Uncertainty Analysis of a Cloud Base Height Measurement System Based on Digital Photography

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Abstract **–** Cloud height is an important parameter in both air safety and weather monitoring. The authors have previously developed a low-cost cloud height measurement system based on stereovision and digital photography. In this paper, the cloud height uncertainty is derived for the previously developed system. The main sources of uncertainty are identified and improvements to the measurement system are proposed.

I. Introduction

Clouds play a major role in the atmospheric climate system [1] and are an important component of the weather. Cloud height as well as wind direction and wind speed are important information for weather forecast and air safety [2]. The latter is especially important for small aerodromes and airplanes that do not have the necessary equipment to perform landings based on instrumental readings alone.
In a previous paper [3] a methodology for measuring cloud height as well as wind speed and direction at cloud level was described. The cloud height determination is based on stereovision [4] by taking two photographs of the overhead sky from two different positions. The wind speed and wind direction are determined by images taken at different times from the same location.
Although the measurement setup was previously developed by the authors, an uncertainty analysis was missing. The main objective of this paper is therefore to provide this uncertainty analysis.
The paper is divided into five Sections, including the Introduction and the Conclusions. In Section II, a summarized description of the measurement system is presented. The uncertainty calculations and basic assumptions are described in Section III. In Section IV an uncertainty analysis of the measurement system is performed to find the main sources contributing to the total uncertainty. Improvements to the system are proposed based on the uncertainty results.

II. Measurement System

The measurement system used to estimate cloud height using digital photography was described in [3]. It is based on stereo-photography where photographs of the overhead clouds are obtained from two different locations. By identifying a common feature of a cloud on both pictures it is possible, by triangulation, to estimate the height at which that cloud feature is located. The underlying principle is presented in Figure 1, where the two camera positions A and B are separated by a distance *d*.

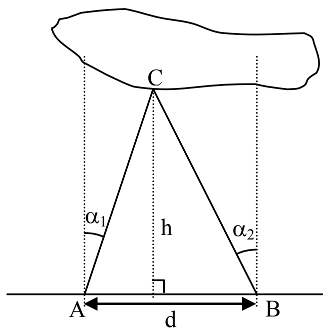

Figure 1. Triangulation method.

After identifying a common feature C in both pictures, angles α_1 and α_2 between the feature and the vertical line at each position, it is possible to estimate the height of that feature through

$$h = \frac{\cos(\alpha_1)\cos(\alpha_2)}{\sin(\alpha_1 + \alpha_2)} d \ . \tag{1}$$

Feature C is identified in both pictures by using image registration techniques [5]. Angles α_j, where $j = 1, 2$, are obtained by counting the number of pixels n_{xj} that the feature is offset from the center of the picture, as shown in Figure 2.

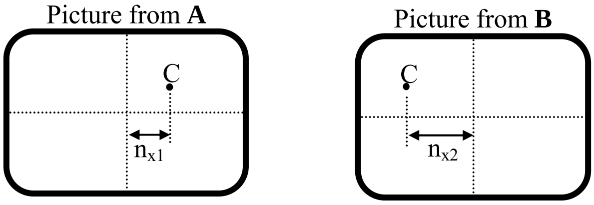

Figure 2. Relative position of feature C in pictures obtained from positions A and B. The horizontal offset of the feature in each photograph is n_{x1} and n_{x2} pixels.

The relation between the offset number of pixels and the angles is given by

$$\alpha_j = \frac{n_{xj}}{N_x}\alpha_h - \alpha_{phys_j} \tag{2}$$

where N_x is the horizontal pixel resolution of the picture and α_h is the horizontal angle of view of the camera. The cameras positions should be levelled with the ground, however in practice there is always a small misalignment relative to the level line. These misalignments are represented by α_{phys_j}, as shown in Figure 3 for camera position A.

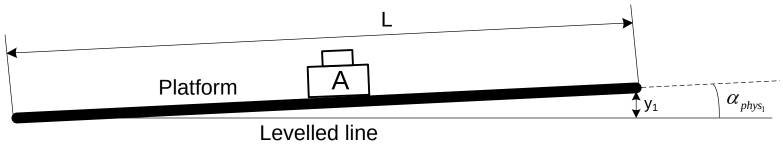

Figure 3. The camera in position A is placed on top of an adjustable platform whose misalignment with the ground level is α_{phys_1}. A similar arrangement is placed at position B.

Due to the availability of only one camera, the photographs from positions A and B cannot be obtained simultaneously. In the time-lapse between the capture of the two photos the clouds move due to the wind and this effect must be taken into account when calculating angles α_j. In fact, the pixel displacement of photograph B shown in Figure 2 needs to be corrected for the displacement due to the wind. Considering a wind speed v_x in the x-direction and in pixel/s, the corrected pixel displacement is

$$n_{x2} = n_{x2}^u - v_x t_{AB} \tag{3}$$

where n_{x2}^u is the pixel displacement measured in picture B, uncorrected for the wind speed and t_{AB} is the time-lapse between pictures A and B. Photograph A is the reference therefore does not need to be corrected for wind induced displacement, resulting in $n_{x1} = n_{x1}^u$.

Wind speed v_x can be estimated by taking an extra picture from position B, and observing the displacement n_{wind_x} of feature C. If the time-lapse between the two pictures is t_{BB} then the wind speed (in pixel/s) is

$$v_x = n_{wind_x} / t_{BB} \ . \tag{4}$$

In the setup, a 6.0 MPixel Olympus SP-500 UZ camera was used. It has 10× optical zoom with a focal distance from 6.3 mm to 63 mm. The photographs were saved in JPEG format with resolution 2816 × 2112 for later image registration in a computer.

III. Uncertainty Calculations

In this section, the uncertainty equations for the cloud height are derived. In most cases, the uncertainty of the physical quantities has an unknown distribution therefore, in conformity with [6], a uniform distribution is considered with the limit values being the maximum possible error.
The cloud height uncertainty $u(h)$ is obtained from (1), resulting in

$$u^2(h) = \left(\frac{\partial h}{\partial \alpha_1}\right)^2 u^2(\alpha_1) + \left(\frac{\partial h}{\partial \alpha_2}\right)^2 u^2(\alpha_2) + \left(\frac{\partial h}{\partial d}\right)^2 u^2(d). \quad (5)$$

The distance d between positions A and B was measured using a 3 m long line, whose maximum error is $\pm 0.5\,\text{mm}$. Since the measurement of d had be split into 10 different steps, the resulting uncertainty is $u(d) = \left(5\times10^{-3}\right)/\sqrt{3}\ \text{m}$. Also, the uncertainties $u(\alpha_j)$ obtained from (2) are

$$u^2(\alpha_j) = \left(\frac{\alpha_h}{N_x}\right)^2 u^2\left(n_{x_j}\right) + \left(\frac{n_{x_j}}{N_x}\right)^2 u^2(\alpha_h) + (-1)^2 u^2\left(\alpha_{phys_j}\right). \quad (6)$$

A. Horizontal Alignment Uncertainty

The horizontal alignment procedure to minimize α_{phys_j} was performed with a laser level on a platform with length L = 41 cm, as shown in Figure 3. The misalignment is given by $\alpha_{phys_j} = \tan^{-1}\left(y_j/L\right)$ where y_j is the vertical distance relative to the level line. Its uncertainty is given by

$$u^2\left(\alpha_{phys_j}\right) = \left(L\left[1+\left(\frac{y_j}{L}\right)^2\right]\right)^{-2} u^2(y_j) + \left(-\frac{y_j}{L^2}\left[1+\left(\frac{y_j}{L}\right)^2\right]^{-1}\right)^2 u^2(L) \quad (7)$$

where a maximum error of $\pm 0.5\,\text{mm}$ was considered on a uniform distribution for y_j and L, resulting in $u(L) = u(y_j) = \left(5\times10^{-4}\right)/\sqrt{3}\ \text{m}$. For a leveled platform $y_j = 0$ which would result in $\alpha_{phys_j} = 0$. Since $y_j = 0$, the second term in (7) disappears and the horizontal alignment uncertainty $u\left(\alpha_{phys_j}\right)$ is solely due to $u(y_j)$. The horizontal alignment uncertainty is therefore $u\left(\alpha_{phys_j}\right) = 0.04^\circ$.

B. Horizontal Angle of View Uncertainty

The camera calibration procedure performed in [3], consisted on taking a photograph of a target of know size placed at a known distance. The camera's sensor width x_S was estimated according to the geometry shown in Figure 4, where $L_T = 27.1\,\text{cm}$, $D = 2.51\,\text{cm}$ and $F = 63.0\,\text{mm}$ (corresponding to maximum zoom), resulting in $x_S = (F/D)L_T = 6.802\,\text{mm}$. With the sensor size it is possible to determine the angle of view for any focal distance F through

$$\alpha_h = 2\tan^{-1}\left(\frac{x_s}{2F}\right). \quad (8)$$

The angle of view uncertainty is thus given by

$$u^2(\alpha_h) = \left\{ F \left[1 + \left(\frac{x_s}{2F} \right)^2 \right] \right\}^{-1} u^2(x_s) + \left[-\frac{x_s}{F^2} \left[1 + \left(\frac{x_s}{2F} \right)^2 \right]^{-1} \right] u^2(F) \tag{9}$$

where the focal distance uncertainty was estimated to be $u(F) = (5 \times 10^{-5}) / \sqrt{3}$ m, according to the accuracy provided by the manufacturer. The uncertainty of the sensor size is, according to the geometry of Figure 4,

$$u^2(x_s) = \left(\frac{F}{D} \right)^2 u^2(L_T) + \left(\frac{L_T}{D} \right)^2 u^2(F) + \left(-\frac{FL_T}{D^2} \right)^2 u^2(D). \tag{10}$$

Considering $u(L_T) = u(D) = (5 \times 10^{-4}) / \sqrt{3}$ m, the uncertainty of the sensor size is $u(x_s) = 7.9\ \mu\text{m}$.

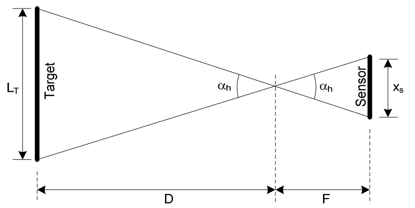

Figure 4. Geometry for the determination of the camera's horizontal angle of view α_h.

C. Pixel Displacement Uncertainty

The final uncertainties that need to be calculated are the uncertainties of n_{xj}. It was considered that the maximum error in the offset is ± 0.5 pixel and a uniform distribution was also assumed. The pixel offset obtained from picture A does not need to be corrected for the wind effect resulting in $u(n_{x1}) = u(n) = 0.5/\sqrt{3}$ pixel. For picture B the uncertainty is, according to (3), given by

$$u^2(n_{x2}) = u^2(n_{x2}^u) + (-t_{AB})^2 u^2(v_x) + (-v_x)^2 u^2(t_{AB}) \tag{11}$$

where $u(n_{x2}^u) = u(n) = 0.5/\sqrt{3}$ pixel. Since the pictures timestamps have only 1 s resolution, the time uncertainty is taken as $u(t_{AB}) = u(t) = 0.5/\sqrt{3}$ s. The only remaining uncertainty to be calculated is the windspeed uncertainty $u(v_x)$, which according to (4) is given by

$$u^2(v_x) = \left(-\frac{n_{wind_x}}{t_{BB}^2} \right)^2 u^2(t_{BB}) + (t_{BB})^{-2} u^2(n_{wind_x}). \tag{12}$$

The time and pixel uncertainties in (12) are considered to be the same as above: $u(t_{BB}) = u(t) = 0.5/\sqrt{3}$ s and $u(n_{wind_x}) = u(n) = 0.5/\sqrt{3}$ pixel.

IV. Results

A clear identification of the main sources of uncertainty is a valuable tool in the improvement of the measurement setup. Therefore, in this section, an uncertainty analysis is presented for the measurements performed in [3], where the distance between the two camera positions was d = 29.0 m. Maximum zoom was used (focal length F = 6.3 mm) corresponding to an angle of view $\alpha_h = 56.7^\circ$ and the time-lapse between pictures was $t_{AB} = 16$ s and $t_{BB} = 6$ s. The cloud height was found to be h = 850 m with a wind speed in the x-direction of $v_x = 3.5$ m/s.

For these parameters the uncertainty given by (5) is $u(h)=88\text{ m}$, with the main contribution resulting from $u(\alpha_2)=0.20^\circ$ followed by $u(\alpha_1)=0.04^\circ$. The effect of $u(d)$ is almost negligible in $u(h)$. The main factor affecting $u(\alpha_1)$ is $u(\alpha_{phys_1})$, while $u(\alpha_2)$ is affected by both $u(\alpha_{phys_2})$ and $u(n_{x2})$. The pixel displacement uncertainty was estimated to be $u(n_{x2})=9.7\text{ pixel}$ which is, mainly, a result of the wind speed uncertainty $u(v_x)=0.6\text{ pixel/s}$.
It is now clear that the cloud height uncertainty can be reduced by improving the horizontal alignment α_{phys_j} and reducing the uncertainty of the estimated wind speed v_x. This can be accomplished by increasing time t_{BB} and reducing the time uncertainty $u(t)$ by computer controlling the camera trigger. From (11) it is possible to see that $u(v_x)$ can also be reduced by decreasing t_{AB}.
A different analysis is presented in Figure 5 where it can be seen that the cloud height relative uncertainty $u(h)/h$ increases as the cloud height increases. The effect of the distance *d* between the two camera positions is also analyzed. Figure 5 shows that the uncertainty decreases as distance *d* increases.

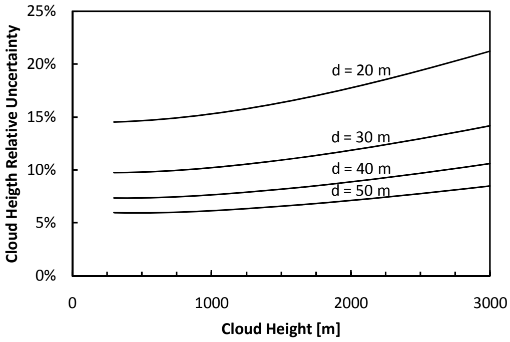

Figure 5. Cloud height relative uncertainty as a function of cloud height.

IV. Conclusions

This paper presents the uncertainty calculations for a cloud base height measurement system based on time lapsed digital photography. The measurement setup was briefly described and its equations and parameters presented. The uncertainty expressions were derived from the system equations and uniform distributions were considered for the uncertainty of basic physical quantities such as time and length.
The uncertainty equations were then used to analyze the effect of each uncertainty component. This analysis revealed that the system can be improved by providing a better horizontal alignment, as well as reducing the time uncertainty.
It was also shown that the cloud height relative uncertainty increases with cloud height, but decreases for longer distances between the cameras positions.

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