag{12}$$

and:

$$M[\ddot{\theta}] + C[\dot{\theta}] + K[\theta] = 0 \tag{13}$$

where the matrices M, C and K are described as:

$$M = \left[\frac{\rho_L L^3}{3} + M_T L^2\right] \;,\; K = \left[K_T - M_T g L - \frac{\rho_L g L^2}{2}\right] \tag{14}$$

In particular, the damping matrix (C_s) is assumed to be of proportional type [20] (i.e., $C = a_0 M + b_0 K$ where $a_0 = \xi_i \times \frac{2\omega_i \times \omega_j}{\omega_i + \omega_j}$ and $b_0 = \xi_j \times \frac{2}{\omega_i + \omega_j}$, where ω_i and ω_j are the structural modal frequencies and ξ_i and ξ_j are the structural damping ratios for modes i and j, respectively) [21]. The matrices M, C and K are expressed in terms of the parameters shown in Table II.

Table II. Parameters of the system

Mass of wind turbine M_T (kg)	Stiffness of tower K_T (N/m)	Height of tower L (m)	Linear density of tower ρ_L (kg/m)	Proportional damping of tower C_s (N/m/s)

1.55×10^5	4.59×10^9	100	2010	2.26×10^8

The fundamental (natural) frequency of the wind turbine is obtained using the software *Matlab®*, with the parameters of M and K as shown in Table II. This natural frequency is expressed as follows:

$$\omega_s = 0.22\ Hz \quad (15)$$

The response of the overall system resulting from an earthquake applied to the base of the tower is presented in the next section, with the purpose of showing the performance of the lateral displacement of the wind turbine considering the system's natural frequency.

VI. PERTURBATION AT THE BASE OF THE WIND TURBINE

The base of the on-shore wind turbine was subjected to a synthetic (simulated) earthquake designed by considering the current soils of the Istmo de Tehuantepec in the state of Oaxaca in Mexico. We consider the first horizontal component of the earthquake (longitudinal plane). This disturbing force $F(t)$ in the base of the structure in terms of the acceleration $\ddot{z}(t)$, is applied to the system to calculate the relative displacement of the wind turbine [22,23]. A simplified mathematical model in terms of the acceleration at the base of the wind turbine subjected to an earthquake is as follows:

$$M_n\ddot{\theta}(t) + C_n\dot{\theta}(t) + K_n\theta(t) = -M_n e\, \ddot{z}(t) \quad (16)$$

where $\theta \in R$ is the vector of the generalized coordinates for the equivalent relative lateral displacement of the wind turbine with respect to the main frame;n is the dimension of the matrix; $\ddot{z}(t) \in R$ is the earthquake in terms of the acceleration applied to the base of the structure; M_n, C_n and K_n are $m \times p$ matrices of the mass, damping and stiffness, respectively. The vector $e^T \in R$ is the influence vector, which represents the displacement of the wind turbine's mass caused by displacement of the soil. The matrices of mass, stiffness and damping of the wind turbine are:

$$M = \left[\frac{\rho_L L^3}{3} + M_T L^2\right] \quad (17)$$

$$K = \left[K_T - M_T gL - \frac{\rho_L\, g\, L^2}{2}\right] \quad (18)$$

The damping matrix is obtained by using Rayleigh's method and considering proportional damping, as discussed in the previous section. Finally, in the following equation, we show the force $F(t)$ in terms of $\ddot{z}(t)$ in the system's dynamics:

$$\left[\frac{\rho_L L^3}{3} + M_T L^2\right][\ddot{\theta}] + \left[K_T - M_T gL - \frac{\rho_L g L^2}{2}\right][\theta] = -\left[\frac{\rho_L L^3}{3} + M_T L^2\right][1][\ddot{z}(t)] \quad (19)$$

To evaluate the performance of the wind turbine, we present a frequency response function obtained from the perturbation in the base. An FFT procedure was carried out to determine the critical vibration modes of the overall structure. In an earlier analysis, we assumed that the lateral displacement of the wind turbine θ was equivalent to the displacement (x) of the cantilever beam (i.e. $\theta \approx x$). The numerical response of the displacement and the frequency of the wind turbine is show in Figs. 3 and 4, respectively. The parameters of the system are shown in Table II.

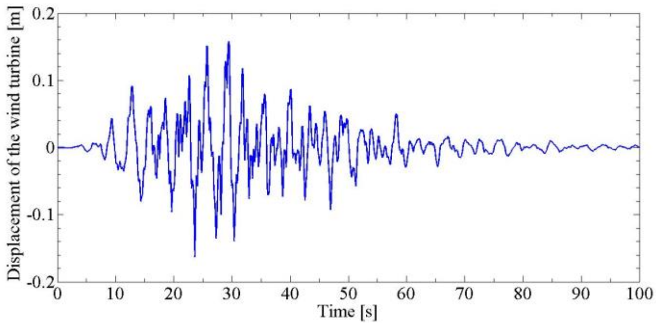


Fig. 3 Lateral displacement of the wind turbine subjected to an earthquake

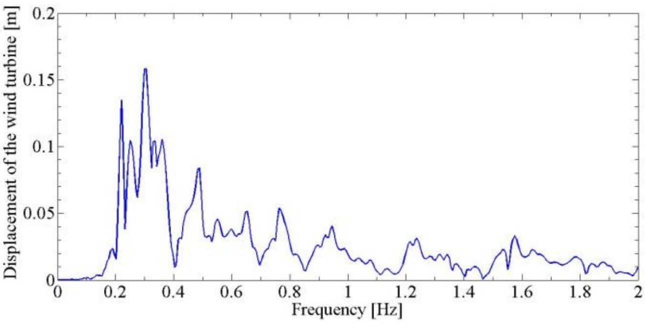


Fig. 4 Frequency response of the wind turbine subjected to an earthquake

Regarding displacement, the lateral response of the system is relatively small. However, the maximum displacement of the system obtained by subjecting it to an earthquake is $\delta_{WE} = 0.1581$ m, contributing in approximately 15.81% of the lateral displacement of the wind turbine. In general terms, δ_{WE} does not present a particularly dangerous displacement of the system. However, in order to preserve the structural integrity throughout the working life of the system, it is necessary to reduce the displacement δ_{WE} through the use of a passive vibration absorber, thus increasing the useful life of the wind turbine.

CONCLUSIONS

The analysis of vibrations in a megastructure such as a wind turbine can be investigated as an inverted pendulum using a simplified mathematical model using its real design parameters to reduce the lateral displacement when it is disturbed by external forces such as an earthquake. This work aimed to carry out such an investigation. The numerical results obtained indicate that the movement of the structure can be reduced indirectly by controlling the mechanical vibrations in the system. Reducing the effects of lateral displacements caused by the wind or earthquake forces (or a combination of both) in the structure can increase its useful life and the safety of the overall system. Future work could focus on the implementation of passive and active vibration controls to reduce the displacement of the structure significantly by using modal controllers such as positive position feedback and positive acceleration feedback.

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