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ESTIMATION OF UNCERTAINTY WITH CORRELATION VALUES IN INTERNATIONAL METROLOGICAL AND ENVIRONMENTAL GUIDES

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Abstract - Peculiarities of the expression of uncertainty estimation with correlation or covariance of input values, using parameters of correlation and covariance in international metrological and environmental guides are shown. The use of international metrological guidance of expression of measurements uncertainty with correlation or covariance of input values in development of new and reconsideration of old international environmental guides is recommended.

I. Introduction

To receive accurate and reliable information about the state of environment the credible evaluation of emission of foul gases is necessary. These gases bring to global ecological problems, in particular to global climate changes. In this case, the important element of analysis obtained in results is uncertainty analysis which must include analysis of main sources of uncertainties, total uncertainty of result with allowance for all influencing values.

Simple cases of uncertainty analysis without correlation or covariance were considered in [1, 2].

Necessity in the analysis of covariance and autocorrelation for the uncertainty estimates of emission foul and greenhouse gases exists. It is importantly to investigate the correlations of the estimated values relevant to emissions as in the context one estimation and the various estimations of emission foul and greenhouse gases.

II. Estimation of uncertainty of input values with correlation and covariance

If correlation exists between input values then this correlation is essential and can not be ignored. Covariance of input values may be estimated experimentally, on condition of possible the change of input correlated values (the estimation of type A covariance), or with the using of necessary data on the correlation changeability of values which relate to this measurement (the evaluation of type B covariance).

In practical cases input values often appear to be correlated, as far as the evaluation of their values is used by the same standards, measuring instruments, standard data and even method of measurement which possess peculiar uncertainty. If direct measurement results are not correlated then calculated value of covariance is expected to be close to zero.

Covariance with the estimations of two input values may be equal to zero or selected as negligible, if: these values are not correlated (values measurement are run repeatedly in various independent experiments, or they present the various estimations of values which made independently); any values may be is accepted as an constant; there is negligible data for calculation of covariance related with the estimations of these values.

In Table 1 the comparison of the estimations of uncertainties of input values with correlation and covariance in metrological and environmental guides are driven.

III. Estimation of total uncertainty with correlation and covariance

Uncertainties aggregation arises of two various processes: aggregation of emissions one gas which complies with the law of propagation of uncertainty; aggregation of emissions bound with several gases. Into second case emission must be result in common scale, and been used for this process consists in the application of Global Warming Potentials (GWP).

TABLE 1. Comparison of estimation of uncertainty in metrological and environmental guides' terms

Metrological guides	Environmental guides
Correlation: two or more input quantities are related [3].	*Correlation*: mutual dependence between two quantities [4, 5].
Covariance: means of the mutual dependence of two random variables [3].	*Covariance*: a measure of the mutual dependence between two variables [4, 5].
Correlation coefficient: the degree of correlation between two input quantities [3].	*Correlation coefficient*: a number laying between −1 and +1 which measures the mutual dependence between two variables which are observed together [4, 5].
Entrance value is non correlated	
Using estimation of uncertainty in accordance with the law of propagation of uncertainty and use equation [3]: $$u_c(y) = \sqrt{\sum_{i=1}^{m}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)} = \sqrt{\sum_{i=1}^{m} u_i^2(y)} \qquad (1)$$ or $u_c(y) = \sqrt{c_1^2 u^2(x_1) + c_2^2 u^2(x_2) + ... + c_m^2 u^2(x_m)}$ (2) where: $u(x_i)$, $u_i(y)$ – standard uncertainty input ($i = 1, m$) and output value; y, x_i – estimations of measurable value Y and input value X_i.	Using estimation of uncertainty Rule A and use equation [4, 5]: $$U_{total} = \frac{\sqrt{(U_1 \cdot x_1)^2 + (U_2 \cdot x_2)^2 + ... + (U_n \cdot x_n)^2}}{x_1 + x_2 + ... + x_n}, \qquad (7)$$ where: U_{total} : uncertainty in the sum of the quantities, %; x_i and U_i : uncertain quantities and uncertainties associated with them, respectively, %.
Entrance value is correlated	
Using estimation of uncertainty in accordance with the law of propagation of uncertainty and use equation [3]: $$u_c(y) = \sqrt{\sum_{i=1}^{m}\left(\frac{\partial f}{\partial x_i}\right)u^2(x_i) + 2\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} u(x_i, x_j)} \qquad (3)$$ or $u_c(y) = \sqrt{\sum_{i=1}^{m} u_i^2(y) + 2\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} c_i c_k u(x_i, x_j)}$ (4) where: $u(x_i, x_j)$ – estimation covariance with two input estimations x_i and x_j; y – estimation of measurable value Y; x_i and x_j – estimation input values X_i and X_j	Not numerical estimated
The degree of correlation between x_i and x_j is characterized by the estimated correlation coefficient [3]: $$r(x_i, x_j) = r(x_j, x_i) = \frac{u(x_i, x_j)}{u(x_i)u(x_j)}, \qquad (5)$$ where: $r(x_i, x_j) = r(x_j, x_i)$; $-1 \le r(x_i, x_j) \le +1$. For $r(x_i, x_j) = \pm 1$ uncertainties contribution is: $$u_{i,j}(y) = \left|u_i(y) \pm u_j(y)\right| = \left|c_i u(x_i) \pm c_j u(x_j)\right| \quad (6).$$ If the estimates x_i and x_j are independent, $r(x_i, x_j) = 0$, and a change in one does not imply an expected change in the other.	A value of correlation coefficient of +1 means that the variables have a perfect direct straight line relation; a value of −1 means that there is a perfect inverse straight line relation; and a value of 0 means that there is no straight line relation [4]. It is defined as the covariance of the two variables divided by the product of their standard deviations.

In Table 2 the comparison of the emission sources with allowance for correlation and covariance of input values in metrological and environmental guides are driven.

TABLE 2. Comparison of uncertainty contribution in metrological and environmental guides' terms

Metrological guides	Environmental guides
Correlation and covariance for entrance value	
For correlation and covariance with type *A* use equation: $$u(\bar{x}_i,\bar{x}_k)=\frac{1}{n(n-1)}\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)(x_{kj}-\bar{x}_k) \qquad (8).$$	The sample covariance of paired sample of random variables X and Y is calculated using the following formula [4, 5]: $$s_{xy}^2=\frac{1}{n}\sum_{i}^{n}(x_i-\bar{x})(y_i-\bar{y}), \qquad (10)$$ where x_i, y_i, $i = 1,...,n$ are items in the sample $\bar{x}$ and $\bar{y}$ are sample means.
For correlation and covariance with type *B* use equation: $$u(x_i,x_k)=\sum_{l=1}^{L}c_{il}\cdot c_{kl}\cdot u^2(Q_l), \qquad (9)$$ where: c_i, c_k – sensitivity coefficients; $u(Q_l)$ – standard uncertainty of variables Q_l, $l=1,2,...,L$.	
Uncertainties contribution every input values and sensitivity coefficient	
Uncertainties contribution $u_i(y)$ every input values X_i to uncertainty $u(y)$ are use equation [3]: $$u_i(y)=\left|c_i\right|u(x_i); \qquad (11)$$ $$c_i=\frac{\partial y}{\partial x_i}=\frac{\partial Y}{\partial X_i}\bigg|_{x_1,x_2,...,x_m}$$ where: c_i – sensitivity coefficients; y – estimation of measurable value Y; $x_1,...,x_m$ – input values ($i=1,m$). For direct measurement all sensitivity coefficients are equal 1.	Sensitivity coefficient λ calculated using the following equation [4]: $$\lambda=\partial E_T/\partial a, \qquad (12)$$ where: E_T – the aggregated emissions; a – input quantity (or parameter). Dispersion of tendency for emission two different time $E(t)$ and $E(t+\Delta t)$ with Δt is use equation: $$\sigma^2(\Delta E)=2\sigma^2_E(1-r(\Delta t)) \qquad (13).$$ where: $r(\Delta t)$ – correlation coefficient

Some variables which are necessary aggregation, do not are Gauss, large dispersion and correlated with other variables are have. In this case the application of Monte-Carlo method for uncertainties aggregation is presented the most preferable.

Alternative variant may be use as a working approach and consists in the estimation of total uncertainty with application of central extreme theorem. Initial conditions for application of central extreme theorem consist in following: the amount of emission means must be large; every single mean does not dominate over total amount; emissions are independent.

This approach is a coarse approach of emission estimation.

On Fig. 1 Estimation uncertainty algorithm according to GUM is present. Uncertainty of the measurement values equation is used:

$$Y=f(X_1,X_2,...,X_m), \qquad (14)$$

where $X_1, ..., X_m$ – entrance value (direct measured value or other value which have an influence on measurement results); m – quantity values; f – functional dependence type [3].

Covariation with type A of entrance value estimation which received from repeated simultaneously observations x_{ij}, x_{kj}, $j = 1, 2,..., n$ use equation (8). Covariation with type B of entrance value estimation use equation (9).

Standard uncertainty estimation calculated using the following equation (GUM-1993 [3]):

$$u_c=\sqrt{\sum_{i=1}^{m}u_i^2}, \qquad (15)$$

where: u_c and u_i – overall uncertainty and its components correspondingly.

On Fig. 2 Estimation uncertainty algorithm according to IPCC-2000 and IPCC-2006 is present.

Overall uncertainty estimation using empirical data (activity data – AD) and expert judgement with used the 95 % confidence interval.
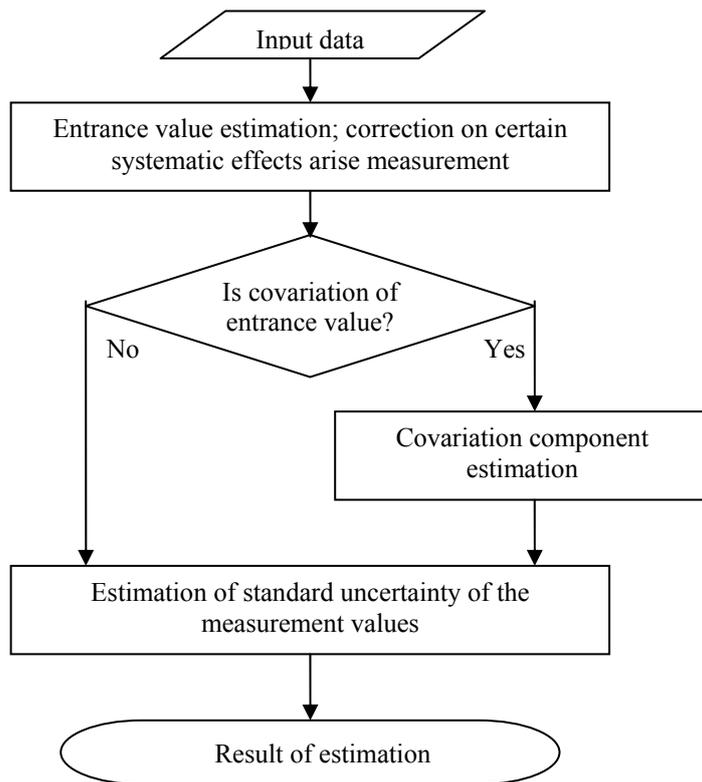
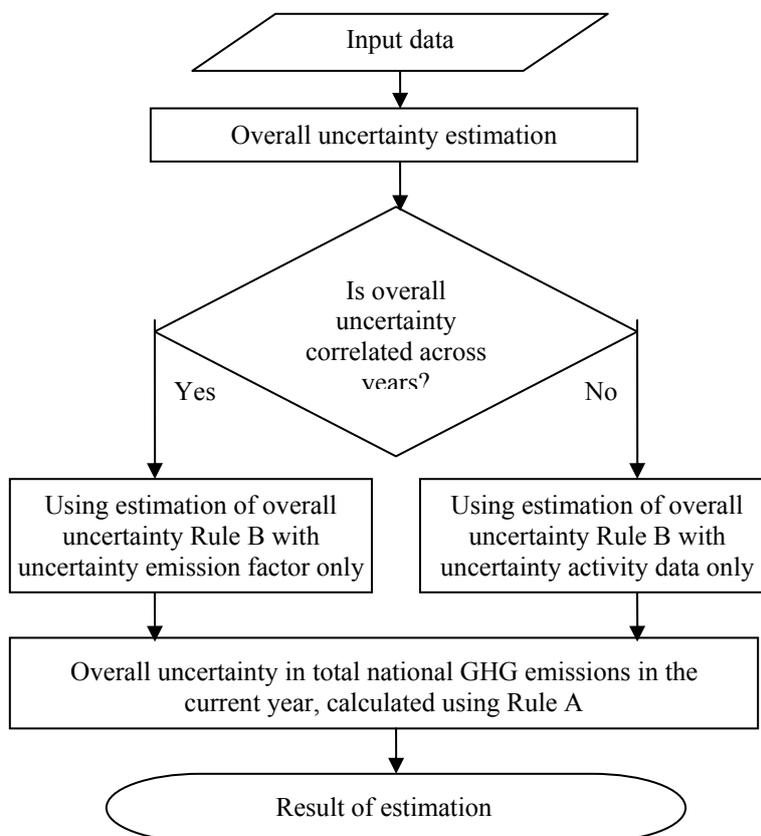Figure 1. Estimation uncertainty algorithm according to GUM

Figure 2. Estimation uncertainty algorithm according to IPCC-2000 and IPCC-2006

If overall uncertainty is correlated across years using estimation of overall uncertainty Rule B with uncertainty emission factor (EF) only (assume AD to be equal 0 %). If overall uncertainty is correlated across years using estimation of overall uncertainty Rule B with uncertainty emission factor (EF) only (assume AD to be equal 0 %).

If overall uncertainty is no correlated across years using estimation of overall uncertainty Rule B with uncertainty AD only (assume EF to be equal 0 %).

Overall uncertainty in total national GHG emissions in the current year, calculated using Rule A (corresponding Type A in accordance with GUM) and use equation

$$U_T = E_i \cdot U_{\sum_B} / \sum E_T , \quad (16)$$

where: $U_{\sum_B}$ – estimation of overall uncertainty Rule B; E_i – GHG emissions from certain source category in CO_2-equivalent, Gg; E_T – total GHG emissions from all source categories in CO_2-equivalent, Gg.

IV. Conclusions

Metrological and environmental guide comparison has shown peculiarities of used terms and uncertainty analysis with correlation and covariance of input values. Environmental guides use more simplified but specific approaches of the uncertainty analysis.

Specific approaches to estimation of the uncertainty in trend of GHG emissions imply realization of long-term observation and also environmental guide's consideration correlation of data across years.

The use of international metrological guidance of expression of measurements uncertainty with correlation or covariance of input values in development of new and

reconsideration of old international environmental guides is recommended.

References

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